Study of a Tank-Pipe Damper System for Seismic Vibration Control of Structures

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SUMMARY:
This paper examines the theoretical model of a tank-pipe (T-P) damper, consisting of two tanks and a connecting pipe with orifice(s), which can control three modal responses of the structure. A transfer function formulation of a base-excited multi-degree-of-freedom (mdof) structure with attached T-P damper is presented. Numerical studies in the frequency domain are carried out to demonstrate the effectiveness of the proposed damper. The supplemental equivalent viscous damping and optimal orifice damping coefficient are studied. Comparisons are made with the liquid column damper (LCD). Results indicate that the T-P damper has lower performance sensitivity and greater robustness. It provides a better option than the LCD for the reduction of responses with significant higher mode participation. Moreover, for ground motions with energy content that excite modes other than the mode to which the LCD is tuned, the tank-pipe damper achieves considerable vibration suppression while the LCD may be rendered practically ineffective.

Keywords: tank-pipe damper, seismic excitation, transfer function, LCD

1. INTRODUCTION

Liquid dampers are popular passive control devices for civil engineering structures due to their low capital and maintenance costs, easy installation, effectiveness even in low vibration amplitudes and for the versatile use of the water in the damper such as for firefighting, etc. There are chiefly two types of liquid dampers, namely sloshing dampers and column dampers. Kareem and Sun (1987) and Fujino et al. (1992) were amongst those who carried out pioneering work on the liquid sloshing damper and expounded the importance of having shallow liquid height and of tuning the sloshing frequency to the structural fundamental frequency. Some researchers like Koh et al. (1994, 1995) and Banerjee et al. (2000) demonstrated the effectiveness of the device in controlling the seismic response of structures. Tait et al. (2008) investigated both unidirectional as well as bidirectional tuned liquid dampers (TLD) and provided performance charts for the same. Tait and Deng (2010) studied different tank geometries of the TLD and reported that a horizontal-cylindrical TLD is the most robust.

The liquid column damper (LCD), which dissipates energy by the movement of an oscillatory column of liquid through orifice(s) provided in the cross section of a U-shaped container, scores over the sloshing damper due to its higher volumetric efficiency, consistent performance over a wide range of excitation levels and a very specific damping mechanism. Originally proposed by Saoka et al. (1988) and experimentally verified by Sakai et al. (1989), it has been extensively investigated for the mitigation of wind-induced vibration such as by Xu et al. (1992), Balendra et al. (1999), Shum and Xu (2004), Wu et al. (2005) and Min et al. (2009), amongst others. The applicability of this device as a seismic vibration control device has been explored by Sun (1994), Won et al. (1996), Reiterer and Ziegler (2005), Ghosh and Basu (2008), etc. Studies have also been undertaken on several variations of the conventional LCD, with either different geometric configuration such as the V-shaped LCD by Gao and Kwok (1997) or with non-uniform column cross-section called the liquid column vibration absorber (LCVA) by Hitchcock et al. (1997), Wu et al. (2009), Konar and Ghosh (2010), etc., with encouraging results.

The present study proposes a simple, innovative liquid damper, the tank-pipe (T-P) damper, which is essentially a combination of the sloshing tank damper and the LCD. It is effective for multi-modal vibration control of structures as it has the benefits of the effects of tuning of the liquid sloshing as well as of the liquid oscillating modes to the structural modes of vibration, along with the orifice damping and the damping due to sloshing. The theoretical model of the T-P damper is presented and a formulation for the displacement transfer functions of a structure, modelled as a multi-degree-of-freedom (MDOF) system equipped with this damper is developed. Numerical studies demonstrate the effectiveness of the proposed damper, especially when the higher modes have a greater participation. The equivalent viscous damping due to the inclusion of the damper is estimated. Comparative studies on the performance of the T-P damper and that of the LCD are also carried out.

