Simultaneous Design of the Low Order Structural Model and the Inverse Lyapunov Semi-Active Control Law

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SUMMARY:

Various semi-active control methods have been proposed for vibration control of civil structures. In contrast to active vibration control systems, all semi-active control systems are essentially asymptotically stable because of the stability of the structural systems (with structural damping) themselves and the energy dissipating nature of the semi-active control devices. By utilizing the above property on the stability of semi-active control systems, a reduced-order structural model and a semi-active control law are simultaneously obtained so that the performance of the resulting semi-active control system becomes good. The semi-active control law in the present study is based on the inverse Lyapunov approach that has been proposed by the authors recently. The control law is a bang-bang type switching control that changes the command of the variable damping coefficient(s) of the semi-active damper(s). In the inverse Lyapunov approach, the Lyapunov matrix is designed so that the control performance of the semi-active control system is improved. Parameters of the reduced-order model and those to determine the Lyapunov matrix are simultaneously searched with Genetic Algorithm (GA). The effectiveness of the proposed approach is shown with a simulation study.

Keywords: Semi-active control, Modeling, Inverse Lyapunov approach

1. INTRODUCTION

Semi-active control of civil structures has been actively studied recent decades with the development of various semi-active control devices. The semi-active control strategy can potentially achieve the superior control performance compared to that of passive control thanks to its adjustability of the damping and/or stiffness coefficient in a real-time manner (Casciati et al., 2006).

As semi-active control devices, variable damping devices and variable stiffness devices have been developed. MR (Magnetorheological) dampers (Dyke et al., 1996; Sodeyama et al., 1997) are one of the representative of variable damping devices. In MR dampers, MR fluids are used as the working fluid and the damping coefficient of MR dampers are controlled by changing the influenced magnetic field provided by an electric magnet. ER (Electrorheological) dampers (Gavin, 2001) are also variable damping semi-active devices whose damping coefficients can be changed with the pair of the ER working fluid and the influenced electrical field. Variable stiffness devices also have been developed with a gas or a hydraulic cylinder (Nasu et al., 2001; John Leavitt et al., 2008) and a spring mechanism (Varadarajan et al., 2004).

In general control system design problem including semi-active control systems, control system designers cannot get the perfectly exact model of the real control object. In other words, we can never obtain the mathematical model whose responses subject to the external input perfectly agrees with those of the real control object. This is because many dynamic characteristics such as nonlinearities of the real control object must be ignored in the modeling process to obtain the simpler model that can be used in the controller design process. Therefore, control system designer generally seeks a simple model whose responses to some external inputs agree with those of the real control object *as much as possible* for the stability of the closed-loop system.

On the other hand, a general semi-active control system that is composed of the structural system with semi-active control devices, e.g., MR dampers etc. is asymptotically stable because of the energy dissipating nature of the control objects (general civil structural systems with the internal damping) and semi-active control devices. From this fact, we do not need to consider the exactness of the model in the sense of the open-loop response as the most important aspect in the modeling process of the semi-active control design. Alternatively we can focus on the resulted control performance of the closed-loop system with the real control object and the semi-active control law obtained based on the model. In other words, we have a freedom to choose model parameters to achieve the good control performance of the closed-loop system with the real control object and the model-based semi-active control law without violating the closed-loop stability.

In the present study, a method for a simultaneous design of the mathematical model and the (model-based) semi-active control law is proposed. The model is defined as the linear parameter varying (LPV) model with a smaller degree of freedom compared to that of the detailed model of the real structural system. A detailed model is obtained so that the structural response to the disturbance input agrees with that of the real structural system with a high degree of accuracy. The detailed model can be nonlinear and have a quite large degree of freedom. The detailed model is used for the performance evaluation of the semi-active control law based on the simple LPV model to be searched. Structural parameters, the mass, damping and stiffness parameters of the LPV model are defined as structural design parameters of the mathematical model for the control design. Those structural design parameters are optimized in the premise of the semi-active control law based on the inverse Lyapunov approach that has been proposed by authors (Hiramoto et al., 2011). In the inverse Lyapunov approach, the Lyapunov matrix that determines the switching semi-active control law is optimized to achieve the good control performance. Structural parameters of the LPV model and each element of the Lyapunov matrix are optimized so that the performance of the semi-active control system with the real structural system and the semi-active control law. Genetic algorithm (GA) is adopted for the optimization.

The rest of the paper is organized as follows: in §2 the problem formulation of the present semi-active control system design is described. The detailed structural mode that is causal but has a high complexity and/or nonlinearities, and its simpler LPV model having structural design parameters are defined. The semi-active control law based on the inverse Lyapunov approach is introduced in §3. The solution procedure for the simultaneous optimization of the structural design parameters of the LPV model and the Lyapunov matrix in the inverse Lyapunov approach is also presented. In §4 a simulation example for 6-dof base-isolated structural system is presented. The conclusion of the present study is given in §5.

