Modal Analysis of Dynamic Behavior of Buildings Allowed to Uplift at Mid-story

T. Ishihara
Building Research Institute, Japan

T. Azuhata
National Institute for Land and Infrastructure Management, MLIT,Japan

M. Midorikawa
Hokkaido University, Japan

SUMMARY:
In this paper, dynamic behavior of buildings allowed to uplift at mid-story is investigated. The system considered is a two dimensional uniform shear-beam model allowed to uplift at mid-story. On account of piece-wise linear characteristics, classical modal analysis is applied to evaluate dynamic behavior during uplift. At first, the equations of motion are derived and eigenproblem is solved. Their free-vibrational responses under gravity are analyzed during the first excursion of uplift following to the first mode vibration in contact phase. Parameters are height/width ratio of whole buildings, height ratio of mid-story uplift system and intensity of vibration. The results show reduction effect due to uplift at mid-story with modal contributions to responses.

Keywords: Uplift at mid-story, Uniform shear beam, Story shear coefficient

1. INTRODUCTION

It has been pointed out that buildings during strong earthquakes have been subjected to foundation uplift (Rutenberg et al. 1982, Hayashi et al. 1999). Some studies dealing with foundation uplift in flexible systems have already been conducted (e.g. Muto et al. 1960, Meek 1975, 1978, Psycharis 1983, Yim et al. 1985). The authors also studied experimentally and analytically from the point of view of utilizing transient uplift motion for reduction of seismic-force response (e.g. Ishihara et al. 2006a, 2006b, 2007, 2008, 2010, Azuhata et al. 2008, 2009, Midorikawa et al. 2006, 2010).

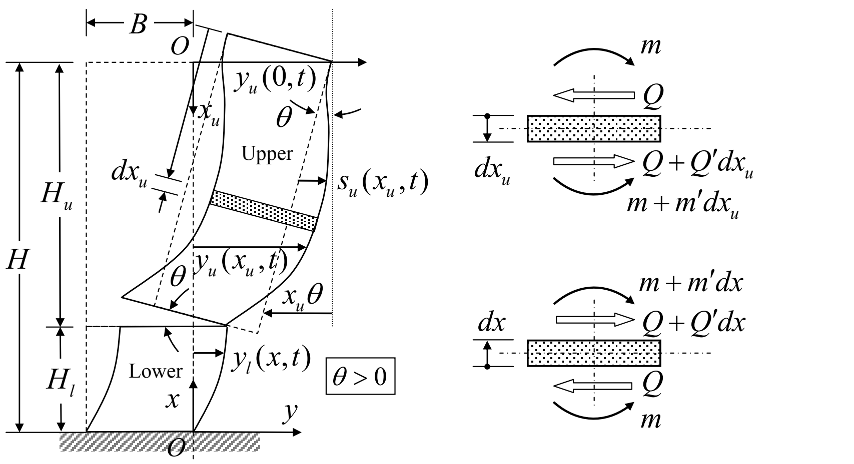
Structures allowed to uplift are sometimes called as “stepping”, and can be recognized as a kind of base isolated structures. Depending on building plans, etc., mid-story isolated system is sometimes more suitable to apply rather than base isolated system for some buildings. By analogy, structural systems allowed to uplift at mid-story may be useful and efficient for reduction of seismic damage of multi-story buildings.

In this paper, dynamic behavior of buildings allowed to uplift at mid-story is investigated. On account of piece-wise linear characteristics, classical modal analysis is applied to evaluate dynamic behavior during uplift.

2. SYSTEM CONSIDERED AND EIGENVALUE ANALYSIS

2.1. System considered

The system considered is a two dimensional uniform shear-beam model allowed to uplift at mid-story as shown in Fig. 2.1 with typical segments of infinitesimal height dx located at the distance x from the base and x_u from the top. Only the upper part of the system is allowed to uplift. Both parts have the same density and stiffness. The base of the system is fixed to the rigid ground. The lower and upper parts of the system are connected each other at height H_l from the base to have the same horizontal displacement. So, no deformation and slippage is allowed between the top of the lower part and the bottom of the upper part. It is assumed that the section is rigid and that displacement is small enough not to need to consider the so-called P–Δ effect. Generally speaking, an ordinary multi-story building does not have the constant story shear stiffness through the height. In slender buildings, deformations due to columns' shortening/elongation should be often considered. Nevertheless we adopt the uniform shear-beam model to make the mathematical expressions of motions as simple as possible and to reduce the number of structural parameters.