2. MODELING OF T-P DAMPER

The T-P damper system (see Fig. 2.1) consists of two upright, circular cylindrical tanks connected at the base by a pipe into which orifice(s) are installed. The system contains liquid, of density ρ, up to a height $\hat{h}$, measured from the centerline of the base pipe. The liquid height from the tank base is given by $\acute{h}[= (\hat{h} + R_p)]$. Let the cross-sectional areas of the left tank, pipe and right tank be represented by $A_l[= \pi R_l^2]$, $A_p[= \pi R_p^2]$ and $A_r[= \pi R_r^2]$ respectively. Here, R_l, R_p and R_r denote the radii of the left tank, pipe and right tank respectively. The distance between the centres of the two tanks is denoted by $\bar{B}$ and the clear length of the pipe between the tanks is given by $\acute{B}[= (\bar{B} - R_l - R_r)]$. The tank walls are assumed to be rigid and the liquid is assumed to be incompressible. Further, liquid displacements are assumed to be small and only linear motion of the liquid inside the tanks is considered.

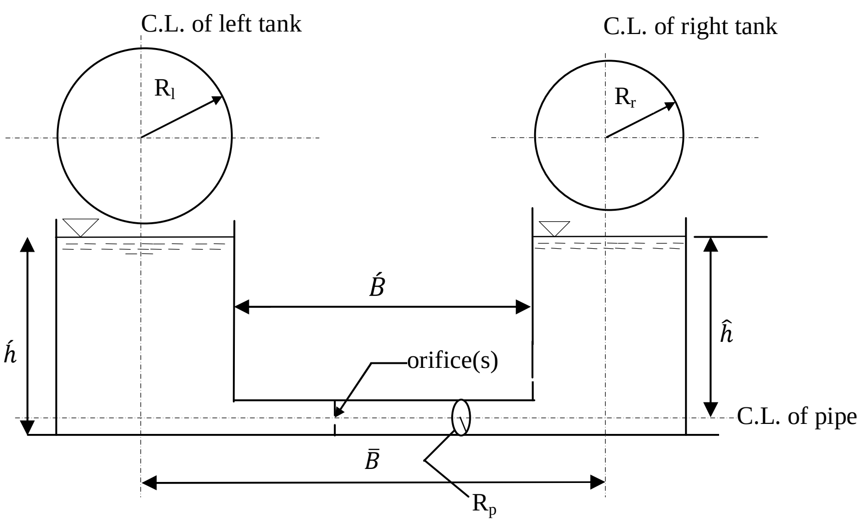

Figure 2.1. Model of T-P damper

It is a common approach in the analysis of laterally excited tanks to separate the hydrodynamic effects into two parts, (i) the "impulsive" component, due to the portion of the liquid accelerating with the tank wall like a rigidly attached mass, and (ii) the "convective " component, caused by the portion of the liquid that experiences sloshing motion (Housner (1957), Veletsos and Tang (1990)). Veletsos and Tang (1990) showed that there is a large separation in the natural frequencies of the impulsive and convective masses and that the two effects can be considered to be uncoupled from each other. Also, the impulsive frequencies are very high as compared to the natural frequencies of the structural systems commonly encountered in civil engineering applications. In the present model of liquid damper proposed, the impulsive response is ignored. As also evinced from the work of Veletsos and Tang (1990), the mass of liquid that participates in the impulsive component as compared to the liquid mass in the sloshing component is greater for tall, slender tanks and less for short, broad tanks. Hence, short, broad tanks are preferred in the proposed liquid damper to achieve pronounced sloshing motion of the liquid in the tanks.

Three components of response are considered while evaluating the hydrodynamic effects of the tank-pipe system. Those are (i) the sloshing response of the liquid in the left tank, (ii) the sloshing response of the liquid in the right tank, and (iii) the response of the liquid oscillating through the pipe from one tank to another, similar to the motion of the column of liquid in a LCD. A simplistic method is employed to analyze the effects of the principal actions of the damper. For the purpose, the above mentioned three components of response are assumed to be uncoupled from each other. Only linearized response of the fundamental liquid sloshing mode of vibration in each tank is considered. These fundamental sloshing modes of vibration for the two tanks are represented by two SDOF systems (see Fig. 2.2), the masses, frequencies and damping ratios of which are given by Veletsos and Tang (1990).