Notations are as follows: t: time, I_n: An n-dimensional identity matrix, $0_{m\times n}$: An $m \times n$ zero matrix, $\mathscr{R}^{m\times n}$ and $\mathscr{R}^m$: The set of $m \times n$ real matrices and m-dimensional real vectors, M^T: The transpose of a matrix M, $\mathrm{RMS}\left(a(t)\right)$ and $\mathrm{PEAK}\left(a(t)\right)$: The RMS (root mean square) and peak values of a signal $a(t)$.

2. PROBLEM FORMULATION

In general structural control problem, the true dynamics of the structural system is unknown. Alternatively a high DOF detailed model that possibly contains some nonlinearities is available using FEM or some modeling methods. In the present study, the dynamics of the detailed model is defined as the following nonlinear state-space form:

$$\dot{x}_f(t) = f\left(x_f(t), w(t), \dot{w}(t)\right) \tag{2.1}$$

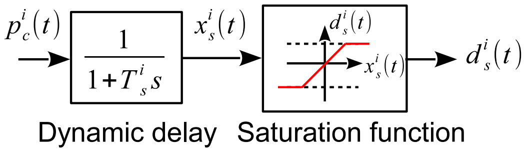

Figure 1. Block diagram of the semi-active control device

where $x_f(t) \in \mathscr{R}^{n_f}$ and $w(t) \in \mathscr{R}^{n_w}$ are the state vector of the structural system and the displacement of the earthquake disturbance, respectively. The displacement and velocity of each storey of the structural system are included in the state vector $x_f(t)$. The causal function $f(x_f(t), w(t), \dot{w}(t))$ is obtained so that the function $f(x_f(t), w(t), \dot{w}(t))$ approximates the dynamic behavior of the real structural system with high accuracy. In the detailed model the dimension of the state vector n_f can become quite high and the function $f(x_f(t), w(t)\,\dot{w}(t))$ can become a complex nonlinear function.

Assume that n_s semi-active control devices whose damping coefficients can be independently changed by their respective command signals are installed on the structural system. In contrast to the structural model, dynamic characteristics of the semi-active dampers are accurately available through a unit testing in the developing phase. The dynamics of the semi-active devices is modeled as the combination of the first-order lag element with saturation function given as

$$\dot{x}_s^i(t) = -\frac{1}{T_s^i} x_s^i(t) + \frac{1}{T_s^i} p_c^i(t), T_s^i > 0, \tag{2.2}$$

$$d_s^i(t) = \mathrm{sat}\left(x_s^i(t)\right),\ i = 1, \ldots, n_s, \tag{2.3}$$

where $x_s^i(t) \in \mathscr{R}$, $p_c^i(t) \in \mathscr{R}$, T_s^i and $d_s^i(t)$, $i = 1, \ldots, n_s$ are the state variable, the command signal, the time constant and the variable damping coefficient of the i-th semi-active damper, respectively. The time constant represents the dynamic delay of the semi-active device. The function $\mathrm{sat}(\cdot)$ is the saturation function defined as the following:

$$\mathrm{sat}(u) = \begin{cases} \overline{u} & (\overline{u} < u) \\ u & (\underline{u} \leq u \leq \overline{u}) \\ \underline{u} & (u < \underline{u}) \end{cases}. \tag{2.4}$$

The maximum and minimum damping coefficients of the i-the semi-active damper are defined as $\overline{d_s^i}$ and $\underline{d_s^i}$, $i = 1, \ldots, n_s$, respectively. The block diagram of the semi-active control device in Eqs. 2.2 and 2.3 is shown as Fig. 1

By combining the detailed model in Eqn. 2.1 and the semi-active control device in Eqs. 2.2 and 2.3, the detailed model of the structural system with n_s semi-active dampers is obtained as the following:

$$\dot{x}_f(t) = g\left(x_f(t)\, w(t), \dot{w}(t), d_s^1(t), \ldots, d_s^{n_s}(t)\right) \tag{2.5}$$

The detailed model in Eqn. 2.5 is an accurate approximation of the unknown dynamics of the real structural system with semi-active control devices. It is useful to simulate structural responses to various type of earthquake disturbances with high accuracy. However it is difficult to obtain a semi-active control law directly from the detailed model of the structural system with semi-active devices in Eqn. 2.5 because of its high dimensionality and nonlinearity. Therefore a simpler model that can be used for the synthesis of the control law is desired[1].

[1] Such simpler models are required also in general control system design problems. Those simpler models are referred to as *models for controller design*.

