Three-Dimensional Geometric Nonlinear Analysis of Structures Consisting of Slack Cables and Bar Elements Subjected to Gravitational and Lateral Loading

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SUMMARY:
Slack cable structures exhibit large displacements and rotations in response to the applied loads before reaching an equilibrium state. Consequently, following any alteration of the applied loads, the structure is essentially unstable and ordinary numerical analysis methods break down due to singularities. In this paper, an effective iterative method is developed for three-dimensional large-displacement analysis of structures consisting of slack cables as the primary load bearing system. This approach takes advantage of substructuring technique that helps reduce the analysis costs associated with slack cables and uses an effective procedure for detecting and limiting the structural instabilities. The proposed analysis method is applied to a number of example structures using a computer program that also serves to generate the initial form of the structure. It is shown that this procedure is an effective approach in the analysis of such structures with high degrees of instability and large displacements.

Keywords: Slack cable, Large displacement, Instability, Numerical analysis, Collapse analysis

1. INTRODUCTION

Tensile cable structures have received closed attention for several decades as a means of construction with minimum use of material. The recent developments of new and improved construction materials further support the use of such structures to cover relatively large spans. On the other hand, the analysis and construction of cable-stayed structures is more challenging than most ordinary structures. Numerical analysis is more problematic particularly when the cables are not pretensioned. In response to the applied loads, large displacements and rotations occur in these structures to reach an equilibrium state that balance the applied loads. That is, following any alteration of the applied loads, the structure is essentially unstable and ordinary numerical analysis methods (e.g. stiffness approach – commonly with tension-only link elements to model the cables) usually break down due to numerical singularities. As a result, the structural form is highly dependent on the applied loads, and unlike other common types of structures, the structural geometry is not known before the analysis. These structures often have irregular shapes and low self-weights which makes them more sensitive to the applied loads. Hence, the first step in the analysis of such structures is finding the initial equilibrium configuration, which in turn affects the load-bearing capacity of the structure. Only after determination of the initial configuration, stiffness matrices of the structural members can be used to describe the structural stiffness to resist the applied loads.

A thorough study of the historical aspects of cable-stayed structures and a literature review of the topic can be found in (Tibert 1999). Cable-suspended roofs (Krishna 1978) considered herein can be divided into the categories of (i) simply suspended cables, (ii) pretensioned cable trusses, and (iii) pretensioned cable nets (Buchholdt 1985). This study focuses on the analysis of simply suspended cables, where pretension is negligible and the cables show significant amounts of sag. Furthermore, the cladding is assumed not to significantly increase the stiffness of the system. These roof systems have a single or double curvature and show little or no stiffness. Most of the literature on this topic are mostly limited to two-dimensional cable loading and deformations (Irvine 1992; Wang et al. 2011). On the other hand, the works on three-dimensional analysis of slack cable structures involve complex formulations or simultaneous solutions of several nonlinear equations (Gosling and Korban 2001; Impollonia et al. 2010), and thus may be restricted to the analysis of relatively small structures.

In this study, an effective and simple iterative method is developed for three-dimensional large-displacement analysis of slack cable-bar structures. The effectiveness of this approach stems from (i) the reduction of the memory and processing costs by treatment of each slack cable element as a substructure, whose point coordinates and end reactions are updated in each iteration, and (ii) the detection of the structural instabilities (that may arise before reaching the equilibrium state) in the stiffness matrix by recognizing degrees of freedom (DF's) with near zero stiffness, and allowing only limited displacement increments in these DF's. The slack cable substructures are treated as centenary members subjected to concentrated loads along their length, and are allowed to deform in three dimensions. The form of each slack cable and its internal forces are found in another iterative procedure within the overall iterations by satisfying the compatibility and equilibrium requirements for that cable. After sufficient number of iterations, this procedure naturally leads to a reconfiguration of the structural elements in a way that the member internal forces will be in equilibrium with the applied loads.

The proposed analysis method is applied to a number of example structures using a computer program written in MATLAB (The MathWorks® Inc 1994-2011) environment. The example structures consist of pin-ended bar elements and both taut and slack cables. A computer procedure is also developed for the initial form-finding of these structures when only subjected to dead gravity loads. Finally, this method is successfully utilized to analyze a large structure of this type – a design alternative for birds' park of Tehran, Iran – with a plan area of more than 50000 square meters. It is shown that this procedure is an effective approach for the analysis of such structures with high degrees of instability and large displacements. Due to its modest memory and processing costs, this approach also paves the way for dynamic analysis of slack cable-bar structures subjected to time varying lateral loads such as earthquake and wind.