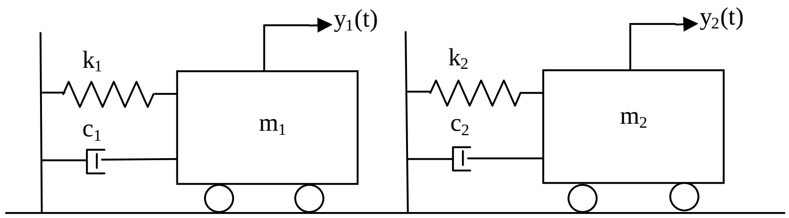

Figure 2.2. Model of fundamental sloshing modes of vibration of tanks

3. FORMULATION OF TRANSFER FUNCTION

Consider a linear *n*-DOF lumped mass stick model of a structural system, subjected to a base excitation, $\ddot{z}_g(t)$. A T-P damper system, is rigidly connected to the top mass of the structure. Let $\{x(t)\}$ denote the displacements along the DOFs of the structure relative to ground. The equation of motion for these DOFs may be written as

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = -[M]\{R\}\ddot{z}_g(t) + \{f(t)\} \tag{3.1}$$

where $[M]$, $[C]$ and $[K]$ respectively represent the mass, damping and stiffness matrices of the structural system, and $\{R\}$ is the influence vector, denoting rigid-body displacements along the DOFs for unit translation of the base. The vector of forces transmitted from the damper to the structure is denoted by $\{f(t)\}$ and is expressed by $\{0 \quad 0 \quad \ldots \quad \vartheta(t)\}^T$, where $\vartheta(t)$ represents the total force transmitted from the three vibration modes of the T-P damper system to the structural system. The expression for $\vartheta(t)$ is given below

$$\vartheta(t) = \{c_1\dot{y}_1(t) + k_1y_1(t) + c_2\dot{y}_2(t) + k_2y_2(t)\} - \rho\big[A_p\acute{B}\{\ddot{u}_p(t) + \ddot{x}_n(t) + \ddot{z}_g(t)\} \\ +\acute{h}(A_l + A_r)\{\ddot{x}_n(t) + \ddot{z}_g(t)\}\big] \tag{3.2}$$

In Eq. (3.2), u_p(t) denotes the displacement of the liquid in the pipe. Assuming the structure to be classically damped, the displacement of the k^{th} DOF of the structure in frequency domain may be expressed as

$$X_k(\omega) = \sum_{j=1}^{n} \phi_k^{(j)} Q_j(\omega) \tag{3.3}$$

where $\phi_k^{(j)}$ is the k^{th} element of the j^{th} structural mode. The terms $X_k(\omega)$ and $Q_j(\omega)$ are the Fourier transforms of $x_k(t)$ and $q_j(t)$ respectively, where $\{q(t)\}$ is the vector of normal coordinates. On Fourier transformation of the modal equations and through appropriate substitutions, the following *n* linear, simultaneous equations are obtained, which provide the input-output relations of the structural

DOFs in the frequency domain.

$$X_k(\omega) - \omega^2 X_n(\omega) \sum_{j=1}^{n} T_k^{(j)}(\omega) = (-\sum_{j=1}^{n} \phi_k^{(j)} H^{(j)}(\omega)\alpha_j + T_k^{(j)}(\omega))\ddot{Z}_g(\omega), \text{ k = 1,2,3....n} \quad (3.4)$$

with

$$T_k^{(j)}(\omega) = \phi_k^{(j)} H^{(j)}(\omega)\phi_n^{(j)} \Big(H_1(\omega)(i\omega c_1 + k_1) + H_2(\omega)(i\omega c_2 + k_2) + \omega^2 \rho A_p H_p(\omega)\frac{\acute{B}^2}{\acute{L}} + \rho A_p \acute{B} + m_1 + m_2\Big) \quad (3.5)$$

In Eq. (3.4), $H^{(j)}(\omega)$ denotes the modal transfer function relating the displacement of the SDOF system of natural frequency ω_j and damping ratio ξ_j, in the j^{th} structural mode, to the ground acceleration. It is given by

$$H^{(j)}(\omega) = 1/(\omega_j^2 - \omega^2 + 2i\xi_j\omega_j\omega) \quad (3.6)$$