In the present study, we do not use the detailed model and use a simpler and lower dimensional linear dynamic system as the model for the controller design. The model for the controller design is defined as the following linear parameter varying (LPV) system:

$$\dot{x}_r(t) = A\left(d_s^1(t), \ldots, d_s^{n_s}(t)\right) x_r(t) + B_d w(t) + B_v\left(d_s^1(t), \ldots, d_s^{n_s}(t)\right) \dot{w}(t), \tag{2.6}$$

where $x_r(t) \in \mathscr{R}^{n_r}$ is the state vector of the model for the control system design. Coefficient matrices A and B_v are functions on variable damping coefficients $d_s^i(t)$, $i = 1, \ldots, n_s$, respectively. In what follows, we refer to the model for the control design in Eqn. 2.6 as the reduced-order model. We assume that the state vector $x_r(t)$ is composed by some portions of measured elements of $x_f(t)$, the state vector of the detailed model in Eqn. 2.5. The semi-active control law is designed for the reduced-order model and is evaluated the control performance with the detailed model in Eqn. 2.5.

The above situation, i.e., the controller is designed for the simple model that is different from the detailed complex model, is frequently found in general control system design problems. In such a case, the accuracy of the reduced-order model compared with the real control object generally becomes one of the most important requirement to keep the stability of the resulted closed-loop system. Therefore the reduced-order model that achieves good agreement of the response to input signals especially in the frequency range below the control bandwidth is highly desirable in general control system design. However, in semi-active control, the control system is always asymptotically stable because of the structural system with the semi-active control devices are always energy dissipative. In fact, the closed-loop system with the real structural system (or the detailed model in Eqn. 2.5) is always asymptotically stable for any command signal (p_c^i, $i = 1, \ldots, n_s$ in Eqn. 2.2) to the semi-active control device. Therefore it is valid to claim that the accuracy of the reduced-order model compared with the real control object in the sense of the open-loop response is no longer the most important issue in the modeling process of the semi-active control design.

Based on the above fact on the stability of semi-active control systems, we propose a new method to *design* the reduced-order model in Eqn. 2.6 so that the closed-loop system with the real control object (or the detailed model in Eqn. 2.5) and the semi-active control law obtained based on the reduced-order model in Eqn. 2.6. In the present study, the reduced-order model the equation of motion of the reduced-order model is parameterized as

$$M_r\ddot{q}_r(t) + \left(D_r + \sum_{j=1}^{n_s} d_s^j(t) E_r^j\right)\dot{q}_r(t) + K_r q_r(t) = F_0 w(t) + \left(F_1 + \sum_{j=1}^{n_s} d_s^j(t) G_r^j\right)\dot{w}(t) \tag{2.7}$$

$$q_r(t) = \begin{bmatrix} q_1^r(t) \\ q_2^r(t) \\ \vdots \\ q_{l_r}^r(t) \end{bmatrix}, \; M_r = \begin{bmatrix} m_1^r & 0 & \cdots & 0 \\ 0 & m_2^r & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & m_{l_r}^r \end{bmatrix}, \; D_r = \begin{bmatrix} d_1^r + d_2^r & -d_2^r & \cdots & 0 \\ -d_2^r & d_2^r + d_3^r & \ddots & \vdots \\ \vdots & \ddots & \ddots & -d_{l_r}^r \\ 0 & \cdots & -d_{l_r}^r & d_{l_r}^r \end{bmatrix},$$

$$K_r = \begin{bmatrix} k_1^r + k_2^r & -k_2^r & \cdots & 0 \\ -k_2^r & k_2^r + k_3^r & \ddots & \vdots \\ \vdots & \ddots & \ddots & -k_{l_r}^r \\ 0 & \cdots & -k_{l_r}^r & k_{l_r}^r \end{bmatrix}, \; l_r = \frac{n_r}{2},$$

where m_i^r, d_i^r and k_i^r, $i = 1, \ldots, l_r$ are the mass, damping and stiffness of the reduced-order model, respectively. The vector $q_r(t)$ is the displacement vector of the reduced-order model whose elements are those of the state vector of the detailed model $x_f(t)$ in Eqn. 2.5. All the elements of the vector $q_r(t)$ and $\dot{q}_r(t)$ are measured by the sensor installed on the real structural system to carry out the semi-active control, i.e., to generate the command signals to all semi-active devices. Matrices F_0 and F_1 are influence coefficient matrices. Matrices E_r^j, G_r^j, $j = 1, \ldots, n_s$ are the constant matrices that have nonzero element depending on the placement of the j-th semi-active device. Then coefficient matrices of the state-space form in Eqn. 2.6 are given as follows:

$$
\begin{aligned}
& x_r(t) = \begin{bmatrix} q_r(t) \\ \dot{q}_r(t) \end{bmatrix},\ A = A_0 + \sum_{j=1}^{n_s} d_s^j(t) A_1^j, \\
& A_0 = \begin{bmatrix} 0_{l_r \times l_r} & I_{l_r} \\ -M_r^{-1} K_r & -M_r^{-1} D_r \end{bmatrix},\ A_1^j = \begin{bmatrix} 0_{l_r \times 2l_r} & \\ 0_{l_r \times l_r} & -M_r^{-1} E_r^j \end{bmatrix} \\
& B_d = \begin{bmatrix} 0_{l_r \times n_w} \\ M_r^{-1} F_0 \end{bmatrix},\ B_v = B_0 + \sum_{j=1}^{n_s} d_s^j(t) B_1^j,\ B_0 = \begin{bmatrix} 0_{l_r \times n_w} \\ M_r^{-1} F_1 \end{bmatrix},\ B_1^j = \begin{bmatrix} 0_{l_r \times n_w} \\ M_r^{-1} G_r^j \end{bmatrix}
\end{aligned} \tag{2.8}
$$