2. ANALYSIS OF SLACK CABLES

A cable analysis method is introduced that helps determine the cable shape, internal forces and deformations, and end reactions. The three-dimensional cable form highly depends on the applied loads and is unknown in the beginning of the analysis. To start the iterative procedure to determine this form, an initial two-dimensional parabolic shape is assumed for the cable. Cables take this shape when are subjected to uniform gravitational loads along the horizontal (Beer et al. 2003). This shape is given by:

$$z' = \frac{wx'^2}{2T_0} = cx'^2 \tag{2.1}$$

where x' and z' are horizontal and vertical cable coordinates measured from the cable lowest point as shown in Figure 2.1, T_0 is the cable tension at this point, w is the uniformly distributed load, and c is the coefficient of the parabola. The cable length measured from the lowest point to an arbitrary point p on the cable can then be obtained from:

$$L_p = \int_0^{x_p} \sqrt{1+\left(\frac{dx'}{dz'}\right)^2}\, dx' \tag{2.2}$$

If p is selected to be the cable support at one end, the above equation yields the cable length at one side of its lowest point, and can be used to determine the total cable length. Special considerations need to be taken into account for the cases when the lowest point of the parabola governing the cable shape falls outside of the cable span. Knowing the cable length (or the desired cable sag) and the coordinates of its end supports, the initial parabolic shape of the cable can be determined using Eqns. (2.1) and (2.2) independent of the amount of uniform load or the cable tension. This constitutes the initial coordinates of the points along the cable and helps determine the initial value of its end reactions. As shown in Figure 2.1a, the initial point coordinates in the local transverse y-direction are all zero, that conform with zero lateral reactions at ends. It is seen that the cable local axes are defined such that the *x*- and *z*-axes lie in a vertical plane including support points, and are horizontal and vertical, respectively.

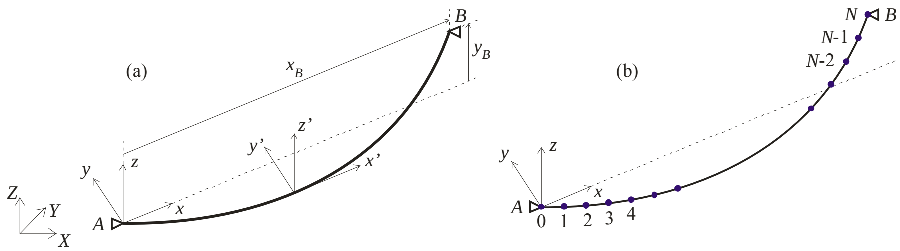

Figure 2.1. (a) Global and local coordinates, (b) Cable substructure in its initial form

2.1 Substructuring

Next, the cable is divided into short straight segments with concentrated loads at joints. These segments can be uniformly spaced, or they can be selected based on the locations of the concentrated loads applied to the cable, if any. Distributed loads can also be replaced with equivalent concentrated loads at these points. In order to avoid the numerical instabilities resulting from small flexural stiffness of the cable and reduce the computational costs, each cable is treated as a substructure that its internal points do not add to the overall DF's of the system. The initial form of the cable substructure is shown in Figure 2.1b.

After determination of the initial point coordinates of the cable in its local coordinates and the applied loads at these points, the cable shape is updated iteratively until the associated compatibility and equilibrium equations are satisfied. Details of this procedure are presented in the next section. The cable internal forces and end reactions can then be determined, which in turn are applied to the supporting structure in the overall analysis procedure.

2.2 Cable Form and Internal Forces

Unless the cable is only subjected to a gravitational uniformly distributed load along the horizontal x-axis, the initial cable form does not satisfy the equilibrium requirements. An iterative procedure is proposed herein that attempts to update the cable form, internal forces and end reactions to satisfy the equilibrium and compatibility requirements. This iterative procedure is described below:

1- Starting from end 0 (support A) shown in Figure 2.1, the initial value of the sum of the forces applied to the cable $\sum\mathbf{F}$ consists only of the load applied from the support at point 0, namely (R_{Ax}, R_{Ay}, R_{Az}). These reactions are calculated in the previous iteration, or with the initial assumption of parabolic shape. These forces must be in equilibrium with the internal force in the first segment of the cable, denoted 01. Hence, knowing the deformed length of segment 01 based on its internal axial force, the new location of point 1 (x_1, y_1, z_1) is determined such that segment 01 lies along the direction of the resultant force $\sum\mathbf{F}=(R_{Ax}, R_{Ay}, R_{Az})$ (Figure 2.2a).

