Inter-Story Drift Spectra for Tall Buildings under Near-Field Earthquakes Based on Non-Uniform Lumped Mass Beam Model

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SUMMARY:
A lumped mass beam model is proposed to estimate the seismic-induced inter-story drift demands in tall shear buildings under near-field earthquakes. Using a modal analysis technique for desired damping ratio and period, and considering a height-wise variation of structural stiffness of the building, the proposed lumped mass shear beam provides an accurate estimate of the drift demands for different stories of the structural models. Drift spectra are constructed for high-rise shear buildings with non-uniform height-wise distribution of their structural stiffness. The mean drift spectra for various types of height-wise stiffness distribution are constructed using near-field strong ground motions. The effect of the ratio of the lateral stiffness at the top stories of the building model to its lateral stiffness at the base level, height-wise variation of the stiffness and number of stories on the resulting drift spectra are investigated.

Keywords: Time History Analysis, Inter-Story Drift, Drift Spectrum, Non-Uniform Beam, Lumped Mass Beam

1. INTRODUCTION

The response spectrum (Biot 1941 and Housner 1947) is an important tool in seismic design of structures. Using modal analysis procedures, the peak elastic response of a multi-degree of freedom (MDOF) system during an earthquake excitation can be approximately determined from the earthquake response spectrum without performing a dynamic time history analysis. The response spectrum analysis can also provide an estimate of the overall displacement demands of the building models. However, extensive research has shown that the inter-story drift ratio has the best correlation with the inflicted damages to buildings under earthquake excitations. Obviously, the distribution of the inter-story drift demands along the height of the buildings is not uniform and therefore cannot be obtained directly from the displacement response spectrum.

On the other hand, the response of the structures under near-field excitations cannot be characterized by a resonance build-up, so the underlying assumptions of the modal analysis method are not satisfied for structures subjected to these types of ground motions (Iwan 1997). Iwan introduced the so-called "drift spectrum" as a new measure of the induced seismic demands on structures for such situations. Iwan's drift spectrum was based on the solution of the problem for non-dispersive damped waves in a one-dimensional continuous medium, as treated by Courant and Hilbert (1962). He used the shear strain in a continuous shear beam as an analogy to the inter-story drift ratio in a building structure.

Using modal analysis of a continuous shear beam, Chopra and Chintanapakdee (2001), showed that the differences between the drifts and the response spectra are not unique for near-field strong ground motions. These differences simply reflect the effects of higher modes on the structural response that are larger due to characteristics of the near-field ground motions. Also, the existing modal combination rules are equally accurate for near-field and far-field ground motions, although their main assumptions are not satisfied by the near-field excitations. Furthermore, Kim and Collins (2002) showed that Iwan's drift spectrum could result in residual drifts for certain near field ground motions with fling step which is not consistent with the assumed linear elastic model.

Miranda and Akkar (2006) extended Iwan's drift spectrum for buildings manifesting flexural or flexural-shear type of behavior. Using Csonka's (1965) flexural-shear beam model consisting of two flexural and shear cantilever beams connected by an infinite number of axially rigid members, they proposed the so-called "generalized inter-story drift spectrum". The proposed drift spectrum was based on modal analysis technique for a continuous beam. They used the derivatives of the mode shapes of the continuous shear-flexural beams to approximate the drift ratios. In a previous study, Miranda and Reyes (2002) concluded that the maximum inter-story drift demands were not significantly influenced by reduction of the stiffness along the height of the structural system. Based on this study, Miranda and Akkar (2006) computed the drift spectra for uniform (stiffness) beams. However, in this paper it will be shown that ignoring the effect of stiffness variation along the structure's height may lead to a large error in estimating the inter-story drift demands. To compute the inter-story drift ratios, Miranda and Akkar used a code-based relationship to estimate the height of the structures from their fundamental period, which could obviously reduce the accuracy of the method. Also, due to difficulties in attaining the required precision in numerically evaluating some exact functions for shear buildings in most computers, they provided approximate equations for the problem.

In this study, a new procedure is introduced for calculating the drift spectra that overcomes these shortcomings. The drift spectra for shear buildings with various types of height-wise stiffness distributions are determined, and the mean spectra are computed using ten near-field strong ground motions. The effects of the ratio of the lateral stiffness at the top story to that of first story, the height-wise stiffness variation and the number of stories on the drift demands are investigated.

2. PROBLEM FORMULATION

Consider the lumped mass shear beam representing an N-story building shown in Fig. 1. The masses of different floors that are assumed to be equal, are lumped at each floor level. The mass matrix [M] can therefore be written as,

$$[M] = m[I] \tag{2.1}$$

in which m is any floor mass and [I] is the identity matrix. The non-uniform distribution of stiffness along the beam model is assumed to be given by,

$$k_i = \left(1 - (1-\delta)(\frac{i-1}{N-1})^{\lambda}\right)k \tag{2.2}$$

where k_i and k are the stiffness of the i^{th} and the first stories, respectively. The parameter δ is the ratio of the lateral stiffness at the top to that of the base of the beam model, and λ is a non-dimensional parameter that controls the variation of the lateral stiffness along the beam. This relationship is similar to the variation used by Miranda and Reyes (2002) for continuous beams. As they noted, values of λ equal to 1 or 2 correspond to linear or parabolic variation of lateral stiffness along the height of the structure respectively, as shown in Fig. 1. All elements of the stiffness matrix of the system can be determined in terms of the first story stiffness k, so this matrix can be written as,

$$[K] = k\ [S] \tag{2.3}$$

in which the non-dimensional matrix [S] becomes a function of δ and λ only.