(a)Whole model (b) Portion of infinitesimal height dx

Figure 2.1. Mathematical model

2.2. Equation of motion and eigenvalue analysis

Equations of motion of the segment (see Fig. 2.1(b)) for free vibration during uplift are as follows:

$$\text{Horizontal:} \quad \rho A dx \cdot \ddot{y} = Q' dx \tag{2.1}$$

$$\text{Rotational:} \quad \rho A dx_u (B^2/3)\ddot{\theta} = -m' dx_u - Q dx_u \tag{2.2}$$

where ρA is the mass per unit height, $y = y(x, t)$ is horizontal displacement, $Q = Q(x,t) = rs'(x,t)$ is shear force, r is shear stiffness(r>0), $s = s(x, t)$ is horizontal displacement due to shear deformation defined as Eqn. 2.3 and 2.4, $\theta(t)$ is rotational angle of the section of the upper part, B is width, m is bending moment. The dots and primes signify differentiation with respect to time t and coordinate x (or x_u in the upper part). Eqn. 2.1 is used for both the upper (x is replaced by x_u) and the lower parts. Eqn. 2.2 is used only for the upper part.

$$\text{Upper:} \quad s_u(x_u, t) = y_u(x_u, t) - y_u(0, t) + x_u\theta(t) \tag{2.3}$$

$$\text{Lower:} \quad s_l(x, t) = y_l(x, t) \tag{2.4}$$

where the subscripts u and l mean the upper and the lower part respectively.

When the system is vibrating in one of its natural modes, the horizontal displacements and rotational angle may be taken in the form $y_u(x_u, t) = Y_u(x_u)q(t)$, $y_l(x, t) = Y_l(x)q(t)$ and $\theta(t) = \Theta q(t)$, where $q(t)$ is generalized coordinate. Let us denote the circular frequency by ω. For free vibration, $\ddot{q} = -\omega^2 q$. Considering the boundary conditions $Q_u(0, t) = 0$ at the top of the upper part and $y_l(0, t) = 0$ at the bottom of the lower part, the mode shapes (eigenvectors) $Y_u(x_u)$ and $Y_l(x)$ are defined as

$$Y_u(x_u) = D_u \cos\frac{\xi x_u}{H} - \Theta\frac{H}{\xi}\sin\frac{\xi x_u}{H} \tag{2.5}$$

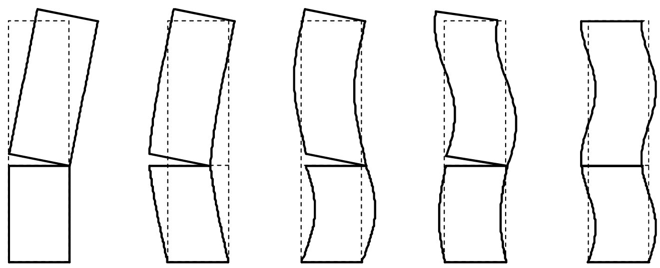
$$Y_l(x) = D_l \sin\frac{\xi x}{H} \tag{2.6}$$

where $\xi \equiv \omega H\sqrt{\rho A/r}$ is dimensionless frequency, H is the total height of the whole system, D_u and D_l are constants. Considering the boundary condition between the upper and the lower parts at height H_l for equilibrium of horizontal force and having the same horizontal displacement with the equation integrating Eqn. 2.2 over the height of the upper part H_u, we can obtain three equations for D_u, D_l and Θ. Let us denote the determinant of these three equations as det A. The frequency equation is

$$H_u \cdot \det A = (1-\cos\xi_u)(\sin\xi - \sin\xi_l) + \left(\frac{B^2}{3H^2}\xi^2 + \frac{\sin\xi_u}{\xi_u} - 1\right)\xi_u \cos\xi = 0 \tag{2.7}$$

where ξ_u is defined as $\xi_u \equiv (H_u/H)\xi$, ξ_l is $\xi_l \equiv (H_l/H)\xi$. We can get the dimensionless natural frequency ξ from Eqn. 2.7.

