Seismic response of reinforced concrete frames considering the influence of high shear stresses by means of a new numerical model

S. Mohr
TEC 4 engineering consulting, Spain

J. Bairán & A. Marí
Universitat Politècnica de Catalunya (UPC), Spain

SUMMARY:
In this paper, a new numerical model is presented which is capable of reproducing the interaction between the normal and shear stresses in static and dynamic analysis of reinforced concrete frame structures. In contrast to other more complex models, the multiaxial problem is approximated using a beam element (1D) and a sectional model. The sectional formulation uses generalized coordinates in order to simulate the state dependent warping, distortion and stress-strain distribution after cracking and up to failure. The formulation is here implemented into a flexibility-based frame element, which allows considering the influence of shear on the dynamic response of frames structures and to evaluate their seismic behaviour. A reinforced concrete frame is analysed using push over and dynamic time history analysis. In order to determine the effects of shear sensible considerations in the seismic response, these results are compared against shear rigid analyses.

Keywords: Shear forces, Dynamic response, Ductility, Sectional formulation, Flexibility formulation

1. INTRODUCTION

The seismic response of reinforced concrete structures is a complex phenomenon due to the presence of alternating normal and tangential stresses, inclined cracks and multi-axial stress states. The recently observed shear failure in the Hanshin Expressway in Kobe (1995) or the Bei-Fong Bridge in Taiwan (1999) highlight the need for analysis and design tools, which adequate capture the seismic response in practical engineering.

Beam-column models are suitable for seismic analysis of frame structures, especially after recent progresses in incorporating the effect of shear stress at the sectional level. However, although some of these models adequately simulate the ultimate load, they often underestimate the structural displacements due to the assumed shear stress or strain distribution at the sectional level. Moreover, such assumption affects the predicted failure mode and achieved ductility, which are of great importance in the seismic behaviour.

In section 2, a simplified model is presented for the accurate consideration of shear forces in strength, deformation, material state and damage. This model was assessed to be suitable for the non-linear analysis of frame structures. The tangential stress and strain distributions in concrete and transverse reinforcements are not predefined, but they are obtained from an inter-fibre equilibrium and compatibility conditions. Therefore, polynomial series are defined in order to represent the warping and distortion of the cross-section. The systematic formulation of the model easily allows varying the level of accuracy by modifying the number and degree of the polynomials considered, thus reproducing both simple classical theories and advanced kinematical distribution when required. The sectional model has been embedded in beam-column elements, which allows static and dynamic structural analysis considering the effect of shear in an efficient way. The model has been verified elsewhere using the results of experimental campaigns over shear sensible structures, (Mohr et al, 2010) and (Mohr, 2011).

The developed model can adequately reproduce the interaction between normal and tangential stresses in structural analysis. In this manner, it is possible to consider the influence of shear stresses in the dynamic response. The influence of shear stresses in the seismic behaviour will be studied in this paper by analysing a 2D RC-frame by means of non-linear static analysis (push-over) and dynamic time-history analysis, including different transverse reinforcement ratio. The same structure is analysed with a standard Navier-Bernoulli model for comparison purposes. The different effects of shear are observed and discussed in the paper showing the capabilities of the proposed model.

2. SECTIONAL FORMULATION

The model presented in this paper was developed by the authors in (Mohr et al, 2010). It keeps the hypothesis presented in (Bairán and Marí, 2007) according to which the full displacement field of any fibre in the cross-section can be approximated by the sum of the plane-section displacement field, similar to the Navier-Bernoulli hypothesis (u^{ps}), and a new displacement field that enables the section to distort and warp (u^{w}):

$$u_x^{ps} = u_0 + z \cdot \Theta_{y_o} \tag{2.1}$$

$$u_z^{ps} = w_0 \tag{2.2}$$

In this model, instead of describing the warping distribution by means of a planar finite element model of the cross-section, it is approximated by means of a polynomial series of increasing order. Therefore, the warping and distortion displacements u_x^w and u_z^w are simulated by finite series of predefined shape functions and coefficients as presented in figure 1, that is:

