The Problem of "Burnt Rubble" in Fire Following Earthquake

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SUMMARY
Catastrophe risk modeling usually deals with the derivation of the probability distribution of the damage to a portfolio of assets due to a given hazard, where damage represents the cost to restore the assets to their pre-damaged condition. In the case of a single peril, such as earthquake ground shaking, the estimation of the damage to an asset comes down to the modeling of its vulnerability - that is, the level of the damage given the intensity of the ground shaking. But there are cases when a single event can cause multi-peril damage to the assets. For example, earthquake ground shaking, fire following and tsunami, hurricane wind and flooding, tornado wind and hail, etc. In such cases, the definition of the damage as the cost to restore the asset to its pre-damaged condition still holds, except the cost should include the total cost required to restore the damage from all perils without double counting. Double counting refers to the overlap of the estimation of damage from multiple perils, e.g. fire damage to a building that has been already destroyed due to earthquake ground shaking. The latter is known as the problem of "burnt rubble". This paper presents an approach for modeling the combined damage from multiple perils without double counting. The developed framework derives the mean and the full probability distribution function of the combined damage for multiple perils. Consideration is given to cases where damage from one peril could impact the vulnerability of the asset to subsequent perils.

Keywords: earthquake, fire following earthquake, catastrophe modeling, insurance

1. INTRODUCTION

Earthquake ground shaking and fire following can be devastating. Problem of burning the rubble arises in catastrophe modeling when each peril is modeled and analyzed independently. One way of doing this would be to modify the portfolio to account for the damage of the primary peril, e.g. shaking, and then analyzing the modified portfolio for the second peril, e.g. fire, where the modified portfolio will have the reduced replacement values accounting for the damage sustained from the primary peril. In most cases, this approach is not practical as it is cumbersome to implement, and because the user may not always have access to the tools used for modeling and simulating the losses for each peril individually.

It is with the above consideration that we present a methodology in this paper that would allow the user to combine the results from the analysis of the individual perils without any need of reanalyzing them. At the same time, the presented methodology accounts for the effect of the 'burnt rubble' and avoids double counting.

2. DAMAGE, DAMAGE RATIO AND VULNERABILITY

Risk analysis usually deals with the derivation of the probability distribution of the damage to a given asset or to portfolio of assets. Where damage represents the monetary payout required to restore the asset to its pre-damaged condition. We denote the random variable representing the damage by D and its probability distribution by $F_D(d)$, where $F_D(d) = P(D \leq d)$ is the probability that the damage, D, is less than or equal to the given value, d. The details of the derivation of $F_D(d)$, is out of the scope of this study.

If the total replacement value of the asset *A* is the constant C_A, then the damage can be expressed as

$$d = C_A \cdot r \tag{2.1}$$

where *r* is the damage ratio. Note that there is a one-to-one mapping between the damage, *d*, and damage ratio, *r*, therefore the probability distribution of the damage ratio, *r*, can be written as

$$F_R(r) = F_D(C_A \cdot r) \tag{2.2}$$

The damage incurred by an asset during a catastrophe event depends on the hazard intensity and asset fragility function. The hazard intensity can be measured and represented by a vector of hazard parameters, $\boldsymbol{z}$, and the fragility function, $F_R(r|\boldsymbol{z})$. The latter is the conditional probability distribution of the damage ratio, *r*, for a given hazard, **z**.

The most commonly used form for defining the damageability of an asset is the vulnerability function which provides the mean damage ratio for a given level of hazard, **z**.

$$V(\boldsymbol{z}) = E[r|\boldsymbol{z}] \tag{2.3}$$

3. COMBINING DAMAGES FROM MULTIPLE PERILS

As it was discussed in the previous section in the case of a single peril, e.g. damage due to earthquake ground shaking, the primary objective is the derivation of the probability distribution $F_D(d)$, where *D* represents the cost required to restore the asset to its pre-damaged condition.