2- Moving to the next segment 12, $\sum\mathbf{F}$ is increased by the force components applied at point 1, and hence must be in equilibrium with the internal force of this segment. Then, knowing the deformed length of segment 12 and the new orientation of the resultant force $\sum\mathbf{F}$, the new location of point 2 is determined. Similarly for the next segment, $\sum\mathbf{F}$ is increased by the force components applied at point 2. This procedure is repeated until the locations of all subsequent points of the cable are updated. Note that using this procedure, the new location of point *N* will not necessarily coincide with the location of the support at far end, and needs to be adjusted as described in Step 4. As an alternative approach, one can leave the position of the last point unaltered at the expense of the calculated length of the cable being different than its actual length, and then use this difference in Step 4 for modification of reactions.

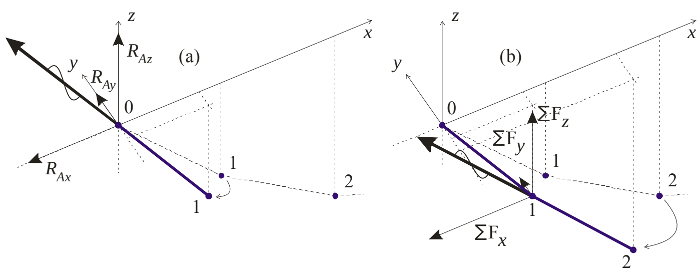

Figure 2.2. Updating the locations of (a) point 1 and (b) point 2 of the cable

3- By using the force equilibrium equations on a section of the cable, or taking the moments about three orthogonal axes passing through the far end, the new amounts of reaction forces at end 0 are recalculated.

4- Using the updated point positions, the deviation of cable end (point *N*) from the far support is determined using:

$$(\Delta x, \Delta y, \Delta z) = (x_N, y_N, z_N) - (x_B, y_B, z_B) \tag{2.3}$$

Then, the reactions (R_{Ax}, R_{Ay}, R_{Az}) applied to the cable at end 0 are modified according to the following rules:

$$(R_{Ax} + c_x, R_{Ay} + c_y, R_{Az} + c_z) \rightarrow (R_{Ax}, R_{Ay}, R_{Az}) \tag{2.4}$$

where the adjustments are given by:

$$c_x = -\gamma \frac{\Delta x}{x_B} R_{Ax}; \quad c_y = \gamma \frac{\Delta y}{x_B} |R_{Ax}|; \quad c_z = \gamma \frac{\Delta z}{x_B} |R_{Ax}| \tag{2.5}$$

In the above equation, γ is a learning gain, whose value can conservatively selected to be 0.1 to avoid divergence of reactions. Furthermore, it is recommended that the magnitudes of the adjustments should not exceed $0.05|R_{Ax}|$. In most cases, one may choose more relaxed limits for the above corrections to speed up the convergence, but this may occasionally destabilize the learning procedure, particularly when the load pattern or support locations are significantly different than what was used to determine the last converged cable shape. The logic behind the development of Eqns. (2.5) is explained later in this section.

5- The iterations are stopped and the reaction at far end is calculated using force equilibrium if the convergence is achieved; otherwise, the next iteration begins with recalculation of point locations as explained in Step 1. The norms that are examined for convergence include the error in the prediction of point *N*, moment equilibrium error, and normalized iterative changes in point coordinates.

To understand how the above procedure updates the cable shape and internal forces towards achieving equilibrium and compatibility, the update rules for point positions and reactions should be studied. In the first two steps, the cable point coordinates are updated such that the internal force in each cable segment is in equilibrium with the applied forces. In the next step, the reactions at cable end 0 are recalculated using the force equilibrium, or the moment equilibrium about cable end point N.