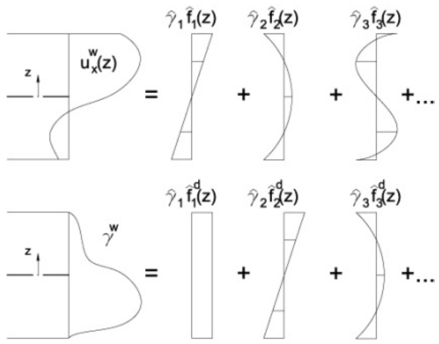


Figure 1. Warping approximation by means of a finite series

$$u_x^w \approx \sum_{i=1}^{n_g} \gamma_i f_i = \gamma \cdot F \tag{2.3}$$

$$u_z^w \approx \sum_{i=1}^{n_e} \varepsilon_i f_i = \varepsilon \cdot F \tag{2.4}$$

where f is a vector which holds the predefined shape functions. n_g is the number of terms used in the series to represent the warping deformation. The predefined shape function are gathered in a vector f_g and the corresponding factors are gathered in γ (n_gx1). In the same way, f_e and ε are obtained (n_ex1), where n_e is the number of terms used in the series to represent the distortion.

The model uses as predefined shape functions polynomial terms of increasing order. The first term in the finite series is given by:

$$f(1,1) = \frac{z}{h} \tag{2.5}$$

The higher order terms in the polynomial series are obtained with general expressions, which can be found in (Mohr, 2011). The coefficients for the predefined shape functions are summarized in a vector a with dimension $n = n_g + n_e$:

$$\vec{a} = \begin{bmatrix} \gamma \\ \varepsilon \end{bmatrix} \tag{2.6}$$

This vector ($\vec{a}$) contains the unknown coefficients which modulate the shape functions in order to adequately approximate the actual distribution of warping and distortion. The components of $\vec{a}$ are state dependent and are determined by means of internal equilibrium consideration:

$$\begin{aligned} \delta\gamma^T \int f_g^d \cdot \tau_{xz} dA &= \delta\gamma^T \int f_g^d \cdot \frac{\partial \sigma_x}{\partial x} dA \\ \delta\varepsilon^T \int f_e^d \cdot \sigma_z dA &= 0 \end{aligned} \tag{2.7}$$

This equilibrium can be solved locally at every cross-section without additional degrees of freedoms in order to consider multi axial stress states. Therefore, the system of equation is solved with a nonlinear iterative procedure. This requires the definition of constitutive equation for steel and concrete in order to evaluate the sectional state. For the further analysis, a uniaxial model is used for the longitudinal and transversal reinforcement bars and assumes a bilinear steel behaviour. The multi-axial concrete stresses are obtained with a model presented in (Bairán and Marí, 2007), which is based on the rotating-smeared crack approach. The sectional formulation is independent form the constitutive equations, so that the material state can also be evaluated with formulations presented in (Vecchio and Collins, 1986) and (Pang and Hsu., 1995) among others.

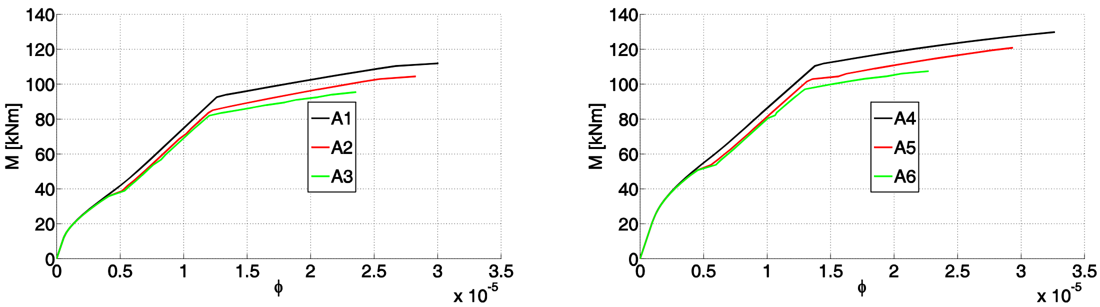
This general approach allows the selection of different terms, n_g and n_e, in order to estimate the warping and distortion deformations respectively. As a consequence the sectional model automatically reproduces Navier-Bernoulli's or Timoshenko's hypotheses by considering zeros in both series, or one warping term and zero distortion in the polynomial series, respectively. It should be noticed, that this model can only be used for a general shaped section symmetric to the z-axis.