However, there are cases when a single event can cause multi-peril damage to the given asset. For example, earthquake ground shaking and fire following, hurricane wind and flooding, tornado wind and hail, etc. In such cases, the definition of the damage to the asset as the total cost required to restore the asset to its pre-damaged condition still holds, except the damage in this case represents the total combined damage, caused by all individual perils, without double counting.

In order to calculate the combined damage from all perils we apply each peril to the asset *A* individually, in consecutive order. The subsequent perils are applied to the residual replacement value of the asset *A* only. This will guarantee that we do not double count for the same damage to asset *A* from multiple perils.

The damage ratio for the *k*-th peril can be expressed in the following form

$$r_{k|1,2,\ldots,k-1} = \frac{d_k}{C_A - \sum_{j=1}^{k-1} d_j} \tag{3.1}$$

where $r_{k|1,2,\ldots,k-1}$ is the conditional damage ratio, given the damages due to the previous perils and is a function of the damage from prior perils.

In general, the damage caused by a particular peril could depend on the damage caused by other perils, e.g. the level of flood damage due to rain would depend on the level of roof damage due to wind. Therefore, the combined damage depends on the order the particular perils impact asset *A*.

Figure 1 illustrates a simple example of the application of two perils, 1 and 2, in consecutive order. The damage from peril 1, can be expressed in terms of the replacement value, C_A, and the damage ratio, r_1, as defined in (2.1).

$$d_1 = C_A \cdot r_1 \tag{3.2}$$

and, using (3.1), the damage from the second peril, can be written in the following form

$$d_2 = (1 - r_1)C_A \cdot r_{2|1} \tag{3.3}$$

where $(1 - r_1)C_A = C_A - \mathrm{d}_1$ is the residual replacement value after the damage caused by peril 1. This residual replacement value is equivalent to the shaded area in Figure 1. Note that since d_2 is the damage incurred only by the residual replacement value of asset *A*, d_1 and d_2 are disjoint sets.

The damage due to the subsequent *k*-th peril can be expressed as

$$d_k = \prod_{j=1}^{k-1} \left(1 - r_{(j|1,2,\dots j-1)}\right) \cdot C_A \cdot r_{k|1,2,\dots,k-1} \tag{3.4}$$