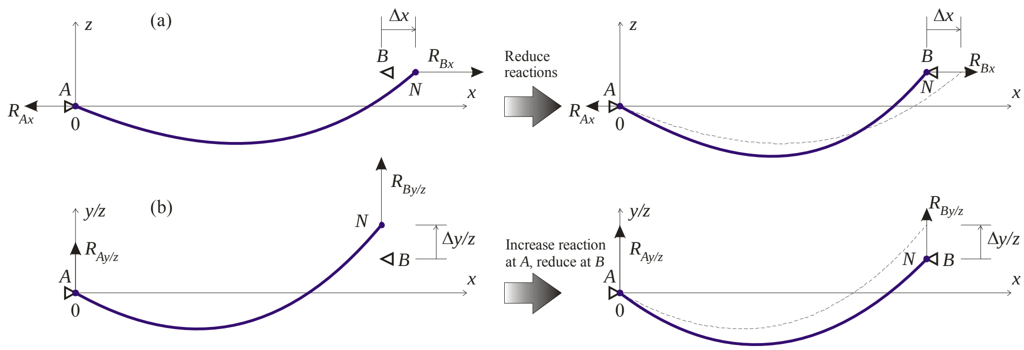

Figure 2.3. Correction of reaction forces to correct the position of point N (a) along x-direction, (b) along y- or z-directions

In the fourth step, the reactions at end 0 are modified to move the end point N closer to the far end support B. For example, Δx in the first equation of Eqns. (2.5) is indicative of error in the prediction of point N along the local x-axis. A positive value of Δx shows that the cable is stretched too much, and it should simply be released to some extent to increase its slack, as shown in Figure 2.3a. For this reason, the first one of Eqns. (2.5) tends to reduce the x-axis reaction in this case, and increase it otherwise. The equations governing the corrections along y- and z-axes also modify the reactions based on errors Δy and Δz, respectively. A positive error shows that the far end of the cable needs to move in the negative direction; that is, the cable should rotate clockwise as demonstrated in Figure 2.3b. To apply a clockwise moment, the reaction at A should algebraically increase, and the reaction at B should in turn decrease. This can be achieved by making a positive correction in the reaction at A using Eqns. (2.5). The performance of the proposed procedure is shown through the analysis of a cable in Section 4 of this paper.

2.3 Special Considerations

The internal forces and reactions have a central role in the success of the analysis procedure described above. For this reason, it is recommended that end 0 of the cable is selected as the end with larger reaction forces. This can done by comparing, say, the x-component of reaction at ends, which is widely used in corrections given by Eqns. (2.5).

Rarely, a special case arises when a cable segment has a near-zero amount of internal force. In this case, the coordinates of the next point cannot be accurately determined using the procedure described in steps 1 and 2. Consequently, the cable can in fact be separated into two or more self-equilibrating statically-determinate sections, separated with zero-force segments. The cable sections immediately connected to the supports can be used to determine the support reactions by summing up the forces applied to that section. The shape of each section can then be determined using the above-mentioned procedure starting from the supported ends. Using a similar procedure, the internal forces and shapes of cable sections between zero-force segments within a cable can also be determined. However, it should be noted that zero-force segments are not fully straight, and their lengths cannot exactly be determined using the coordinates of their end points. One can only check the distance between the segment ends, which should not exceed the length of the segment for compatibility. This is usually the case for these segments, as otherwise, their internal force would not be zero. Consequently, if a cable has more than one zero-force segment (extremely rare), the exact position of the intermediate sections relative to cable ends would not be clear, and the calculated overall cable length will be equal to or smaller than its actual length. On the other hand, this will not cause any issues in the overall structural analysis since only the end reactions are required for this purpose.

3. OVERALL ANALYSIS PROCEDURE

A large-displacement analysis procedure is presented herein that can handle the numerical instabilities resulting from small flexural stiffness of cables and unknown structural geometry. In response to the applied loads, this procedure allows for gradual changes in the structural form (and hence, the locations of cable end supports) and limits the amounts of displacements in DF's with near zero stiffness, until the external loads are balanced.

3.1 Analysis Start Point

The initial form of the structure is usually obtained when the structure is subjected to its self weight. In order to start the analysis, the locations of end supports for primary cables (cables that are directly connected to supports or column tops) are temporarily fixed and a parabolic shape is assumed for all slack cables. Knowing the initial locations of cable supports, their shapes and internal forces are determined according to the procedure described in the preceding section. Then, if the primary cables support any other cables (called herein the secondary cables), their shape are used to update the support locations of those cables. Next, the secondary cables are analyzed with updated support locations, which in turn, alter their end reactions applied to the supporting primary cables. The forces applied to the primary cables are then updated accordingly, and the above procedure is repeated until the displacement increments become sufficiently small. At this point, the equilibrium is achieved in the slack cable system, but the cable supports have not yet moved to their actual positions. It may be possible to leave the achievement of this equilibrium state to the next step, where the structural geometry is allowed to change; however, several simulations have shown that this may result in a considerable increase in the number of iterations necessary to achieve equilibrium.