The ability of the sectional model can be shown by analysing the column section of the frame presented in figure 4. Therefore, several loading scenarios, sectional kinematics and transversal spacing have been considered in sectional analysis using the presented sectional scheme. A total of six analyses have been performed, which are defined in table 1 assuming a shear span (M/V) of 1.0 m. The analyses A1 and A4 considers only the normal stress interaction due to the selection of $n_g = n_e = 0$ and simulate a shear rigid behaviour. Consequently the spacing of the transversal reinforcement has no effect on the obtained response. The other analyses consider the same moment - shear relation, but the interaction of normal and shear stresses is activated by selecting $n_g = n_e = 6$ and the transversal reinforcement spacing affects the sectional response. The resulting moment curvature diagrams for the six analyses are the presented in figure 2.

The presented sectional model considers in a realistic way the influence of the shear stresses and needs the transversal reinforcement in order to equilibrate the compression strut. The result is a stress increment in the longitudinal reinforcement. Consequently the considered spacing of 60 and 150 mm results in different first yield bending moments and post yield stiffness of the section. It can be observed, that the adequate consideration of shear stresses has a significant influence on the moment - curvature response, which should also affect the curvature ductility used in the seismic design of RC members.

Table 1. Definition of sectional analysis

Name	Axial load	Section kinematics	Spacing transversal reinforcement
A1	0.0 kN	$n_g = 0$; $n_e = 0$	not considered
A2	0.0 kN	$n_g = 6$; $n_e = 6$	s = 60 mm
A3	0.0 kN	$n^g = 6$; $n_e = 6$	s = 150 mm
A4	-200.0 kN	$n_g = 0$; $n_e = 0$	not considered
A5	-200.0 kN	$n_g = 6$; $n_e = 6$	s = 60 mm
A6	-200.0 kN	$n_g = 6$; $n_e = 6$	s = 150 mm



a) Sectional response without applied axial load b) Sectional response with applied axial load

Figure 2. Moment curvature diagrams

The sectional model has been embedded in shear flexible beam elements. The resulting structural model allows the consideration of shear normal stress interaction in static and dynamic structural analysis. The conceptual idea is presented in figure 3 and detailed information about the structural model (Mohr et al, 2010) and (Mohr, 2011). In these references, the sectional and structural formulation has been verified by reproducing displacements, stresses and strains in the concrete and in the reinforcements.

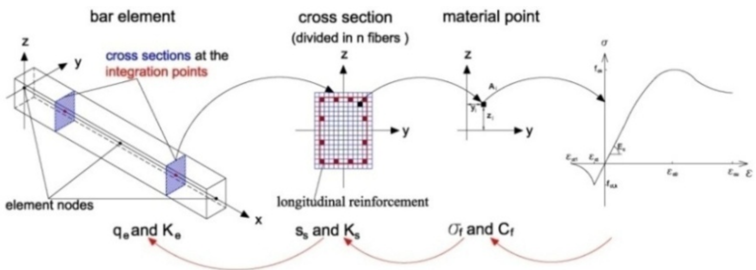


Figure 3. Computational model of beam column elements with cross section

3. EFFECTS IN PUSH OVER ANALYSIS

The influence of shear stresses in the curvature response, observed in the previous sectional analyses, can be considered in structural analysis. Therefore a two story frame, presented in figure 4, will be analysed considering a shear flexible behaviour (SF) by selecting six terms in warping and distortion series ($n_g=n_e=6$). In addition, a shear rigid model (SR) is provided as numerical reference ($n_g=n_e=0$), this is equivalent to a Navier-Bernoulli formulation. Both analyses consider as permanent loads the self-weight of the frame and a distributed load of q_z = 20kN/m on both first and second story. The assumed concrete and steel strength are the following: concrete f_{ck} = 30 MPa f_{ct} = 2 MPa and steel f_y = 400 MPa. Ultimate compression strain for unconfined concrete is taken as ε_{cu}=0.004; ultimate strain in steel is ε_{su}=0.1. Both numerical models for the two story frame will be loaded with a linear lateral load pattern, so that the lateral load at node 5 is two times the one at node 3.