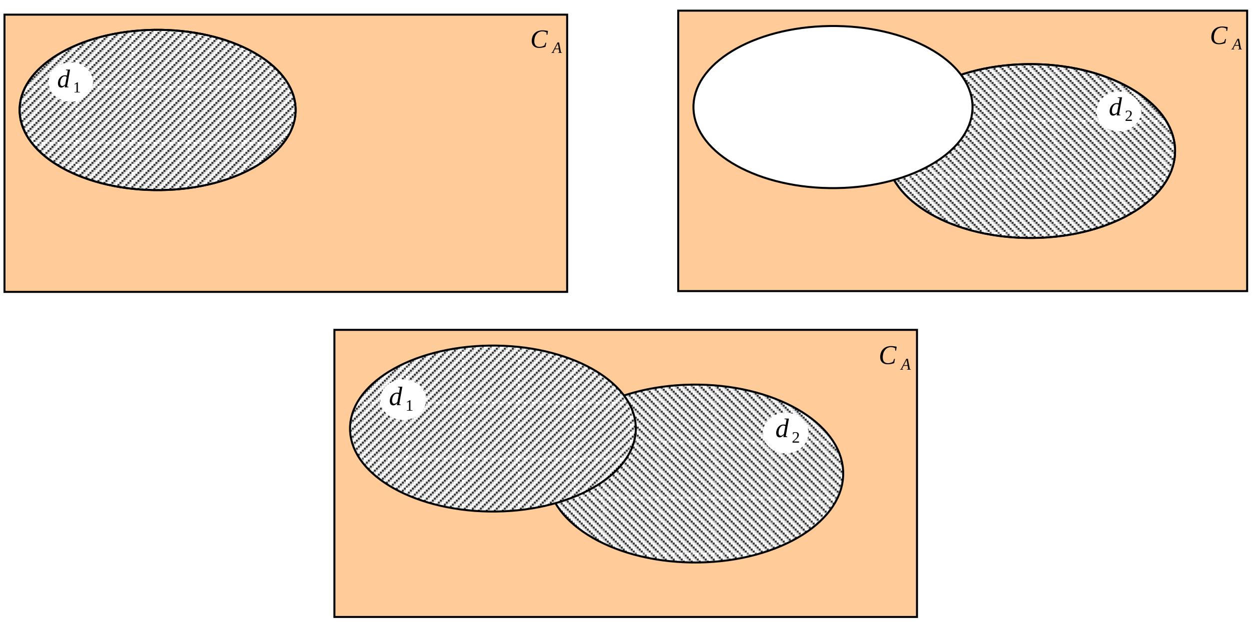

Figure 1. Venn diagrams for peril 1 and 2.

And the total combined damage from all *m* perils is,

$$d_{[m]} = \sum_{k=1}^{m} d_k = \left(r_1 + \sum_{k=1}^{m} r_{k|1,2,\dots,k-1} \cdot \prod_{j=1}^{k-1} \left(1 - r_{(j|1,2,\dots j-1)}\right) \right) \cdot C_A \tag{3.5}$$

In the above equation and hereafter the square bracketed indices, [*m*], indicate the combination of all the perils from 1 to *m*.

Note that (3.5) can be written in the form of (3.2)

$$d_{[m]} = r_{[m]} \cdot C_A \tag{3.6}$$

where

$$r_{[m]} = r_1 + \sum_{k=1}^{m} r_{k|1,2,\dots,k-1} \cdot \prod_{j=1}^{k-1} \left(1 - r_{j|1,2,\dots,j-1}\right) = 1 - \prod_{k=1}^{m} \left(1 - r_{k|1,2,\dots,k-1}\right) \tag{3.7}$$

is the combined damage ratio from all *m* perils.

Assuming that the damage caused by one peril has no impact on the vulnerability of the asset to the subsequent perils, that is

$$r_{k|1,2,\dots,k-1} = r_k \tag{3.8}$$

the combined damage ratio, given in (3.7), will simplify to

$$r_{[m]} = 1 - \prod_{k=1}^{m} (1 - r_k) \tag{3.9}$$

For the most common case, when m= 2, (3.7) and (3.9) simplify to

$$r_{[2]} = r_1 + r_{2|1} - r_1 \cdot r_{2|1} \tag{3.10}$$

and

$$r_{[2]} = r_1 + r_2 - r_1 \cdot r_2 \tag{3.11}$$

respectively.

Note that (3.9) and (3.11) are true if and only if the damage due to one peril has no impact on the vulnerability of the asset due to the other perils.

4. MEAN AND VARINACE OF THE COMBINED DAMAGE RATIO

Since the combined damage and combined damage ratio are functions of random variables, namely, functions of the damage and damage ratio of the individual perils, using principals of probability theory, one can derive the probability distributions of the combined damage and damage ratio. However, in most cases in practice the evaluation of the mean and the variance of the combined damage or the combined damage ratio will suffice.

Based on Ang et al. (1984), Augusti et al. (1984) it can be shown that using series expansion of the definition of the combined damage ratio in (3.7), and retaining only the linear elements, one can obtain the first order approximation of the mean and the variance of the combined damage ratio due to the *m* perils as

$$E[r_{[m]}] = 1 - \prod_{k=1}^{m} \left(1 - E[r_{k|1,2,\ldots k-1}\right) \tag{4.1}$$

and

$$\mathrm{Var}[r_{[m]}] = (\nabla_{\mathbf{r}})^T_{\mathbf{r}=E[\mathbf{r}]} \cdot \mathbf{\Sigma}_{\mathbf{r}\,\mathbf{r}} \cdot (\nabla_{\mathbf{r}})_{\mathbf{r}=E[\mathbf{r}]} \tag{4.2}$$