Also in Eq. (3.4), α_j is the j^{th} modal participation factor and $\ddot{Z}_g(\omega)$ is the Fourier transform of the ground acceleration. In Eq. (3.5), $H_1(\omega)$ and $H_2(\omega)$ denote the transfer functions relating the displacement respectively of m_1 and m_2 (see Fig. 2.2), to the acceleration at the base of the tanks. They are expressed as

$$H_m(\omega) = 1/(\Omega_{\mathrm{m}}^2 - \omega^2 + 2i\zeta_m\Omega_{\mathrm{m}}\omega), \quad \text{m = 1,2} \quad (3.7)$$

where Ω_1, ζ_1 and Ω_2, ζ_2 are respectively the frequencies and damping ratios of the SDOF systems corresponding to m_1 and m_2. Further in Eq. (3.5), $H_p(\omega)$ may be defined as the transfer function relating the displacement of the liquid oscillating through the pipe between the tanks, modelled as a SDOF oscillator, to the base acceleration modified by the factor $(\acute{B}/\acute{L})$, with the expression for $\acute{L}$ given by $[\acute{B} + \hat{h}\{(A_p/A_l) + (A_p/A_r)\}]$. $H_p(\omega)$ is expressed as

$$H_p(\omega) = 1/(\omega_{\mathrm{p}}^2 - \omega^2 + 2i(\acute{C}_p/\acute{L})\omega) \quad (3.8)$$

where $\omega_p[= \sqrt{g\{(A_p/A_l) + (A_p/A_r)\}/\acute{L}}]$ is the natural frequency of liquid oscillation through the pipe between the tanks. $\acute{C}_p[= \sigma_{\dot{u}_p}\varsigma/\sqrt{2\pi}]$ represents the equivalent linear damping coefficient for the nonlinear orifice damping as in the LCD, where ς is defined as the coefficient of head loss controlled by the opening ratio of the orifice(s) and $\sigma_{\dot{u}_p}$ is the standard deviation of the liquid velocity $\dot{u}_p(t)$. This is obtained with the assumption that $u_p(t)$ is a Gaussian process (Xu et al. (1992)).
Once the displacement transfer functions are obtained from the solution of Eq. (3.5), the transfer functions of other response quantities such as floor acceleration or base moment may be readily determined. The response rms value can be numerically evaluated by computing the square root of the area under the curve of the corresponding power spectral density function (PSDF) (Newland (1993)).

4. NUMERICAL STUDY

The performance of the T-P damper is investigated by considering an example structure, modeled as a 3-DOF system. The masses lumped at the bottom, middle and top levels are equal to 25 x10^5 *kg*, 20 x10^5 *kg* and 15 x10^5 *kg* respectively. The connecting elements between the masses have identical stiffness equal to 1.0 x10^7 *N/m* each. The natural frequencies of this system are calculated as $\omega_1[= 1.05\, rad/s]$, $\omega_2[= 2.74\, rad/s]$ and $\omega_3[= 4.01\, rad/s]$ respectively A damping ratio of 1% is assumed in all the three modes of the structure. A T-P damper system containing water is attached to the top mass of the structure. The dimensions of the damper are fixed from the consideration of tuning

ω_p, ω_l and ω_r to ω_1, ω_2 and ω_3 respectively. The ratio of the mass of total water in the damper is taken as 5% of the top mass of the structure. Geometric feasibility is also a consideration in fixing the damper dimensions. In the present example study, the radii of the left tank, right tank and pipe are obtained as 2.287*m*, 1.122*m* and 1.046*m* respectively, while $\acute{B}$ is calculated as 8.26*m*. As short and broad tanks are preferred over tall and slender tank, a unit value is assigned to the ratio $\acute{h}/R_l$.
The seismic excitation at the base of the structure is characterized by a white noise PSDF of intensity $1cm^2/s^3$. The orifice damping coefficient, ς, is obtained by minimizing the rms value of the displacement of the top mass of the structure. The optimum value, ς_{opt}, for the present example system is 3.0. By employing the formulation in Section 3, the transfer functions relating the displacement of the top mass of the structure, relative to ground, to the input base acceleration, with and without T-P damper, are presented in Fig. 4.1. The figure clearly indicates the effect of tuning and damping in the three modes of the structure. The transfer functions for the base moment (not shown here) are also evaluated using the same value of ς. For this purpose, the bottom, middle and top mass of the structure are assumed to be lumped at a height of $3m$, $6m$ and $9m$ from the ground respectively. The percent reductions in the rms responses of displacement and base moment due to the T-P damper are indicated in Table 4.1 and show that appreciable response reduction is possible by using this damper system.