The column and beam sections of this frame are presented in figure 4 and define the longitudinal reinforcement arrangement in the SR and SF analysis. In addition, the SF analysis requires the definition of the transversal reinforcement. The spacing of the stirrups outside the plastic hinge region is 150 mm for the beam and column elements. In the plastic hinge region the column and beam sections consider a smaller spacing of 60 and 80 mm respectively. All sections have a transversal reinforcement diameter of 10 mm, obtained by neglecting the concrete contribution (V_c=0.0 kN) in the shear design, as required in seismic provisions for ductile frames. This assumption should ensure that several longitudinal reinforcement bars yield before the stirrups.

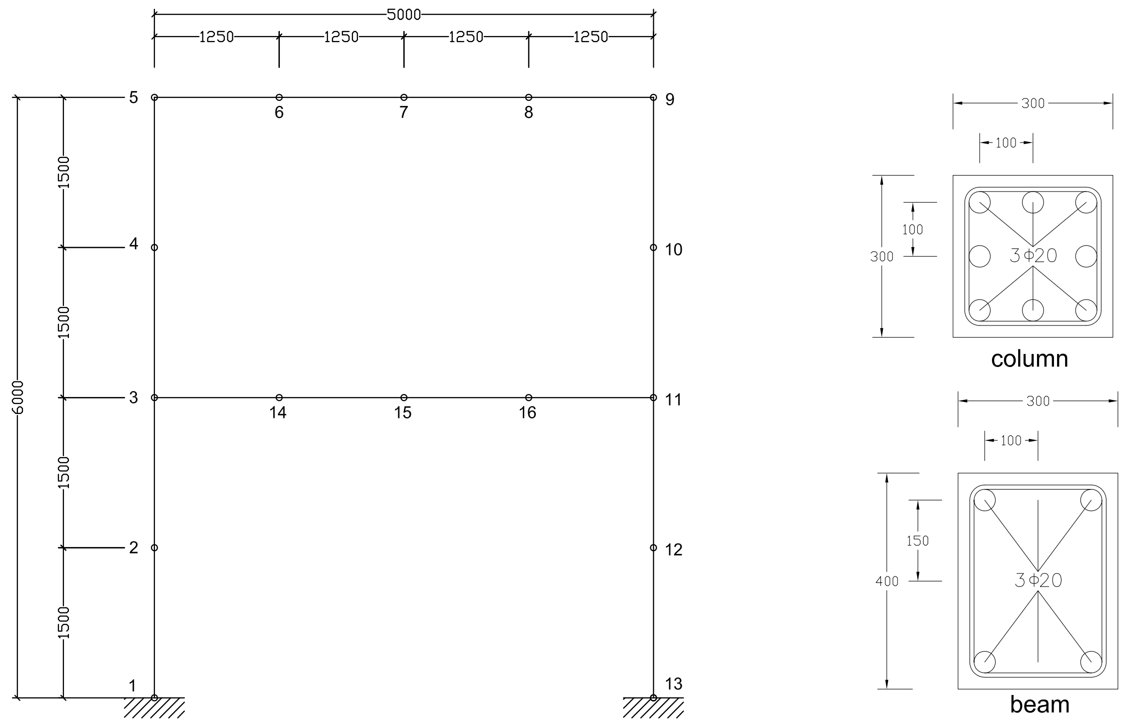


Figure 4. Two story reinforced concrete frame [mm]

The results of the push-over analyses are presented in figure 5 for two section kinematics. These curves show the relationship between the applied lateral load and the corresponding lateral displacement at node 5 (roof). In figure 5, it can be noticed that the shear rigid analysis (SR) defines an upper bound for the lateral load that can be resisted by the structure. The shear flexible analysis shows a lower resisted lateral load than the shear rigid one.

A simplified bilinear representation of the push-over curve can be used to quantify the strength and ductility characteristics of the frame. This curve is determined using the condition of equal areas under the load–displacement curves. The obtained values for the three analyses are summarized in table 2. In addition, this table includes displacement ductility of the frame ($\mu_d = d_{max}/d_{ys}$) and the displacements where the first longitudinal (d_y^l) and transversal (d_y^t) bar yielded in the push over analyses. The ductility of the structure is overestimated in the shear rigid analysis, since smaller values are obtained considering the effect of shear.