where

$$(\nabla_{\mathbf{r}})^T_{\mathbf{r}=E[\mathbf{r}]} = \begin{bmatrix} \prod_{k=2}^{m} \left(1 - E[r_{k|1,2,\ldots k-1}]\right) \\ \prod_{\substack{k=1 \\ k\neq 2}}^{m} \left(1 - E[r_{k|1,2,\ldots k-1}]\right) \\ \prod_{\substack{k=1 \\ k\neq 3}}^{m} \left(1 - E[r_{k|1,2,\ldots k-1}]\right) \\ \vdots \\ \prod_{k=1}^{m-1} \left(1 - E[r_{k|1,2,\ldots k-1}]\right) \end{bmatrix} \tag{4.3}$$

In the above equation, $(\nabla_{\mathbf{r}})_{\mathbf{r}=E[\mathbf{r}]}$ is the gradient of the vector of damage ratios $\mathbf{r}$, evaluated at the mean point $E[\mathbf{r}]$ **,** and $E[r_{k|1,2,\ldots k-1}]$ represents the mean damage ratio due to the k-th peril given the damage ratio of the prior $k-1$ perils, and . $\mathbf{\Sigma}_{\mathbf{r}\,\mathbf{r}}$ is the covariance matrix of the damage ratios.

4.1 Shake and Fire Following Earthquake

First we note that the probability that a building will self-ignite is at least an order of magnitude smaller than that of it burning down due to fire spread that has started in some other building in the neighborhood. This is the main reason why the fire following earthquake losses are most often observed in dense urban areas.

Given the above observation it is safe to assume that the events of shake damage and fire following damage for a given building are statistically independent.

$$r_{\text{fire}\,|\,\text{shake}} = r_{\text{fire}} \tag{4.4}$$

That is, we are assuming that the effect of shake damage on the vulnerability of the building to fire hazard is negligible.

Note that although we have assumed that the fire vulnerability of the building is indifferent to its sustained shake damage, the losses due to shake and fire can still be statistically correlated. We will denote the correlation between the shake and fire losses by ρ.

Given that we have analyzed the building for shake and fire following earthquake and obtained the respective means and variances of the damage ratios, $E[r_{\text{shake}}]$, $\mathrm{Var}[r_{\text{shake}}]$ and $E[r_{\text{fire}}]$, $\mathrm{Var}[r_{\text{fire}}]$, respectively, then the first order approximation of the mean combined damage ratio is,

$$E[r_{\text{fire + shake}}] \quad = 1 - (1 - E[r_{\text{shake}}])(1 - E[r_{\text{fire}}]) \\ = E[r_{\text{shake}}] + E[r_{\text{fire}}] - E[r_{\text{shake}}]E[r_{\text{fire}}] \tag{4.5}$$

and the first order approximation of the variance of the combined damage ratio is

$$\text{Var}[r_{\text{shake + fire}}] = (1 - E[r_{\text{fire}}])^2 \, \text{Var}[r_{\text{shake}}] + (1 - E[r_{\text{shake}}])^2 \, \text{Var}[r_{\text{fire}}] + \\ +2\rho \, (1 - E[r_{\text{fire}}])(1 - E[r_{\text{shake}}]) \, \text{Var}[r_{\text{shake}}]^{\frac{1}{2}} \, \text{Var}[r_{\text{fire}}]^{\frac{1}{2}} \tag{4.6}$$

where $E[.]$ and $Var[.]$ can be derived from (4.1) and(4.3), respectively. We note the following limits for the mean and variance of the combined damage ratio

$$\lim_{E[r_{\text{shake}}]\to 1} \text{E}[r_{\text{fire + shake}}] = 1 \qquad \lim_{E[r_{\text{shake}}]\to 1} \text{Var}[r_{\text{fire + shake}}] \to 0 \tag{4.7}$$

and

$$\lim_{E[r_{\text{fire}}]\to 1} \text{E}[r_{\text{fire + shake}}] = 1 \qquad \lim_{E[r_{\text{fire}}]\to 1} \text{Var}[r_{\text{fire + shake}}] \to 0 \tag{4.8}$$