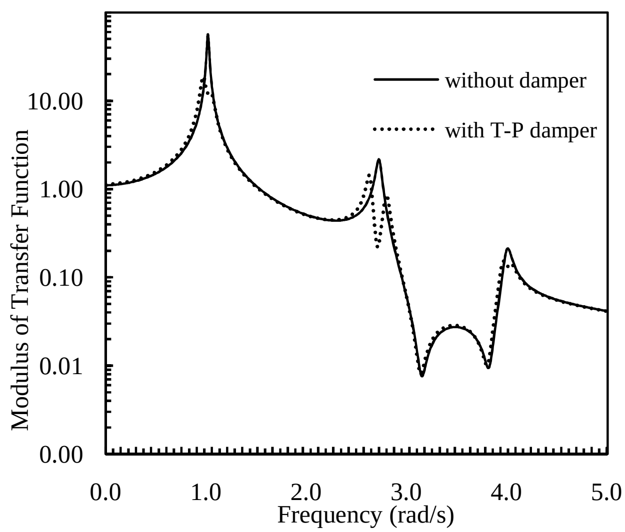

Figure 4.1. Displacement transfer function of top mass, with and without T-P damper (white noise input)

Next, the input is changed to a band limited white noise of spectral intensity of $1cm^2/s^3$ ranging between $0.5s[= 12.57rad/s]$ and $2.5s[= 2.51rad/s]$. This range of excitation is chosen as the frequency analysis of recorded accelerograms indicates high energy content in the short period range. The transfer function curves for the top mass displacement and the base moment are again evaluated and the corresponding percent reductions in the rms response values are given in Table 4.1. It may be noted that the fundamental mode of the structure is not excited as the fundamental frequency lies outside the frequency range of the base input and so the reduction in response is possible only through control of the higher modes, which is achieved by the multi-modal tuning capacity of the T-P damper system.
To further highlight the advantage of the multi-modal tuning that is possible in the T-P damper, a comparison is made of its performance with that of the LCD. The LCD is taken to have the same ratio of water mass to top mass of structure as that of the T-P damper and is tuned (with a tuning ratio equal to unity) to the most dominant mode of the structure, which is the fundamental, while considering white noise input of intensity $1cm^2/s^3$ to the structure. Also, a commonly used value of 0.9 (Xu et. al. (1992)) is considered for the ratio of the horizontal length to the total length of the liquid column tube. As in the case of the T-P damper, the coefficient of orifice damping for the LCD determined by minimizing the rms value of the displacement response of the top mass of the structure is obtained as

13.0. Using the transfer function formulation for the LCD-MDOF structural system given by Paul and Ghosh (2009), the transfer functions for the displacement of the top mass of the structure (see Fig. 4.2) and for the base moment, without damper and with LCD, are evaluated. In Fig. 4.2, the transfer function curve for LCD exhibits the effect of tuning and damping only in the fundamental mode of the structure. The rms response reductions (see Table 4.1) are found to be greater in the case of LCD as compared to that for the T-P damper. This may be attributed to the greater mass of liquid tuned to the fundamental mode, in case of the former control device. The reductions in the rms values of the same response quantities of the structure-LCD system in case of the band-limited white noise are also evaluated and given in Table 4.1. It is seen that in this case the LCD is practically ineffective as a control device. It may thus be inferred that the performances of the LCD and the T-P damper are comparable in case of white noise excitations while the T-P damper performs significantly better in case of band limited excitations affecting only the higher modes.