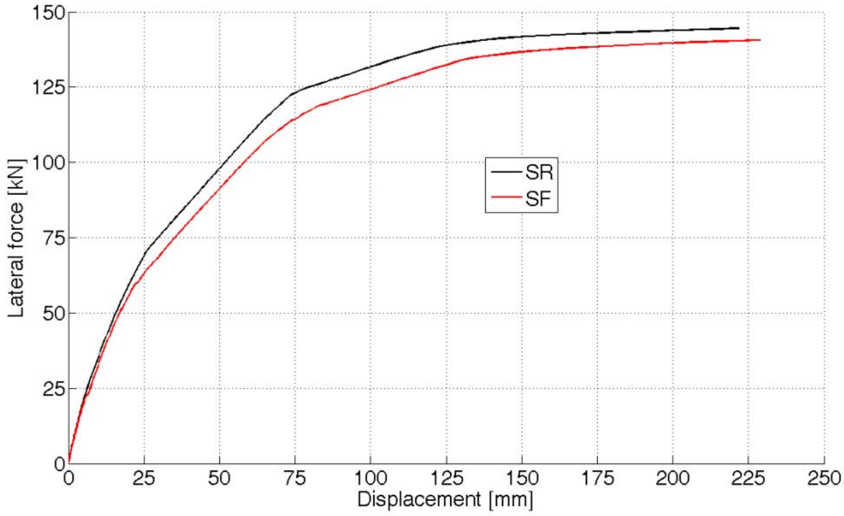


Figure 5. Load – displacement curve for the shear rigid and flexible structural model

The large advantage of the presented formulation is that different beam theories can be simulated using the same element formulation, section discretization and constitutive models. Consequently, the only difference between SR and SF analyses is the considered section kinematics.

Table 2. Summarized values for the idealized load displacement curve and displacement ductility

Model	d_y^l [mm] (longitudinal reinforcement)	d_y^t [mm] (transversal reinforcement)	d_{ys} [mm] (bilinear representation)	d_{max} [mm] (max. displacement)	P_{max} [kN] (max. lateral force)	μ_d (displacement ductility)
SR	25.9	-	80.3	221.9	144.5	2.8
SF	24.7	126.1	87.7	229	140.5	2.6

4. EFFECTS IN DYNAMIC ANALYSIS

In this section, the two storey frame is subjected to a dynamic excitation. Therefore, structural properties such as geometry, materials and permanent loads are taken identical to the previous push-over analysis. In addition, the mass matrix of the system has to be computed for dynamic analysis. The self-weight of the beams and columns are considered in a consistent mass matrix with additional entries considering the mass of the distributed load.

The time dependent load will subject the frame to large cycles in the ultimate limited range. The study is aimed to estimate the maximum structural response and possible failure due to the seismic action. In this paper, no elastic structural damping is considered in the dynamic equilibrium. However, hysteretic damping will be activated due to yielding of several reinforcement bars.

The frame will be subjected to a real earthquake excitation, in this study the North-South component of "El Centro" has been considered to conduct this analysis. The peak ground acceleration of this earthquake record is 0.32g and the acceleration history is presented in figure 6.

The dynamic response of the two storey frame is obtained considering again a shear rigid (SR) and flexible (SF) behaviour with a stirrup diameter d_t = 10mm for the SF analysis. Therefore, the generalized α method, presented in (Chung and Hulbert., 1993), is used, which requires the definition of several parameters in order to control the numerical damping properties of the solution algorithm. These parameters are selected as $\alpha_f = 0.1$, $\alpha_m = -0.1$, $\beta_N = 0.36$ and $\gamma_N = 0.7$ using the recommendation made in (Erlicher, Bonaventura and Bursi, 2002).