Fig. 2.2 shows examples of mode shapes for an uplifting phase. The first mode is rigid mode with zero frequency where the upper part rotates as a rigid block. The fifth mode's shape and frequency is nearly equal to those of the fourth mode for contact phase.



(a) 1st (b) 2nd (c) 3rd (d) 4th (e) 5th

Figure 2.2. Mode shape for an uplifting phase (H/B=4, H_l/H=0.4)

3. INITIAL VELOCITY ANALYSIS

To grasp the fundamental characteristics of dynamic behavior of rocking motion accompanied by uplift, initial velocity analysis is carried out utilizing the modal equations. Suppose that the system initially at rest is subjected to the impulsive horizontal forces having the same distribution shape along the whole height as that of the first mode in contact phase (i.e. fixed condition). The system begins to vibrate only in the first mode. If the system oscillates enough, the upper part of the structure begins to rock accompanied by uplift motion when the overturning moment reaches the resisting moment due to its self-weight, $M_u gB/2$, where M_u is the total mass of the upper part and g is gravitational acceleration. The analysis is conducted between the instant of initiation of uplift and that of landing, that is, the first excursion or half cycle of uplifting behavior. As a basic study, ground motions and damping of the system are neglected.

Parameters of the system are the height/width ratio of whole system H/B and the height ratio of mid-story uplift system H_l/H. The first natural frequency in contact phase (or shear wave velocity) is included in dimensionless formulation of equation of motion. Parameter for the intensity of oscillation is represented by the maximum base shear coefficient C_{Bf} where the system is not allowed to uplift. Only 3 kinds of parameters, H/B, H_l/H and C_{Bf}, are used in this study.

3.1. Critical base shear coefficient at the initiation of uplift

Base shear coefficient at the initiation of uplift for mid-story uplift system $C_{Bf1,cr}$ can be expressed as

$$C_{Bf1,cr} = F \cdot C_{Bf1,cr0} \tag{3.1}$$

$$F = \frac{1 - H_l/H}{1 - \sin\{(\pi/2)(H_l/H)\}} \tag{3.2}$$

$$C_{Bf1,cr0} = \frac{\pi B}{4H} \tag{3.3}$$

where $C_{Bf1,cr0}$ is the corresponding base shear coefficient for system allowed to uplift at the base (i.e. H_l=0).

Fig. 3.1 shows the function F in Eqn. 3.2. For example, F=1.16 for H_l/H=0.2 and F=1.46 for H_l/H=0.4. Mid-story uplift system is not so easy to uplift compared to base uplift system.

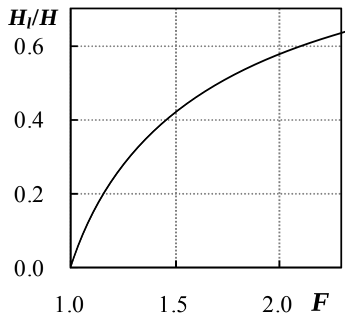


Figure 3.1. F in Eqn.3.2

3.2. Equation of motion

At first, let us introduce the dimensionless time τ and dimensionless pseudo acceleration a_δ (Meek 1978) as follows:

$$\tau \equiv \omega_{f1} t \tag{3.4}$$

$$a_\delta \equiv \omega_{f1}{}^2 \delta / g \tag{3.5}$$

where ω_{f1} is the first natural circular frequency in contact phase (or fixed condition not allowed to uplift) and δ is an arbitrary displacement.

We can express the horizontal displacements and rotation by superimposing the modal responses, i.e. $y_u = \sum Y_{uj} q_j$, $y_l = \sum Y_{lj} q_j$ and $\theta = \sum \Theta_j q_j$,where subscript j means jth mode. Substituting these equations into Eqns. 2.1 and 2.2, adding the effect of gravity $-\rho A dx_u \cdot g\, B/2$ (provided θ>0) to the right-hand side of Eqn. 2.2, and integrating and adding the Eqns. 2.1 and 2.2, considering the orthogonal property of the modes, the dimensionless equation of motion for mode j for an uplift phase can be derived as,

$$\frac{d^2 a_{yu0j}}{d\tau^2} + \left(\frac{\xi_j}{\xi_{f1}}\right)^2 a_{yu0j} = -\frac{D_{uj} \cdot Sv_j}{M_j} \tag{3.6}$$