Table 4.1. Response reductions with T-P damper and with LCD

Input	Response quantity	Structure alone	Structure with T-P damper	Structure with LCD	Percent response reduction with T-P damper	Percent response reduction with LCD
White noise	rms value of top mass displacement (cm)	10.4	6.4	5.3	38.5	48.5
	rms value of base moment (kNm)	3113.6	1836.6	1552.3	41.0	50.1
Band-limited white noise	rms value of top mass displacement (cm)	0.64	0.48	0.638	25.6	0.3
	rms value of base moment (kNm)	191.0	128.6	189.5	32.7	0.8

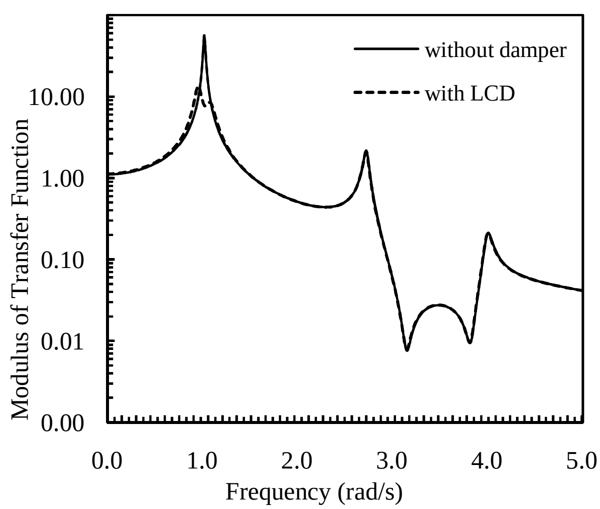

Figure 4.2. Displacement transfer function of top mass, with and without LCD (white noise input)

A further study is done on the control of the base shear of the structure. It is known that the higher modes of a structure affect the force responses more than the displacement response of the structure. In order to simulate the behavior of a structure with significant higher mode participation in the structural response, the same example structure is considered, but with different values of the modal damping ratios. It is assumed that the damping ratio in the first mode is equal to 0.05 while the damping ratio in the second and third modes is each equal 0.002. Again a white noise input of intensity 1cm^2/s^3 is considered. The orifice damping coefficient is determined corresponding to the minimum value of the base shear and is obtained as 4.0 for the T-P damper and as 67.0 for the LCD. The transfer function curves, relating the base shear to the input acceleration, are plotted in Figs. 4.3 and 4.4. The rms value of the base shear for the structure without damper is evaluated as 271.8 *kN* while that with the T-P damper and with the LCD are obtained as 214.7 *kN* and 227.5 *kN* respectively. This means that the T-P damper achieves a response reduction of 21% while the LCD reduces the response by 16%.

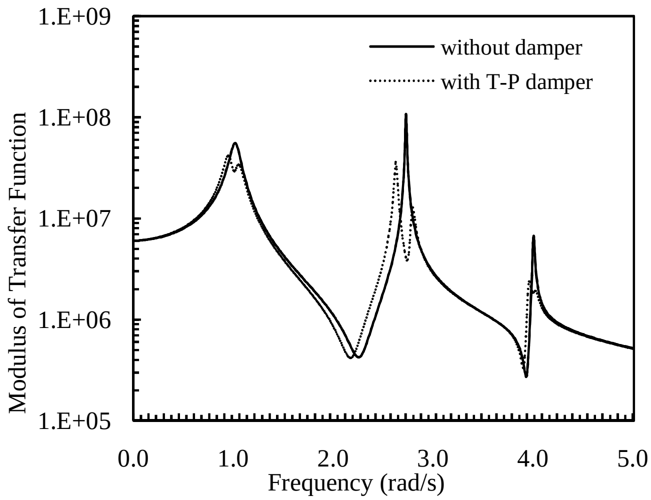

Figure 4.3. Transfer Function of base shear, with and without T-P damper (white noise input)

Next, an estimate is made of the amount of supplemental damping obtained due to the inclusion of the T-P damper, in terms of the increase in the equivalent viscous damping ratio of the structure. A comparison is also made with the LCD. The methodology adopted for the evaluation of the equivalent viscous damping ratio is as follows:

Step 1: The rms value of a chosen response parameter of the example MDOF structure is determined for a base input characterized by a given PSDF.

Step 2: An equivalent SDOF system is considered with damping ratio equal to the modal damping ratio of the MDOF structure and the same base input as in Step 1. The natural frequency of the SDOF system is determined such that it has the same rms value of the response parameter as calculated in Step 1.

Step 3: The MDOF structure with T-P damper is subjected to the same base input and the rms value of the response parameter is evaluated from the transfer function formulation given in Section 3.