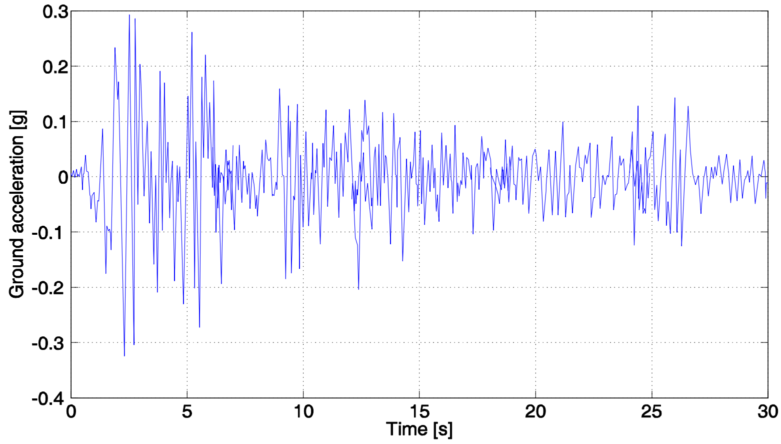


Figure 6. Ground acceleration "El Centro" N-S

Figure 7 shows the horizontal displacement responses for the second floor. This result shows different oscillations for the SR and SF analyses and maximum displacements are predicted at different times (-99.7 mm for SR at 11.7 s and -80.7 mm for SF at 2.2 s). The consideration of shear stresses changes the stiffness of the frame and consequently the internal force redistribution and period of the structure.

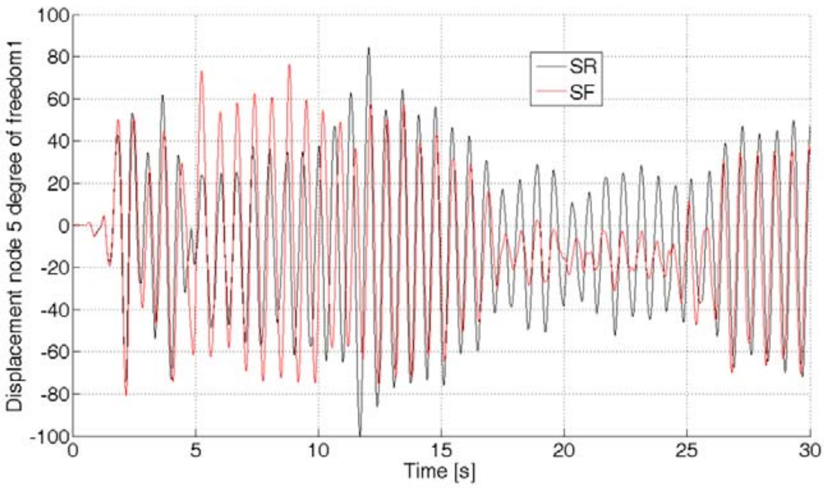


Figure 7. Second floor displacement response (node 5)

The differences in the displacement response between SR and SF analyses can also be observed in the calculated base shear, which is presented in figure 8. This figure indicates again that different base shear histories are obtained, due to the change of stiffness and period as previously stated. The maximum base shear values are -134.7 kN for SR and -124.5 kN for SF. By comparing, the maximum base shears obtained in the dynamic analyses with the values estimated in the push over analyses (see table 1), it can be observed that the two story frame has been loaded in the ultimate limited range by the considered ground acceleration. The first story drift for the SR analysis was estimated at 11.7 s, which coincides with the maximum horizontal displacement shown in figure 7.