Note that matrices A and B_v are linear functions on the variable damping coefficient of the semi-active control device, respectively. With the reduced-order system defined as Eqs. 2.6 and 2.8, the semi-active control system design problem in the present study is formulated as the following:

Find the parameter of the reduced-order model in Eqn. 2.6 and the semi-active control law based on the reduced-order model. The parameter of the reduced-order model and the semi-active control law are searched such that the control performance of the semi-active control system with the real structural system (or the detailed model in Eqn. 2.5) and the semi-active control law obtained from the reduced-order model.

3. SOLUTION PROCEDURE

3.1 SEMI-ACTIVE CONTROL LAW

As the semi-active control law in the present paper, the inverse Lyapunov approach (Hiramoto et al., 2011) for the reduced-order system is adopted. For the reduced-order system in Eqs. 2.6 and 2.8, define the following Lyapunov function:

$$
V(t) = x_r^T(t) P x_r(t),\ P = P^T \geq 0 \tag{3.1}
$$

where the matrix P is the Lyapunov matrix. With Eqn. 2.8, the time derivative of the Lyapunov function $V(t)$ is given by

$$
\begin{aligned}
\dot{V}(t) &= \dot{x}_r(t) P x_r(t) + x_r^T(t) P \dot{x}_r(t) \\
&= \left(A x_r(t) + B_d w(t) + B_v \dot{w}(t)\right)^T P x_r(t) + x_r(t) P \left(A x_r(t) + B_d w(t) + B_v \dot{w}(t)\right) \\
&= x_r^T(t) \left(A^T P + PA\right) x_r(t) + 2 w^T(t) B_d^T P x_r(t) + 2 \dot{w}^T(t) B_0^T P x_r(t) \\
&+ \sum_{j=1}^{n_s} d_s^j(t) \left[x_r^T(t) \left\{ \left(A_1^j\right)^T P + P \left(A_1^j\right) \right\} x_r(t) + 2 \dot{w}^T(t) \left(B_1^j\right)^T P x_r(t) \right].
\end{aligned} \tag{3.2}
$$

Then the variable damping coefficients $d_s^j(t),\ j = 1, \ldots, n_s$ that minimize $\dot{V}(t)$ are obtained as follows:

$$
d_s^j(t) = \begin{cases} \overline{d_s^j} & x_r^T(t) \left\{ \left(A_1^j\right)^T P + P \left(A_1^j\right) \right\} x_r(t) + 2 \dot{w}^T(t) \left(B_1^j\right)^T P x_r(t) < 0 \\ \underline{d_s^j} & x_r^T(t) \left\{ \left(A_1^j\right)^T P + P \left(A_1^j\right) \right\} x_r(t) + 2 \dot{w}^T(t) \left(B_1^j\right)^T P x_r(t) \geq 0 \end{cases},\ j = 1, \ldots, n_s \tag{3.3}
$$

Note that the switching control law in Eqn. 3.3 is remained unchanged for any symmetric positive semidefinite matrices P. The timing of the switching can be changed by choosing a different matrix

P. In many studies on semi-active control of civil structural systems based on Lyapunov function, e.g., Gavin (2001), the Lyapunov matrix P is chosen as the following:

$$P = \frac{1}{2}\begin{bmatrix} K_r & 0_{n_r \times n_r} \\ 0_{n_r \times n_r} & M_r \end{bmatrix} \tag{3.4}$$

For the Lyapunov matrix P in Eqn. 3.4, the Lyapunov function $V(t)$ in Eqn. 3.1 becomes the following total energy of the structural system at a certain instant t:

$$V(t) = \frac{1}{2}\left(\dot{q}_r^T(t) M_r \dot{q}_r(t) + q_r^T(t) K_r q_r(t)\right). \tag{3.5}$$

In the inverse Lyapunov approach, the Lyapunov matrix P in Eqn. 3.1 is dealt with as the design parameter in the semi-active control design. The Lyapunov matrix is searched over a certain range so that the control performance of the semi-active control system is optimized. The control performance is evaluated for the closed-loop system with the real structural system (or the detailed structural model) and the semi-active control law in Eqn. 3.3 that is obtained based on the reduced-order model in Eqs. 2.6 and 2.8.