Step 4: For the equivalent SDOF system with natural frequency calculated in Step 2, the equivalent viscous damping ratio is numerically evaluated such that the rms value of the response parameter matches that of Step 3. The difference in the damping ratio thus evaluated from the damping ratio of the SDOF system in Step 2 denotes the increase in the viscous damping ratio due to the inclusion of the T-P damper.

Steps 3-4 are repeated with the LCD instead of the T-P damper.

It may be noted that for white noise input and displacement as the chosen response parameter, the natural frequency of the equivalent SDOF system may be determined by using the standard closed form equation given by Newland (1993), else it has to be computed numerically.

Here, a 3-DOF example structure with natural frequencies as considered earlier is taken for a numerical illustration. The modal damping ratio is taken as 2%. The specifications of the T-P damper and LCD are as before. Different intensity levels of white noise seismic input are considered and for each case the values of the orifice damping coefficients chosen are those which minimize the rms values of the displacement of the top mass of the structure. Fig. 4.4 compares the optimal orifice damping coefficient of the T-P damper and of the LCD against the intensity of the white noise PSDF. It is observed that the variation in the optimal orifice damping coefficient is far less in case of the T-P damper as compared to the LCD. This indicates that the performance of the T-P damper is more robust since for a passive damper it is not possible to change the orifice diameter and hence the orifice damping coefficient once the damper has been installed.

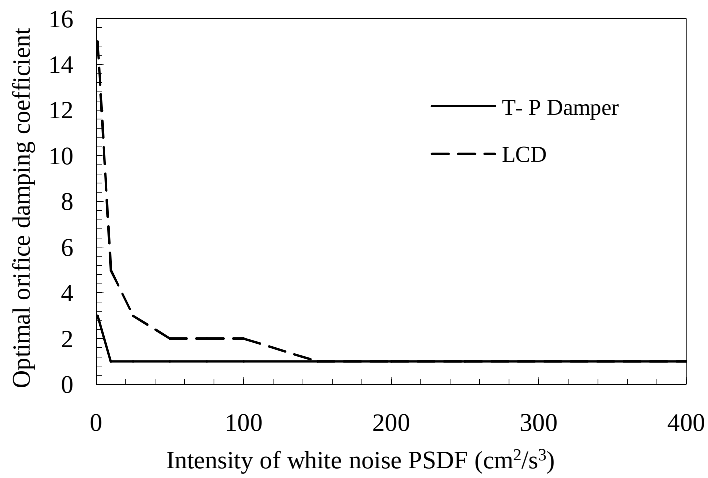

Figure 4.4. Variation in the orifice damping coefficient for the T-P damper and the LCD for different levels of seismic input

Considering the displacement of the top mass of the structure as the response parameter, the percent increase in the equivalent viscous damping ratio due to the T-P damper and the LCD for different levels of seismic input are evaluated and presented in Fig. 4.5.

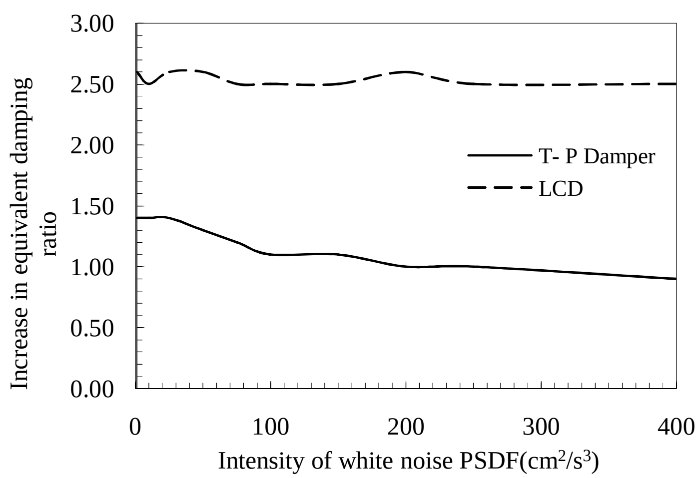

Figure 4.5. Increase in the equivalent viscous damping ratio of T-P damper and LCD for different intensities of white noise input, considering displacement as response parameter

The LCD is seen to provide higher supplemental damping than the T-P damper. The study is repeated by considering the base shear of the structure as the response parameter. Both white noise as well as band-limited white noise ranging between 0.5*s* and 2.5*s*, of varying intensities, are considered to characterize the seismic base input. The results are presented in Figs. 4.6 and 4.7. It is seen that for the white noise input, the T-P damper provides an additional viscous damping ratio of 1.1% to 1.7% while the LCD supplements the structural viscous damping by 2.8%. However, in case of the band-limited white noise, the T-P damper provides an increase of 5.7% to 6.5% in the equivalent viscous damping ratio across a wide range of different intensities while the LCD is rendered almost non-functional. So in this case the T-P damper is more effective as it can exercise control over the higher modes as well as over the fundamental.