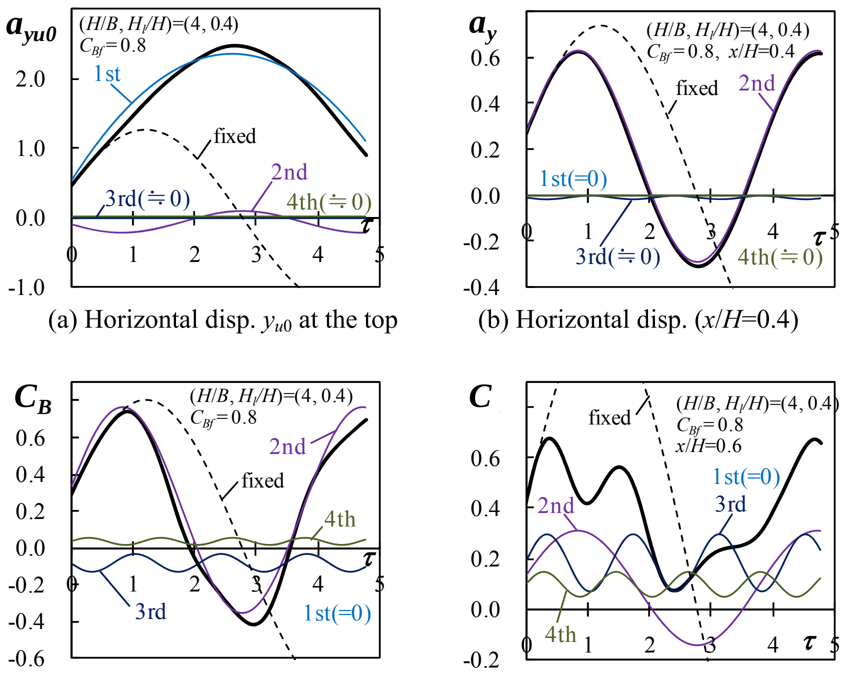
where $a_{yu0j} \equiv \omega_{f1}{}^2 y_{u0j}/g$, $y_{u0j} \equiv D_{uj} q_j$ is the horizontal displacement at the top of the jth mode,

$\xi_{f1} \equiv \omega_{f1} H\sqrt{\rho A/r}$, $Sv_j \equiv (H_u/H)(MB/2)\Theta_j$, $M \equiv \rho AH$ is the total mass of the whole system, M_j is the generalized mass of the jth mode. Note that the values of right-hand side of Eqn. 3.6 are independent of how the mode is normalized. The analytical solution of Eqn. 3.6 can be found easily.

For the representation of the results in the next section, we assume $t = \tau = 0$ at the instant of initiation of uplift and $\theta > 0$ after lift-off. Initial conditions of modes for an uplift phase can be derived using the orthogonal properties of modes with $\theta(0) = 0$ and $(d\theta/d\tau)_{\tau=0} = 0$. Up to 10 modes are used to calculate the responses.

3.3. Time histories and modal contributions

Fig. 3.2 shows examples of time histories of responses during the first uplifting excursion with the modal contributions. Thick lines show the responses calculated by the sum of modal responses up to 10 modes, and thin lines show modal contributions from the first to the fourth modes. The dotted lines show the corresponding response with fixed condition (i.e. not allowed to uplift). In the case of the figs., we assume H/B=4, H_l/H=0.4 and C_{Bf}=0.8. In this case, the duration of the first uplifting phase is τ=4.8 in the dimensionless time.