3.2 SIMULTANEOUS DESIGN OF THE REDUCED-ORDER MODEL AND THE SEMI-ACTIVE CONTROL LAW

In the present study, the reduced-order model in Eqn. 2.7 and the Lyapunov matrix in Eqn. 3.1 are simultaneously searched so that the performance of the semi-active control system is optimized. Design parameters of the reduced-order model are m_i^r, d_i^r, k_i^r in Eqn. 2.7. In optimizing the Lyapunov matrix P in Eqn. 3.1, each element of the following Cholesky factor of the matrix P is employed as design parameters to be searched:

$$P = LL^T \tag{3.6}$$

By taking L, not P itself, as the design parameter the Lyapunov matrix P is always symmetric and positive semi-definite.

The objective function to evaluate the control performance of the semi-active control system is defined as

$$J = \sum_{k=1}^{n_e} J_k,\ J_k = \sum_{l=1}^{4} J_k^l \tag{3.7}$$

$$J_k^1 = \sum_{n=1}^{n_f}\left\{\lambda \frac{\mathrm{RMS}\left({}^k r_n^{on}\right)}{\mathrm{RMS}\left({}^k r_n^{0}\right)} + (1-\lambda)\frac{\mathrm{RMS}\left({}^k r_n^{s}\right)}{\mathrm{RMS}\left({}^k r_n^{on}\right)}\right\},$$

$$J_k^2 = \sum_{n=1}^{n_f}\left\{\lambda \frac{\mathrm{RMS}\left({}^k a_n^{on}\right)}{\mathrm{RMS}\left({}^k a_n^{0}\right)} + (1-\lambda)\frac{\mathrm{RMS}\left({}^k a_n^{s}\right)}{\mathrm{RMS}\left({}^k a_n^{on}\right)}\right\},$$

$$J_k^3 = \sum_{n=1}^{n_f}\left\{\lambda \frac{\mathrm{PEAK}\left({}^k r_n^{on}\right)}{\mathrm{PEAK}\left({}^k r_n^{0}\right)} + (1-\lambda)\frac{\mathrm{PEAK}\left({}^k r_n^{s}\right)}{\mathrm{PEAK}\left({}^k r_n^{on}\right)}\right\},$$

$$J_k^4 = \sum_{n=1}^{n_f}\left\{\lambda \frac{\mathrm{PEAK}\left({}^k a_n^{on}\right)}{\mathrm{PEAK}\left({}^k a_n^{0}\right)} + (1-\lambda)\frac{\mathrm{PEAK}\left({}^k a_n^{s}\right)}{\mathrm{PEAK}\left({}^k a_n^{on}\right)}\right\},\ 0 \le \lambda \le 1,$$

where ${}^k r_n^*$, ${}^k a_n^*$, $k = 1,\ldots,n_e$, $n = 1,\ldots,n_f$ are the relative displacement between neighboring n-th and $n-1$-th storeys and the absolute acceleration of n-th storey, respectively. Those structural responses are obtained for the control system that is composed of the detailed model in Eqn. 2.5 and the designed semi-active control law by using the k-th earthquake wave as the disturbance input. Superscripts $* = s$,

Table 1. Structural Parameters of the 6-dof Structural System

Floor	m_i [kg]	d_i [kNs/m]	k_i [kN/m]
1	6800	7.45	231.50
2	5897	67	33732
3	5897	58	29093
4	5897	57	28621
5	5897	50	24954
6	5897	38	19059

on and 0 show quantities with semi-active control, passive on (all variable damping coefficients of the semi-active devices are kept their maximum values throughout the earthquake event) or those without semi-active control devices, respectively. The scalar λ is an weighting factor to adjust the amount of the performance improvement of the structural responses between two situations, that is, the passive on case compared to the case without semi-active control devices and the semi-active control case compared to the passive on case.

Design parameters of the reduced-order model and the Lyapunov matrix in the inverse Lyapunov approach are optimized so that the objective function J is minimized. In the present study, Genetic Algorithm (GA) is adopted for the optimization.

4. SIMULATION EXAMPLE

As a simulation example, a semi-active control system design for a 6-dof base-isolated structure (Kelly et al., 1987; Romallo et al., 2002) in Fig. 2 (a) is considered. In the 6-dof structural system, a semi-active damper with $T_s^1 = 0.02$ [s], $\overline{d_s^1} = 20 \times 10^3$ [Ns/m], $\underline{d_s^1} = 0$ [Ns/m] (in Eqs. 2.2 and 2.3) is installed in the isolated storey. The detailed model of the structural system in Eqn. 2.5 is defined as the following:

$$\dot{x}_f(t) = A^f x_f(t) + B_d^f w(t) + B_v^f \dot{w}(t), \tag{4.1}$$

$$x_f(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix},\ q(t) = \begin{bmatrix} q_1(t) & \dots & q_6(t) \end{bmatrix}^T,\ A^f = A_0^f + d_s^1(t) A_1^1,$$