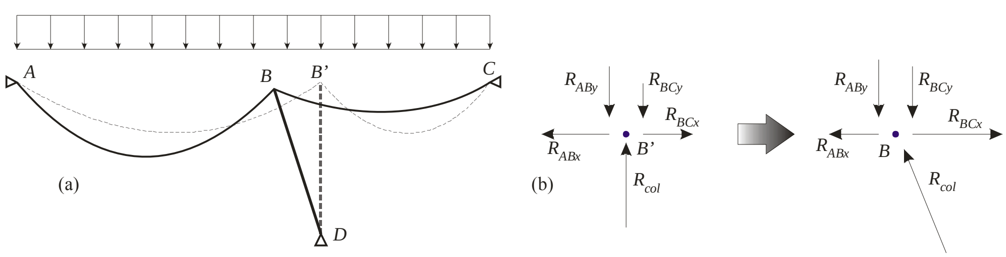

Figure 3.1. (a) Analysis of a two-dimensional slack cable system in the cases of cable supports fixed and released, (b) Equilibrium at point B

3.2 Determination of the Equilibrium State

After achieving equilibrium in the cable system, the column tops are released to allow them to move to their equilibrium locations. In these structures, the columns are usually pin-ended, as otherwise the horizontal cable reactions will lead to extremely large moments at their bases. A two-dimensional example of such movement is shown schematically in Figure 3.1. It is illustrated that since the cable to the left of the column is longer (and hence has a larger tension) the initially-vertical column tilts to the left, so the cable sags in the left and right cables are increased and reduced, respectively. This reduces the horizontal component of cable reaction in cable *AB* and increases that *BC*. This continues until the sum of cable end reactions and column compressive force cancel out as shown in Figure 3.1b.

As mentioned earlier, pin ended columns supported by slack cables do not have any lateral stiffness, and the external forces are balanced only by a reconfiguration of the structure. For this reason, a stiffness matrix consisting of DF's at column tops is essentially singular. To get around this issue, the following procedure is proposed.

1- The displacement vector $\mathbf{u}_c$ consisting of column top displacements is initialized to zero, or the values obtained from the last converged analysis, if any.
2- The unbalanced load vector $\mathbf{f}_u$ is calculated as the sum of forces $\mathbf{f}_c$ applied to the column tops

(from primary cables or directly-applied concentrated loads) and the column internal forces applied to their top nodes.

3- A stiffness matrix $\mathbf{K}$ is formed for the DF's of column top nodes based on the current orientation of the columns and the current state of tensioned or straight cables. Here, the slack cables are assumed to provide no stiffness for the system.

4- The eigenvalues λ_i and eigenvectors $\boldsymbol{\varphi}_i$ of the stiffness matrix are obtained to determine its singularities. In this step, care must be taken about the effect of truncation errors on the symmetry of the stiffness matrix, to avoid imaginary values in the results.

5- The stiffness matrix and unbalanced force vectors are transformed to modal coordinates, where the stiffness matrix is diagonal.

$$\mathbf{K}_m = \boldsymbol{\Phi}^T \mathbf{K} \boldsymbol{\Phi} = \mathrm{diag}(\lambda_i) \tag{3.1}$$

$$\mathbf{f}_{um} = \boldsymbol{\Phi}^T \mathbf{f}_u \tag{3.2}$$

in which $\boldsymbol{\Phi}$ is the modal matrix consisting of eigenvectors of $\mathbf{K}$.

6- In the modal coordinates, the incremental modal displacements $\delta\mathbf{u}_{um}$ are determined by dividing each modal unbalanced force by the corresponding nonzero modal stiffness. If the modal stiffness is obtained to be zero (showing an unstable DF), the modal displacement in that DF is allowed to increase in the direction of its modal unbalanced force by a small amount, say υ. It is recommended to take υ as the average of incremental modal deformations in stable DF's, to avoid abrupt changes in structural geometry.