(a) Horizontal disp. y_{u0} at the top (b) Horizontal disp. (x/H=0.4)

(c)Base shear coefficient (d) Story shear coefficient (x/H=0.6)

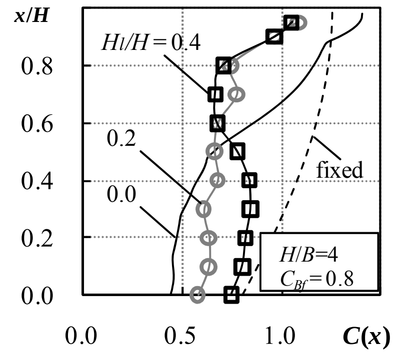
Figure 3.2. Time histories and modal contributions (H/B=4, H_l/H=0.4, C_{Bf}=0.8)

Horizontal displacement at the top (Fig.3.2(a)) is mainly dominated by the first mode and larger than that of fixed condition. On the other hand, horizontal displacement at x/H=0.4 (Fig.3.2(b)), where the uplift allowed, is dominated by the second mode.

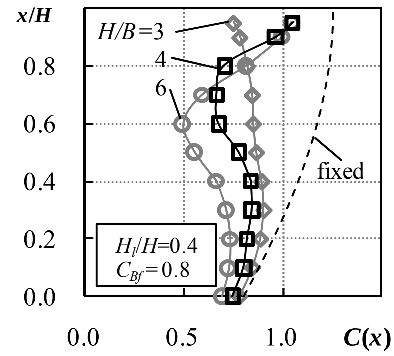
In Figs. 3.2(c) and (d), the first mode contribution is zero because the story shear is calculated from the deformation of the structure and the first mode is rigid mode (see Fig.2.2(a)). Base shear coefficient (Fig.3.2(c)) is dominated by the second mode and slightly less than that of fixed condition. Force reduction effect due to uplift is not so large for base shear. Fig.3.2(d) shows the story shear coefficient in the upper part at x/H=0.6 (x_u/H=1-x/H=0.4). The story shear is affected by higher modes including the third and fourth modes. The maximum story shear coefficient is much smaller than that of fixed condition.

3.4. Maximum story shear coefficient along the height

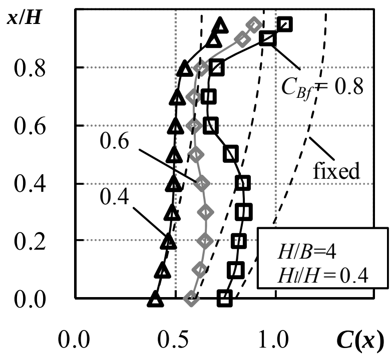
Fig. 3.3 shows the maximum story shear coefficient along the height. The ordinate of the graphs is dimensionless height x/H and the abscissa is the maximum story shear coefficient C. The dotted lines show the corresponding results under fixed condition vibrating only in its first mode.



(a) Effect of H_l/H (H/B=4, C_{Bf}=0.8)



(b) Effect of H/B (H_l/H =0.4, C_{Bf}=0.8)



(c) Effect of C_{Bf} (H/B=4, H_l/H =0.4)

Figure 3.3. Maximum story shear coefficient along the height

Fig. 3.3(a) compares the effect of the height ratio of mid-story uplift system, H_l/H. In the region about x/H<0.5, the story shear coefficients are increased with increase of the height ratio H_l/H, whereas those in x/H>0.5 are decreased.

Fig. 3.3(b) compares the effect of the height/width ratio H/B. The story shear coefficients are decreased with increase of H/B except the region near the top. In the case of H/B=6, the distribution of story shear coefficients along the height has a sharply bent shape.

Fig. 3.3(c) compares the effect of intensity of vibration represented by C_{Bf}. The force reduction effect due to uplift cannot be seen enough near the base and the top. The minimum story shear coefficient along the height appears in the region around x/H=0.6, which is a little bit higher region than mid-story uplift system installed at H_l/H=0.4.

4. CONCLUSIONS

In this paper, dynamic uplifting behavior of buildings allowed to uplift at mid-story is investigated by means of classical modal analysis. Uniform shear-beam model is used as a representation of multi-story buildings allowed to uplift. The dynamic behavior during an uplift excursion is clarified by conducting the initial velocity analysis as the sum of modal responses to be able to calculate analytically. From the results, the conclusions are summarized as follows:

(1) Horizontal displacements in the upper part of buildings are dominated mainly by the first (rigid body) mode, whereas those in the lower part are affected higher modes.
(2) Story shears are reduced compared to those without uplift, especially in the upper part.

ACKNOWLEDGEMENT
This work was supported by JSPS KAKENHI (24560707, 21560607, 22360219).

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