as well as

$$\lim_{E[r_{\text{shake}}]\to 0} \text{E}[r_{\text{fire + shake}}] = \text{E}[r_{\text{fire}}] \qquad \lim_{E[r_{\text{shake}}]\to 0} \text{Var}[r_{\text{fire + shake}}] \to \text{Var}[r_{\text{fire}}] \tag{4.9}$$

and

$$\lim_{E[r_{\text{fire}}]\to 0} \text{E}[r_{\text{fire + shake}}] = \text{E}[r_{\text{shake}}] \qquad \lim_{E[r_{\text{fire}}]\to 0} \text{Var}[r_{\text{fire + shake}}] \to \text{Var}[r_{\text{shake}}] \tag{4.10}$$

In the above limits we have made use of the following lemma.

Lemma: If the random variable *X* is defined in the interval $\lfloor a, b \rfloor$, then

$$\lim_{E[x]\to a} \text{Var}[x] = 0 \qquad \text{and} \qquad \lim_{E[x]\to b} \text{Var}[x] = 0 \tag{4.11}$$

where $E[x]$ is the mean of the random variable *X*.

Proof: By definition

$$E[x] = \int_a^b x \, f_X(x) \, dx \tag{4.12}$$

Therefore $E[x] = a$ or $E[x] = b$ if and only if $f_X(x) = \delta(x - a)$ or $f_X(x) = \delta(x - b)$, respectively. Where $\delta(t)$ is the Direc delta function. That is when the entire probability mass is concentrated at the respective end of the given interval. Hence, if $E[x] = a$, then

$$Var[x] = \int_a^b (x - a)^2 \, f_X(x) \, dx = \int_a^b (x - a)^2 \, \delta(x - a) dx = (a - a)^2 = 0 \tag{4.13}$$

It is easy to see that similar result can be obtained for $E[x] = b$.

5. APPLICATION IN CATASTROPHE MODELING – AN EXAMPLE

Catastrophe modeling is the process of using computer-assisted calculations to estimate the monetary damage that could be sustained by an asset or portfolio of assets due to a catastrophic event such as earthquake. Most available catastrophe models are based on catalogs of hypothetical events (e.g. earthquakes) that have the potential to cause damage to a given asset or a portfolio of assets under consideration. Most common output of the analysis is the report of the losses of individual assets or a combination thereof (policy, zip code, county, etc) due to a given peril (e.g. earthquake ground shaking, fire following earthquake, etc.) for the events in the above mentioned hypothetical events catalog.

As an illustration of the presented methodology we have randomly simulated shake and fire losses for 30 sites, which are located in three zip codes and two counties.

Table 5.1 Event Loss,($), Table for Damage due to Ground Shaking.