7- The calculated incremental modal displacements are transformed to the global coordinates system using:

$$\delta\mathbf{u}_u = \boldsymbol{\Phi}\, \delta\mathbf{u}_{um} \tag{3.3}$$

8- The system geometry is updated based on the current values of column top displacements $\mathbf{u}_c = \sum \delta\mathbf{u}_u$, and the internal forces of columns and fully tensioned cables are recalculated.

9- Based on the new locations of cable supports, the cable forms are updated using a procedure similar to what described in Section 3.1, and the cable support reactions are recalculated.

10- The above procedure is repeated from step 2, unless convergence is achieved. The convergence norms include (i) the norm of the displacement increments with respect to overall displacements, (ii) the norm of unbalanced forces with respect to the overall forces applied to column tops, and (iii) the norm of the variation of unbalanced forces with respect to the overall forces applied to the column tops. The convergence is achieved when these norms become smaller than the predetermined tolerances.

It is seen that the above procedure tends to update the cable support points and the reactions applied to the supports simultaneously until the applied loads are balanced with the internal forces. To further stabilize this analysis procedure, it is possible to transfer the cable reactions to columns gradually through several increments. In this case, the complete transfer of forces must also be checked in each iteration as a convergence criterion. If after a considerable number of iterations, no further reduction in the unbalanced forces is observed, this may be a sign of an unstable structure, where an equilibrium configuration cannot be achieved.

3.3 Updating the Equilibrium State in Response to External Loads

Any modification of external loads may result in significant displacements and change of the structural geometry. Due to nonlinear behavior of the structures considered herein, the order of the application of the loads may alter the final equilibrium state, and hence, must be selected realistically. Similar to other nonlinear analysis procedures, the loads must be gradually modified from the previous equilibrium state towards their target values using suitable increments considering convergence and analysis speed. The equilibrium state should be updated in each load increment using the procedure described in the preceding section, until the incremental loads sum up to their desired values.

4. ANALYSIS EXAMPLES

Several numerical analyses have been carried out to verify the accuracy and efficiency of the proposed analysis procedure. A few illustrative examples are presented below.

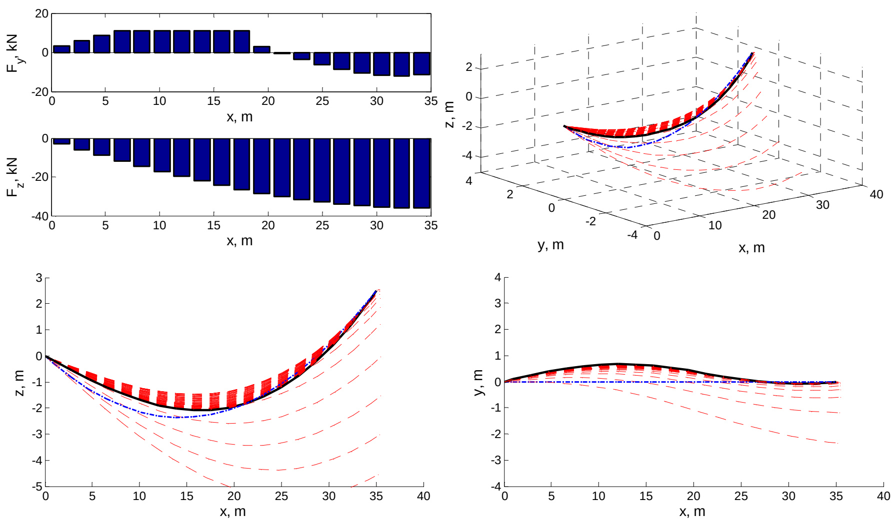

Figure 4.1. Cable loads along *x*-axis and progressive determination of cable form

4.1 Analysis of a Single Cable in Three Dimensions

In order to illustrate the behavior of the proposed procedure for the analysis of cables, the progressive determination of an example is illustrated in Figure 4.1. The span length of the 36-meter-long cable is taken to be 35 meters, support *B* being 2.5 meters higher than support *A*. This cable is treated as a substructure with 20 segments. The load system applied to the cable at its internal nodes is also shown in Figure 4.1. As illustrated, the cable form is initially taken to be parabolic (shown by the dash-dot curve). In the first iteration, a relatively large change in the cable form is seen, resulting from the large differences of reactions at *A* from what are needed for equilibrium and compatibility. Then, the proposed procedure attempts to gradually modify the form (dashed curves) towards its final configuration (solid curve) using the learning procedure described in Section 2.2. With a convergence tolerance of 0.001 and a learning gain of 0.5, the number of required iterations is 65. The number of iterations required to determine the cable shape in subsequent analysis steps will be considerably smaller, since they usually comprise smaller changes in cable form than those of the first step.