$$A_0^f = \begin{bmatrix} 0_{6\times6} & I_6 \\ -M_f^{-1}K_f & -M_f^{-1}D_f \end{bmatrix},\ A_1^1 = \begin{bmatrix} \multicolumn{2}{c}{0_{6\times12}} \\ 0_{6\times6} & -M_f^{-1}E_f^1 \end{bmatrix},$$

$$B_d^f = \begin{bmatrix} 0_{6\times1} \\ M_f^{-1}F_f \end{bmatrix},\ B_v^f = B_0^f + d_s^1(t) B_1^1,\ B_0^f = \begin{bmatrix} 0_{6\times1} \\ M_f^{-1}F_1^f \end{bmatrix},\ B_1^1 = \begin{bmatrix} 0_{6\times1} \\ M_f^{-1}G_1^1 \end{bmatrix},$$

$$M_f = \begin{bmatrix} m_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m_6 \end{bmatrix},\ D_f = \begin{bmatrix} d_1+d_2 & -d_2 & \cdots & 0 \\ -d_2 & d_2+d_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -d_6 \\ 0 & \cdots & -d_6 & d_6 \end{bmatrix},$$

$$K_f = \begin{bmatrix} k_1+k_2 & -k_2 & \cdots & 0 \\ -k_2 & k_2+k_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -k_6 \\ 0 & \cdots & -k_6 & k_6 \end{bmatrix}.$$

Structural parameters of the 6-dof building are given in Table 1.

As the reduced-order model in Eqn. 2.6, a 2-dof system in Fig. 2 (b) is assumed. The absolute displacement and velocity of the 1st and 6th storeys are employed as measurement signals of the reduced-order model. Note that the order of the reduced-order model and the placement of two sensors are determined

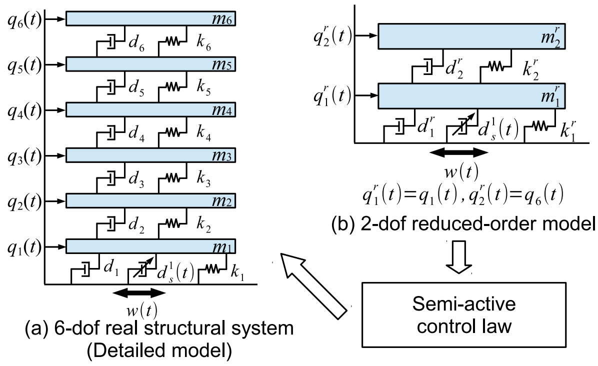

Figure 2. 6-dof detailed structural model and 2-dof reduced-order model

by trial and error so that the control performance of the resulted semi-active control system is optimized. Then design parameters of the reduced-order model in Eqn. 2.6 are m_i^r, d_i^r and k_i^r, $i = 1,2$. The state vector of the reduced-order model is defined as the following:

$$x_r(t) = \begin{bmatrix} q_1(t) & q_6(t) & \dot{q}_1(t) & \dot{q}_6(t) \end{bmatrix}^T \quad (4.2)$$

As design parameters of the semi-active control law based on the inverse Lyapunov approach, all the elements of the Cholesky factor of the Lyapunov matrix P in Eqn. 3.1 are defined. The Cholesky factor L is parameterized as

$$L = \begin{bmatrix} l_1 & 0 & 0 & 0 \\ l_2 & l_3 & 0 & 0 \\ l_4 & l_5 & l_6 & 0 \\ l_7 & l_8 & l_9 & l_{10} \end{bmatrix}, \ l_1, l_3, l_6, l_{10} \geq 0, \quad (4.3)$$

where l_j, $j = 1, \ldots 10$ are design parameters of the semi-active control law based on the inverse Lyapunov approach.

Design parameters in the reduced-order model and the matrix L in Eqn. 4.3 are optimized such that the performance index obtained is minimized. Structural responses are obtained for four earthquake disturbances, i.e., EL Centro NS (1940), BCJL1 (artificial), Hachinohe NS (1968) and JMA Kobe NS (1995) waves. All earthquake waves are scaled so that their peak ground accelerations (PGA) become 4.0 $[\mathrm{m/s^2}]$.

The optimization is carried out under the above mentioned setting with $\lambda = 0.5$ in Eqn. 3.7. Results for the El Centro NS (1940) wave are shown in Fig. 3, Tables 2 and 3. In the result, NC, Pon and SA denotes cases without the semi-active damper, passive on and the semi-active control based on the inverse Lyapunov Approach, respectively. For the El Centro NS (1940) earthquake the case SA shows the best result compared to cases NC and Pon both in the sense of the relative displacement between neighboring two storeys and the absolute acceleration of each storey.

Moreover, a simulation is conducted for the Taft NS (1952) wave with PGA $= 4.0$ $[\mathrm{m/s^2}]$ to show the performance robustness of the optimization result. Note that the earthquake disturbance is not employed in the optimization to obtain the reduced-order model and the Cholesky factor of the Lyapunov matrix. Results for the Taft NS (1952) wave are shown in Fig. 4, Tables 4 and 5, respectively. As in the case of El Centro NS (1940), the semi-active control system achieves the better control performance than those in cases NC and Pon. The result shows that the proposed semi-active control system based on the reduced-order model is also effective for unknown forthcoming earthquake disturbance.