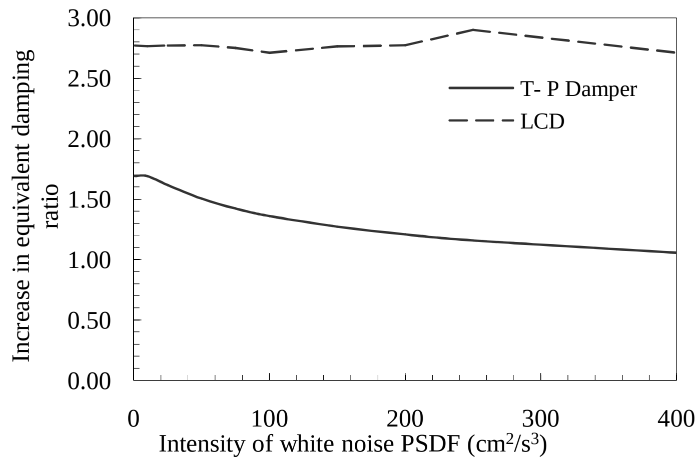

Figure 4.6. Increase in the equivalent viscous damping ratio of T-P damper and LCD for different intensities of white noise input, considering base shear as response parameter

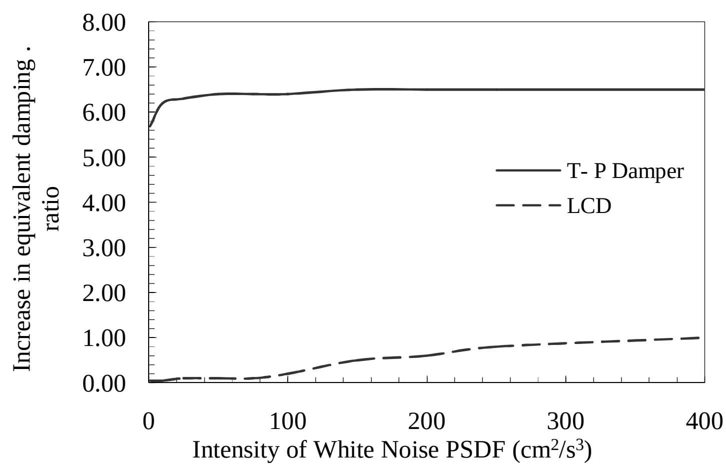

Figure 4.7. Increase in the equivalent viscous damping ratio of T-P damper and LCD for different intensities of band-limited white noise input, considering base shear as response parameter

5. CONCLUSIONS

A simple yet effective liquid damper, the T-P damper, combining the effects of the tank damper and the LCD, is proposed for controlling the seismic vibrations of structures. With appropriate mathematical modelling of the damper, a frequency domain formulation is developed for the transfer functions describing the input-output relation of a base-excited *n*-DOF structure with an attached T-P damper. Numerical studies indicate that for the control of responses which are dominated by a single mode, the performance of the T-P damper is comparable, though on the lower side, to that of the LCD. But in case of responses for which contributions from the second and third modes are not negligibly small, as may be in the case of force responses, the T-P damper, by dint of its multi-modal tuning capability, provides a better option. Especially in the case of ground motions with predominant energy content outside the frequency to which the LCD is tuned, the T-P damper achieves considerable vibration suppression, while the LCD is rendered practically ineffective. These observations are also reflected in the supplemental damping provided in terms of the equivalent viscous damping of the T-P damper and of the LCD. The optimal orifice damping coefficient of the T-P damper also shows little variation for a wide range of intensities of white noise PSDF. Thus, overall the proposed T-P damper presents a more robust damping performance as compared to the LCD.

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