Site	zip	Value	Mean Loss			Loss Ratio			Loss Standard Deviation		
			Event 1	Event 2	Event 3	Event 1	Event 2	Event 3	Event 1	Event 2	Event 3
1	1	59	27.9	33.7	27.4	0.47	0.57	0.46	42.5	45.1	43.8
2	1	335	73.7	284.2	9.8	0.22	0.85	0.03	66.0	288.6	14.3
3	1	85	25.6	78.1	36.4	0.30	0.92	0.43	28.6	28.9	20.9
4	1	326	302.4	178.4	288.7	0.93	0.55	0.89	77.3	64.2	139.6
5	1	268	96.3	21.7	77.4	0.36	0.08	0.29	100.2	7.1	95.5
6	1	219	110.7	205.1	95.0	0.51	0.94	0.43	26.0	96.9	39.0
7	1	117	18.5	44.6	87.5	0.16	0.38	0.75	11.6	28.4	50.1
8	1	191	190.7	173.5	111.8	1.00	0.91	0.59	12.8	132.7	170.5
9	1	196	118.2	105.2	101.5	0.60	0.54	0.52	113.2	84.7	22.8
10	2	336	160.7	313.6	297.5	0.48	0.93	0.89	124.9	144.2	190.0
11	2	67	37.5	40.7	58.5	0.56	0.61	0.87	54.9	9.4	75.7
12	2	200	117.2	20.1	167.1	0.59	0.10	0.84	187.3	18.3	145.6
13	2	314	72.6	237.5	162.2	0.23	0.76	0.52	103.6	70.5	180.9
14	2	241	72.4	118.3	43.8	0.30	0.49	0.18	65.9	161.7	60.4
15	2	179	123.6	64.7	40.1	0.69	0.36	0.22	144.3	108.4	31.7
16	2	215	80.9	84.2	197.4	0.38	0.39	0.92	98.9	83.2	227.5
17	2	115	9.0	22.6	49.7	0.08	0.20	0.43	9.5	36.6	16.5
18	2	245	222.2	152.2	175.2	0.91	0.62	0.72	228.8	232.9	270.3
19	2	414	112.2	405.2	43.7	0.27	0.98	0.11	163.0	270.2	73.7
20	2	436	362.3	69.0	329.4	0.83	0.16	0.76	390.3	47.9	176.5
21	2	161	151.7	42.8	136.5	0.94	0.27	0.85	33.6	58.5	87.2
22	2	165	80.8	94.5	69.5	0.49	0.57	0.42	101.1	143.6	115.5
23	2	133	110.4	40.8	47.2	0.83	0.31	0.36	71.5	48.6	20.3
24	2	433	130.6	314.6	18.9	0.30	0.73	0.04	208.1	414.7	11.9
25	3	449	11.0	339.5	229.8	0.02	0.76	0.51	11.5	401.7	74.6
26	3	115	110.6	42.6	10.5	0.96	0.37	0.09	18.9	52.4	9.5
27	3	185	98.0	71.8	27.9	0.53	0.39	0.15	29.4	77.7	31.0
28	3	363	174.2	133.8	318.8	0.48	0.37	0.88	152.0	209.8	376.8
29	3	299	231.2	84.5	97.7	0.77	0.28	0.33	98.6	118.6	67.9
30	3	423	165.3	174.6	322.0	0.39	0.41	0.76	128.4	190.2	377.7
	Totals	7,284	3,598	3,992	3,679						

Table 5.2 Event Loss Table for Damage due to Fire Following Earthquake.

Site	zip	Values	Mean Loss			Loss Ratio			Loss Standard Deviation		
			Event 1	Event 2	Event 3	Event 1	Event 2	Event 3	Event 1	Event 2	Event 3
1	1	59	16.2	32.9	-	0.27	0.56	-	7.8	14.4	-
2	1	335	280.7	256.7	236.6	0.84	0.77	0.71	130.5	504.7	349.0
3	1	85	-	20.9	-	-	0.25	-	-	41.2	-
4	1	326	-	-	91.4	-	-	0.28	-	-	146.8
5	1	268	79.9	-	-	0.30	-	-	112.5	-	-
6	1	219	-	138.7	-	-	0.63	-	-	155.4	-
7	1	117	-	-	-	-	-	-	-	-	-
8	1	191	160.8	164.6	87.3	0.84	0.86	0.46	218.4	255.8	166.1
9	1	196	89.8	-	-	0.46	-	-	86.6	-	-
10	2	336	-	-	298.4	-	-	0.89	-	-	211.7
11	2	67	-	-	10.8	-	-	0.16	-	-	17.3
12	2	200	149.6	-	-	0.75	-	-	288.8	-	-
13	2	314	-	-	120.0	-	-	0.38	-	-	205.6
14	2	241	-	-	120.5	-	-	0.50	-	-	177.8
15	2	179	-	-	9.7	-	-	0.05	-	-	13.8
16	2	215	-	3.1	-	-	0.01	-	-	2.7	-
17	2	115	-	108.4	39.3	-	0.94	0.34	-	29.8	57.6
18	2	245	45.1	-	-	0.18	-	-	29.7	-	-
19	2	414	207.0	-	348.6	0.50	-	0.84	293.9	-	297.7
20	2	436	-	296.7	334.1	-	0.68	0.77	-	540.2	409.1
21	2	161	-	5.9	-	-	0.04	-	-	15.7	-
22	2	165	-	41.4	-	-	0.25	-	-	74.2	-
23	2	133	-	-	76.7	-	-	0.58	-	-	67.1
24	2	433	328.9	-	291.0	0.76	-	0.67	198.8	-	630.0
25	3	449	390.4	234.4	69.8	0.87	0.52	0.16	702.2	344.7	70.1
26	3	115	69.9	-	24.2	0.61	-	0.21	155.4	-	58.6
27	3	185	4.8	-	-	0.03	-	-	9.8	-	-
28	3	363	-	69.6	195.3	-	0.19	0.54	-	48.6	175.8
29	3	299	253.7	-	-	0.85	-	-	239.6	-	-
30	3	423	-	26.0	399.0	-	0.06	0.94	-	45.1	431.1
Totals:		7,284	2,077	1,399	2,752						