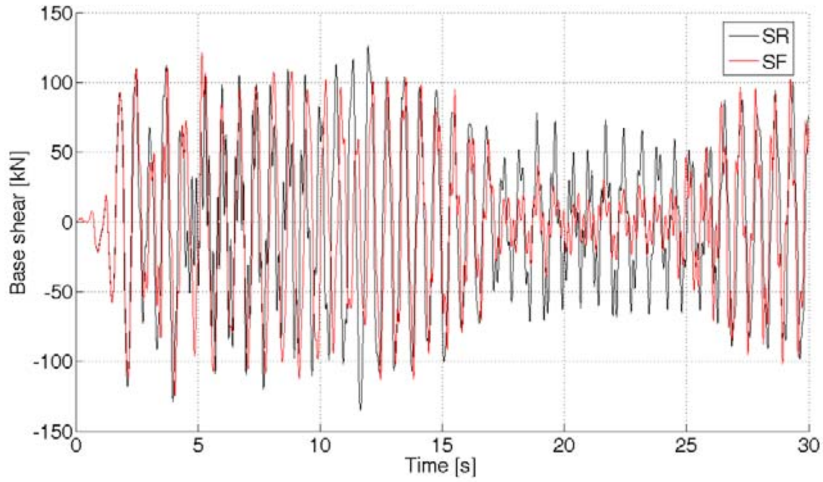


Figure 8. Calculated base shear history

The large rotation at the bottom of the left column results in large plastic strain in the external reinforcement layer. Figure 9 shows the strain histories of this layer for the SR and SF analyses. The largest rotations and long bar strains are estimated in the first floor beam. There large rotations are desired in seismic design in order dissipate energy in these plastic hinge regions. Long bar strain responses for the left end of the first story beam are presented in figure 10 for both models. In the first seconds of the seismic excitation, the stress increment in the longitudinal reinforcement can be observed in the estimated strains for the SF analysis. After the yielding of several longitudinal bars, the shear flexible analysis shows a larger damping effect, due to inclined cracks that result by the consideration of shear.

It is noticed that, although concrete contribution to shear strength is neglected in design of the transversal reinforcement, yielding of stirrups take place after several cycles and large displacement; which indicates the degradation of shear resistance mechanism in cyclic large magnitude excitations. One example is presented in figure 11 for the left plastic hinge region of the beam at section mid height. The transversal strain response shows that the presented sectional and structural model is suitable to investigate in further studies the axial shear bending interaction in dynamic analysis.

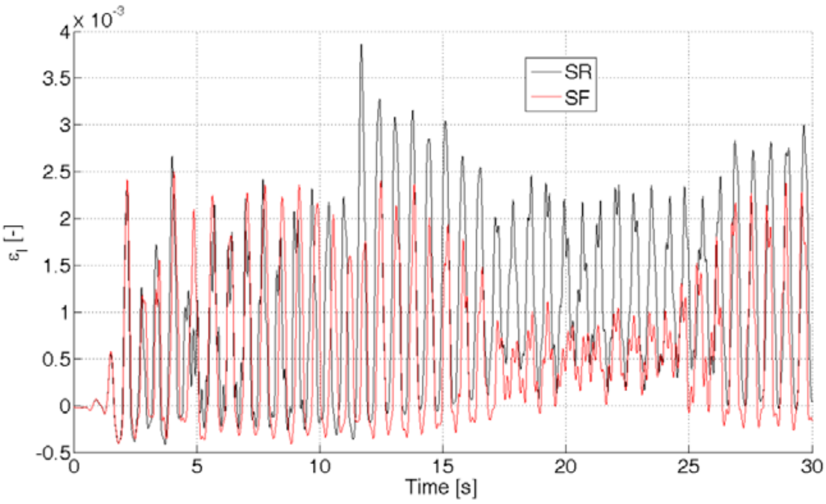


Figure 9. Strain response for the external longitudinal bars at the bottom of the left column

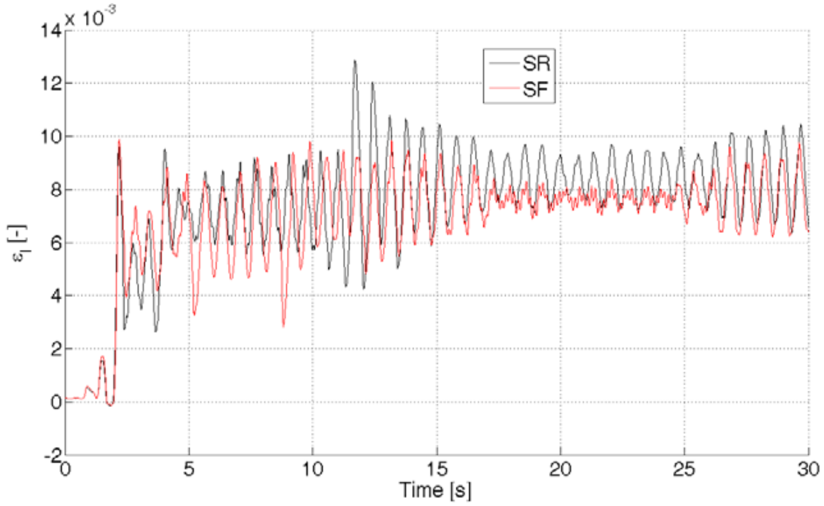


Figure 10. Strain response for long bar in the first floor beam (left plastic hinge region)