Table 2. Results for El Centro NS (1940) earthquake wave (PGA=4.0 [m/s^2]): RMS value.

Quantity	NC	Pon	SA	SA/NC	SA/Pon
r_1 [mm]	12.8	8.8	8.5	0.67	0.97
r_2 [$\times 10^{-2}$mm]	7.28	6.39	5.50	0.76	0.86
r_3 [$\times 10^{-2}$mm]	6.83	6.20	5.35	0.78	0.86
r_4 [$\times 10^{-2}$mm]	5.27	5.00	4.32	0.82	0.86
r_5 [$\times 10^{-2}$mm]	4.08	4.08	3.53	0.87	0.86
r_6 [$\times 10^{-2}$mm]	2.71	2.85	2.49	0.92	0.87
$\ddot{q}_1$ [m/s^2]	0.081	0.078	0.068	0.84	0.87
$\ddot{q}_2$ [m/s^2]	0.082	0.074	0.065	0.79	0.88
$\ddot{q}_3$ [m/s^2]	0.083	0.072	0.064	0.77	0.88
$\ddot{q}_4$ [m/s^2]	0.084	0.074	0.066	0.78	0.88
$\ddot{q}_5$ [m/s^2]	0.085	0.082	0.071	0.83	0.87
$\ddot{q}_6$ [m/s^2]	0.087	0.092	0.081	0.92	0.87

Table 3. Results for El Centro NS (1940) earthquake wave (PGA=4.0 [m/s^2]): Peak value.

Quantity	NC	Pon	SA	SA/NC	SA/Pon
r_1 [mm]	65.1	48.6	44.4	0.68	0.91
r_2 [$\times 10^{-2}$mm]	36.6	36.2	28.5	0.78	0.79
r_3 [$\times 10^{-2}$mm]	34.5	35.4	27.7	0.80	0.78
r_4 [$\times 10^{-2}$mm]	26.8	28.3	22.3	0.83	0.79
r_5 [$\times 10^{-2}$mm]	20.9	22.3	17.7	0.85	0.79
r_6 [$\times 10^{-2}$mm]	13.9	15.2	12.2	0.88	0.80
$\ddot{q}_1$ [m/s^2]	0.44	0.42	0.36	0.83	0.87
$\ddot{q}_2$ [m/s^2]	0.44	0.38	0.36	0.81	0.93
$\ddot{q}_3$ [m/s^2]	0.43	0.38	0.34	0.79	0.91
$\ddot{q}_4$ [m/s^2]	0.42	0.43	0.38	0.92	0.90
$\ddot{q}_5$ [m/s^2]	0.44	0.46	0.37	0.85	0.80
$\ddot{q}_6$ [m/s^2]	0.45	0.49	0.40	0.88	0.80

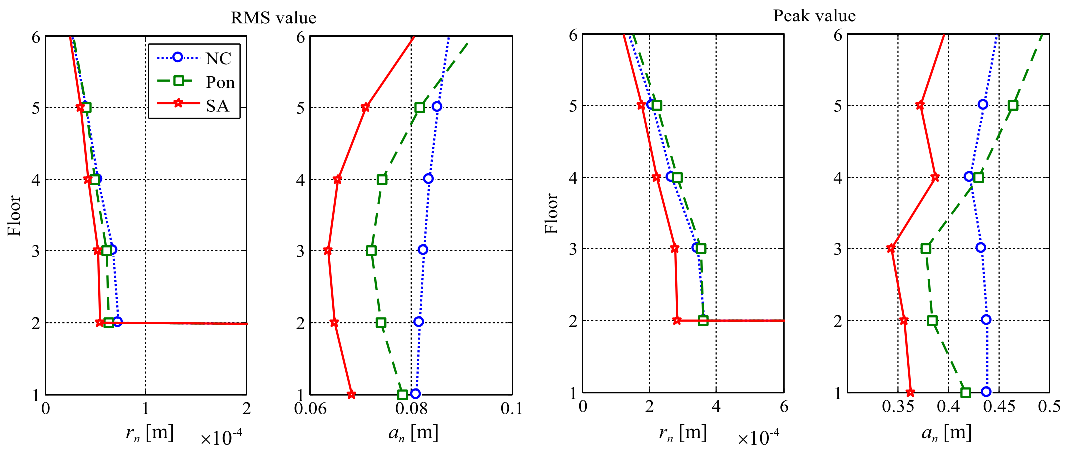

Figure 3. Result for El Centro NS (1940) earthquake wave

5. CONCLUSION

In this study, a new semi-active control methodology based on the reduced-order model and the model-based semi-active control law has been proposed. As the semi-active control law, the inverse Lyapunov approach for the reduced-order model is adopted. Parameters of the reduced-order model are employed as the design parameters so that the control performance of the resulted semi-active control system (composed of the detailed structural model and the semi-active control law based on the reduced-order model) is optimized. Design parameters in the reduced-order model and parameters of the Lyapunov matrix are simultaneously optimized with Genetic Algorithm. The effectiveness of the present approach is shown with the simulation example of the semi-active control for the 6-dof base-isolated structure with 2-dof reduced-order model.