4.2 Analysis of a Slack-Cable-Suspended Roof

The irregular hexagonal roof system shown in Figure 4.2 consists of seven compression-only pin-ended columns located on a slope with lengths ranging from 24 to 30 meters. Each of the six columns on the perimeter of the structure are restrained with a pair of fully taut cables that anchor the column tops to the ground. These columns are tilted with a slope of 10% to the sides to increase the stability and load-bearing capacity of the structure. The center column is initially assumed to be vertical. The column tops are interconnected with primary slack cables, which in turn support several uniformly spaced secondary cables that span between primary ones. This structure is in fact a typical module of a design candidate for a bird park in Tehran (having 15 hexagonal units covering an area of more than 50000 m^2 – only one unit is presented herein for brevity and clarity of the results).

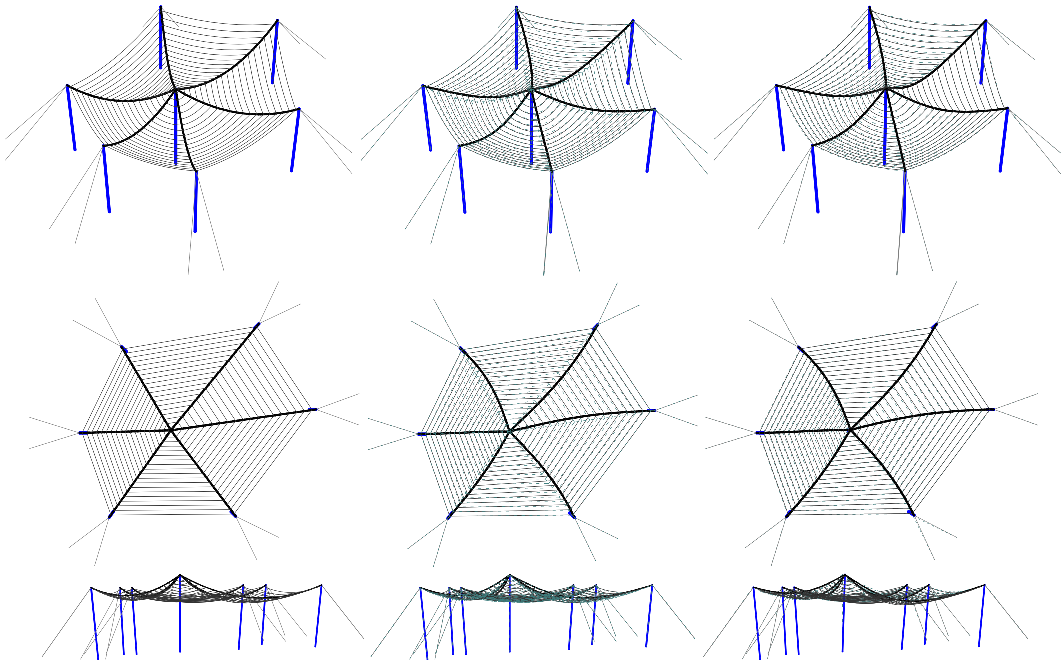

Figure 4.2. Progressive analysis of a slack cable roof system (from left to right: initial cable forms, updated cable forms with supports fixed, and final form with column tops released)

Figure 4.2 shows the results of analysis of the slack cable roof system subjected to a uniform gravitational dead load of 250 N/m^2. Note that each step is plotted against a dashed outline of the previous step to highlight the displacements corresponding to that step. In the first step, all cables are assumed to have a parabolic shape in a vertical plane, and their lengths are specified to achieve a near-uniform sag in the cables, being 9% of the span length for primary cables and 6% for secondary cables. Next, the cable forms are updated while preventing the translation of their supports; as a result, the primary cables deform in three dimensions in response to the unsymmetrical loads applied from secondary cables. In this step, the designer may choose to revise cable lengths to alleviate the uneven sag of the cables as a result of lateral movements of primary cables, noting that this usually intensifies the lateral cable displacements. Finally, the columns are released to gradually move towards achieving equilibrium and balancing the external loads. This usually leads to a lower energy state, in which the lateral displacements of cables are reduced. The amount of displacement at the top of the central column was calculated to be 95.3 cm, mostly towards east.