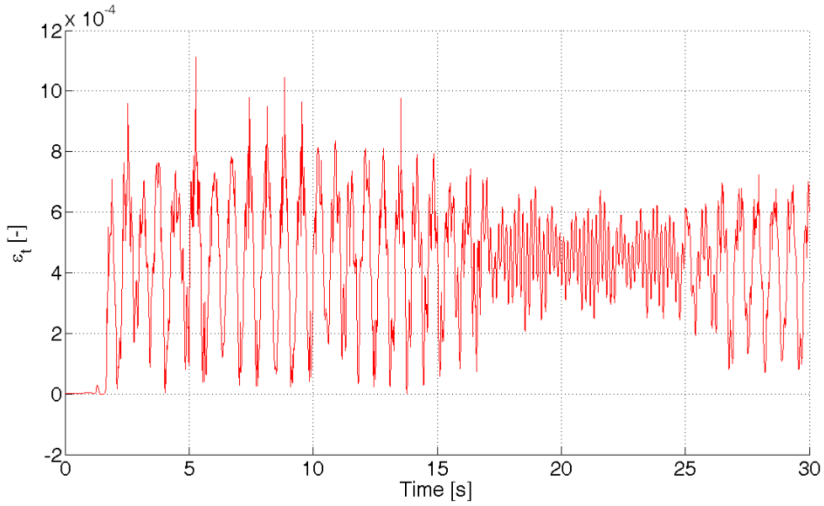


Figure 11. Strain response for stirrup (plastic hinge region of the beam)

5. CONCLUSIONS

The presented results show the influence of axial-shear-bending interaction in the static and dynamic response of reinforced concrete structures. The consideration of shear stresses in the section internal equilibrium affects in several ways in the seismic response of reinforced concrete frames. The effect of shear provokes a stress increment in longitudinal reinforcement, which is adequately captured with the presented formulation. In addition, the presence of large shear stresses changes the stiffness of the frame and consequently the natural frequency of the system. This effect has been observed in the displacement and base shear responses considering shear rigid and flexible structural behaviour. The adequate simulation of the shear changes the internal force redistribution and affects the ductility of the structure. It can be concluded, that load - displacement curves obtained with a shear rigid analysis only define an upper bound for the lateral load.

AKCNOWLEDGEMENT

This study was conducted with the support of the research projects “Assessment of deteriorated, repaired and strengthened structures” (BIA2009-11764) and “Rehabilitation of Roads and Freeways: REHABCAR (IPT-370000-2010-29), founded by the Spanish Ministry of Science and Innovation.

LIST OF SYMBOLS

Latin letters

$\vec{a}$	Vector containing warping distorsion coefficients
d_y^l	First yield displacement longitudinal bars
d_y^t	First yield displacement transversal bars
d_{max}	Maximum lateral displacement
d_{ys}	Yield displacement in bilinear push over curve estimation
f	Predefined shape functions
F	Vector of predefined shape functions
f_{ck}	Concrete compression strength
f_{ct}	Concrete tension strength
f_y	Steel yield strength
h	Section height
M	Bending moment
n_e	Number of terms for the warping series
n_g	Number of terms for the distorsion series
P_{max}	Maximum lateral force in push over curve
u^{ps}	Plane section displacement field
u^w	Warping-distorsion displacement field
u_0	Normal displacement
V	Shear force
w_0	Transversal displacement
x	Local axis along the beam
z	Local axis orthogonal to beam axis x

Greek letters

γ	Coefficients for the sectional warping displacement
ε	Coefficients for the sectional distorsion displacement
Θ_{y0}	Rotation angle around y-axis
μ_d	Displacement ductility
σ	Stress
τ_{xz}	Shear stress

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