Using (4.5) and (4.6) the combined losses for ground shaking and fire following without double counting for the effect of the "burnt rubble" for all 30 sites are listed in Table 5.3, below. For this example, we have assumed the correlation between the losses due to ground shake and fire following earthquake to be 0.5.

Table 5.3 Event Loss Table for Combined Damage due to Ground Shaking and Fire Following Earthquake.

Site	zip	TIV	Mean Loss			Loss Ratio			Loss Standard Deviation		
			Event 1	Event 2	Event 3	Event 1	Event 2	Event 3	Event 1	Event 2	Event 3
1	1	59	36.4	47.8	27.4	0.62	0.81	0.46	31.1	20.9	43.8
2	1	335	292.6	323.1	239.4	0.87	0.96	0.71	102.4	102.0	338.8
3	1	85	25.6	79.8	36.4	0.30	0.94	0.43	28.6	22.1	20.9
4	1	326	302.4	178.4	299.2	0.93	0.55	0.92	77.3	64.2	101.9
5	1	268	147.5	21.7	77.4	0.55	0.08	0.29	100.7	7.1	95.5
6	1	219	110.7	213.9	95.0	0.51	0.98	0.43	26.0	36.9	39.0
7	1	117	18.5	44.6	87.5	0.16	0.38	0.75	11.6	28.4	50.1
8	1	191	191.0	188.6	148.0	1.00	0.99	0.77	2.0	29.7	115.4
9	1	196	153.8	105.2	101.5	0.78	0.54	0.52	70.3	84.7	22.8

10	2	336	160.7	313.6	331.7	0.48	0.93	0.99	124.9	144.2	32.2
11	2	67	37.5	40.7	59.9	0.56	0.61	0.89	54.9	9.4	63.6
12	2	200	179.1	20.1	167.1	0.90	0.10	0.84	128.5	18.3	145.6
13	2	314	72.6	237.5	220.2	0.23	0.76	0.70	103.6	70.5	149.6
14	2	241	72.4	118.3	142.4	0.30	0.49	0.59	65.9	161.7	148.6
15	2	179	123.6	64.7	47.7	0.69	0.36	0.27	144.3	108.4	31.9
16	2	215	80.9	86.0	197.4	0.38	0.40	0.92	98.9	82.0	227.5
17	2	115	9.0	109.7	72.1	0.08	0.95	0.63	9.5	24.0	34.5
18	2	245	226.4	152.2	175.2	0.92	0.62	0.72	186.7	232.9	270.3
19	2	414	263.1	405.2	355.5	0.64	0.98	0.86	229.2	270.2	266.6
20	2	436	362.3	318.8	411.1	0.83	0.73	0.94	390.3	455.0	108.2
21	2	161	151.7	47.1	136.5	0.94	0.29	0.85	33.6	57.6	87.2
22	2	165	80.8	112.2	69.5	0.49	0.68	0.42	101.1	112.2	115.5
23	2	133	110.4	40.8	96.7	0.83	0.31	0.73	71.5	48.6	44.1
24	2	433	360.3	314.6	297.2	0.83	0.73	0.69	147.6	414.7	602.5
25	3	449	391.8	396.7	263.9	0.87	0.88	0.59	685.0	209.6	71.7
26	3	115	113.3	42.6	32.5	0.99	0.37	0.28	9.5	52.4	53.8
27	3	185	100.2	71.8	27.9	0.54	0.39	0.15	29.0	77.7	31.0
28	3	363	174.2	177.8	342.6	0.48	0.49	0.94	152.0	172.3	175.4
29	3	299	288.7	84.5	97.7	0.97	0.28	0.33	56.4	118.6	67.9
30	3	423	165.3	189.8	417.3	0.39	0.45	0.99	128.4	180.5	105.1
	Totals:	7,284	4,803	4,547	5,074						