Next, in addition to the above-mentioned dead load, a uniform lateral load of 100 N/m^2 in the positive global *X*-direction is applied to all of the cables. The analysis results in Figure 4.3 show the lateral deformations in the secondary cables as well. In this case, the displacement of the central column from its originally-vertical position is obtained to be 140.5 cm. As a result, the sag of the cables to the east of the central column significantly increase. In order to further demonstrate the versatility of the proposed procedure to handle very large deformations and resolve numerical instabilities, a collapse scenario is assumed for the considered structure. After reaching the equilibrium state for dead load (Figure 4.2), it is assumed that one of the anchorage cables of the southwest column is removed and the structure is reanalyzed. The results shown in Figure 4.4 demonstrate a large displacement at the top of the column with a failed cable: 291.1 cm and 43.5 cm in the east and south directions, respectively. The displacement of the central column towards east also increases to 119.2 cm. It should be noted that reaching the equilibrium state in these conditions that comprise sudden changes in the internal force system may take several hundred iterations using the proposed procedure.

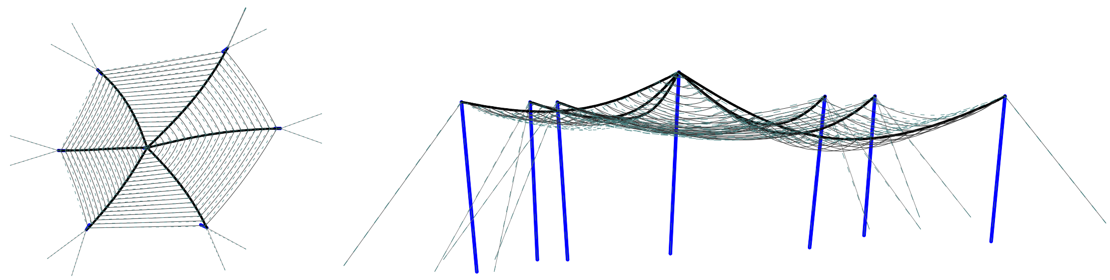

Figure 4.3. Example slack cable roof system subjected to lateral load from west to east

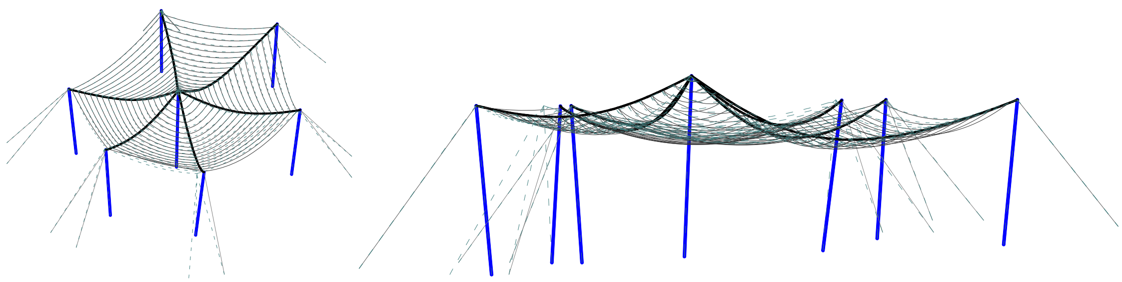

Figure 4.4. Extreme loading scenario: gravitational dead load with removal of one of the anchorage cables

5. CONCLUSIONS

An effective iterative approach is proposed for the analysis of highly unstable structures consisting of slack cables and bar elements. A specialized substructuring technique is used for the analysis of slack cables to reduce the processing costs and memory requirements, and to alleviate the cable instability issues before reaching the equilibrium state. The procedure to determine the cable form and internal forces uses sensible learning relations that progressively adapt the cable form to satisfy the equilibrium and compatibility requirements. Furthermore, the singularities in the overall structural level are addressed by recognizing the unstable DF's and restricting the corresponding displacement increments. Using these procedures, the structural form corresponding to load combinations consisting of gravitational and lateral loads can be accurately determined. The proposed method is successfully applied to the analysis of several slack-cable structures that exhibit a considerable degree of instability. It is shown that this method can be used in the analysis of structures with very large rigid-body displacements and rotations, such as when collapse scenarios are examined. Further study is underway for the application of this method to dynamic analysis of slack-cable systems.

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