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Table 4. Results for Taft NS (1952) earthquake wave (PGA=4.0 [m/s^2]): RMS value.

Quantity	NC	Pon	SA	$\frac{SA}{NC}$	$\frac{SA}{Pon}$
r_1 [mm]	80.7	48.7	48.1	0.60	0.99
r_2 [$\times 10^{-2}$mm]	44.9	26.5	25.6	0.57	0.96
r_3 [$\times 10^{-2}$mm]	41.7	24.8	23.9	0.57	0.97
r_4 [$\times 10^{-2}$mm]	31.9	19.0	18.4	0.58	0.97
r_5 [$\times 10^{-2}$mm]	24.4	14.7	14.2	0.58	0.96
r_6 [$\times 10^{-2}$mm]	16.0	9.68	9.39	0.59	0.97
$\ddot{q}_1$ [m/s^2]	0.51	0.30	0.29	0.58	0.97
$\ddot{q}_2$ [m/s^2]	0.51	0.30	0.29	0.57	0.97
$\ddot{q}_3$ [m/s^2]	0.51	0.30	0.29	0.58	0.97
$\ddot{q}_4$ [m/s^2]	0.51	0.30	0.29	0.57	0.97
$\ddot{q}_5$ [m/s^2]	0.52	0.31	0.30	0.58	0.97
$\ddot{q}_6$ [m/s^2]	0.52	0.31	0.30	0.59	0.97

Table 5. Results for Taft NS (1952) earthquake wave (PGA=4.0 [m/s^2]): Peak value.

Quantity	NC	Pon	SA	$\frac{SA}{NC}$	$\frac{SA}{Pon}$
r_1 [mm]	199	138	134	0.68	0.97
r_2 [$\times 10^{-2}$mm]	110	82.9	75.9	0.69	0.92
r_3 [$\times 10^{-2}$mm]	102	80.3	71.2	0.70	0.89
r_4 [$\times 10^{-2}$mm]	78.7	63.7	56.2	0.71	0.88
r_5 [$\times 10^{-2}$mm]	60.7	50.5	44.3	0.73	0.88
r_6 [$\times 10^{-2}$mm]	40.0	33.9	30.7	0.77	0.90
$\ddot{q}_1$ [m/s^2]	1.28	0.98	0.87	0.67	0.89
$\ddot{q}_2$ [m/s^2]	1.27	0.93	0.87	0.69	0.89
$\ddot{q}_3$ [m/s^2]	1.26	0.90	0.89	0.70	0.99
$\ddot{q}_4$ [m/s^2]	1.25	0.96	0.88	0.70	0.99
$\ddot{q}_5$ [m/s^2]	1.27	1.04	0.92	0.72	0.88
$\ddot{q}_6$ [m/s^2]	1.29	1.10	0.99	0.76	0.90

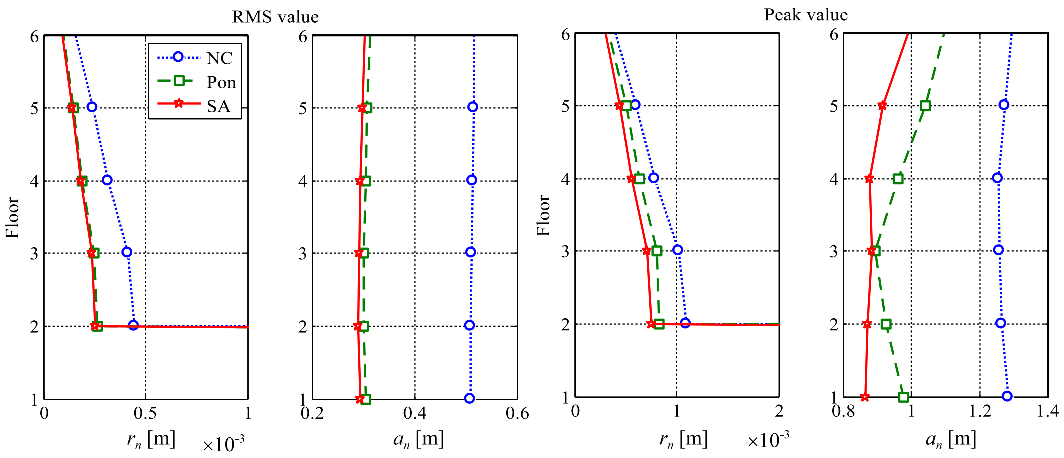

Figure 4. Result for Taft NS (1952) earthquake wave

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