There are cases where only the aggregate results are available, for example at zip code or county level. One might still use the above presented method to combine the losses from ground shaking and fire following, but have to be aware that they could be different from the site by site combined results.

For this illustrative example we have compared the mean losses of the combined shake and fire for the same portfolio using results at the site, zip code and county level. For this example, the error introduced by using the aggregate results is between -3.0% and 1.2%.

Table 5.4 Combining Shake at Fire at zip code Aggregation Level.

Zip		Shake			Fire			Shake + Fire		
	TIV	Event 1	Event 2	Event 3	Event 1	Event 2	Event 3	Event 1	Event 2	Event 3
1	1,796	964	1,124	836	627	614	415	1,254.5	1,353.9	1,057.6
2	3,654	1,844	2,021	1,837	731	456	1,649	2,206.0	2,224.2	2,657.0
3	1,834	790	847	1,007	719	330	688	1,199.3	1,024.3	1,317.1
	7,284	3,598	3,992	3,679	2,077	1,399	2,752	4,660	4,602	5,032
Results from Site-by-Site Analysis from Table 6.3								4,803	4,547	5,074
Difference between aggregate and site-by-site results								-3.0%	1.2%	-0.8%

6. SUMMARY AND CONCLUSIONS

A method for combing monetary losses from earthquake ground shaking and fire following that avoids double counting, also known as ‘the problem of burnt ruble’ has been presented. The advantage of the presented methodology is that it allows to combined losses due to earthquake ground motion, “shake”, and fire following earthquake, “fire”, when one has access only to the event loss table, i.e. the losses for the list of hypothetical events.

It has been demonstrated that the method can be applied to aggregate results with acceptable accuracy. The resulting combined event loss tables for “shake+fire” can be used to derive loss exceedance probabilities and other statistics, that is out of the scope of this paper.

REFERENCES

Ang, A. H.-S., and Tang, W. H. (1984). Probability concepts in engineering planning and design, Vol. II: Decision, risk, and reliability, Wiley, New York, USA.

Augusti, G., Baratta, A., and Casciati, F., (1984). Probabilistic Methods in Structural Mechanics, Chapman & Hall, London, UK.

Der Kiureghian, A., and De Stefano, M. (1991). Efficient algorithm for second-order reliability analysis. *J. Eng. Mech*., **117:12**, 2904–2923.

Der Kiureghian, A., Lin, H. Z., and Hwang, S. J. (1987). Second-order reliability approximations. *J. Eng. Mech*., **113:8**, 1208–1225.

Grigoriu, M. (1983). Approximate analysis of complex reliability analysis. *Struct. Safety*, **1:4**, 277–288.

Hasofer, A. M., and Lind, N. C. (1974). Exact and invariant second moment code format. *J. Engrg. Mech. Div*., **100:1**, 111–121.

Hong, H. P. (1996). Point-estimate moment-based reliability analysis. *Civ. Eng. Syst*., **13**, 281–294.

Thoft-Christensen, P. and Murotsu, Y., (1986). Structural Reliability Theory and Its Applications, Springer Verlag, Berlin, Germany.