Substituting Eqns. 2.1 and 2.3 into the characteristic equation of the system yields,

$$\det(-\omega_i^2[M] + [K]) = 0 \quad , \quad i = 1, 2, ..., N \tag{2.4}$$

or,

$$\det(-\omega_i^2 \frac{m}{k}[I]+[S])=0 \quad , \quad i=1,2,....,N \tag{2.5}$$

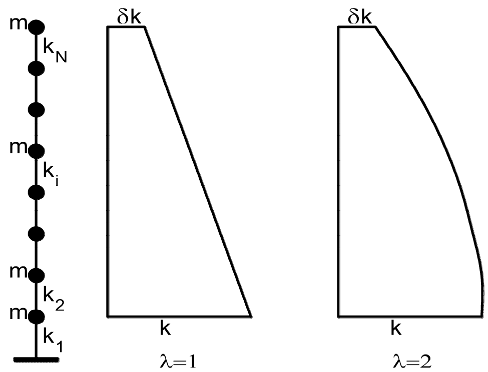

Figure 1. The lumped mass, non-uniform beam model and variation of stiffness along its height.

where ω_i is the i^{th} natural frequency of the structural model. The mode shapes (eigenvectors), ϕ_i, can then be determined from,

$$(-\omega_i^2 \frac{m}{k}[I]+[S])\{\phi_i\}=0 \quad , \quad i=1,2,...., N \tag{2.6}$$

Defining the non-dimensional parameter 'β_i' by,

$$\beta_i^2=\omega_i^2 \frac{m}{k} \tag{2.7}$$

Eqns 2.5 and 2.6 can be written as,

$$\det(-\beta_i^2[I]+[S])=0 \tag{2.8}$$

and,

$$(-\beta_i^2[I]+[S])\{\phi_i\}=0 \tag{2.9}$$

Eqn. 2.9 shows that buildings with uniform mass and a similar variation pattern for stiffness, with no dependency on the value of stiffness, mass and height, would have identical mode shapes. Using Eqn. 2.7 for the1st and the i^{th} modes of vibration produces,

$$\frac{\beta_i^2}{\beta_1^2}=\frac{\omega_i^2}{\omega_1^2} \tag{2.10}$$

Therefore, the i^{th} circular frequency or period can be obtained from the frequency or the period of the first mode as follows,

$$\omega_i = \frac{\beta_i}{\beta_1}\omega_1 \qquad , \qquad T_i = \frac{\beta_1}{\beta_i}T \tag{2.11}$$

This means that, if the fundamental period of vibration of a shear building with particular stiffness variation is known, its higher mode natural periods can be estimated using Eqn. 2.11. For a given ground motion time history, the maximum inter-drift distribution for an N-story building with the height-wise stiffness variation described by Eqn. 2.2, can be obtained from its fundamental period of vibration. The procedure is simple; first the matrix [*S*] for the specified values of δ and λ is generated, and then, by solving Eqns. 2.8 and 2.9, the non-dimensional parameters β_i and mode shapes $\{\phi_i\}$ can be obtained. The periods of higher modes can then be determined from Eqn. 2.11. The masses are equal at all stories so the modal participation factors will be determined. If the period T_i of any mode is known, the response history of the equivalent SDOF system, $D_i(t)$, for selected modal damping ratio ξ_i, under any earthquake record can be determined by solving the equation of motion of that mode. The drift of the j^{th} story can be calculated from:

$$Drift_j = \sum_{i=1}^{N}\{U_i(j)\} - \{U_i(j-1)\} = \sum_{i=1}^{N}\Gamma_i[\phi_i(j) - \phi_i(j-1)]D_i(t)\ , \quad j = 1,2,...,N \tag{2.12}$$

in which $U_i(j)$ and Γ_i are the displacement of the j^{th} floor in the i^{th} mode, and the i^{th} modal participation factor, respectively. Now, consider two tall buildings with the same first mode natural period *T*. In a continuous beam model, as an approximate model of a tall building, the maximum inter-story drift ratios (MIDR) can be obtained using the following relation (Miranda and Akkar 2006),

$$\mathrm{MIDR(x,t)} \approx \frac{1}{H}\sum_{i=1}^{m}\Gamma_i\,\phi_i'\,D_i(t) \tag{2.13}$$

Where ϕ_i' is the first derivative of the i^{th} mode shape ϕ_i with respect to the non-dimensional height parameter *x*. Eqn. 2.13 can be rewritten as,

$$\mathrm{MIDR(x,t)} \times \mathrm{H} \approx \sum_{i=1}^{m}\Gamma_i\phi_i' D_i(t) \tag{2.14}$$

It is also shown in Eqns. 2.9 and 2.10 that all buildings with the same fundamental period of vibration and uniform mass and a similar variation pattern for stiffness would have identical mode shapes and higher periods of vibration with no dependency on the value of stiffness, mass and height parameters. So it is reasonable to assume that uniform mass, tall buildings with same natural periods and similar variation pattern for stiffness have nearly the same mode shapes. Hence form Eqn. 2.14 it is concluded that:

$$MIDR_1 \times H_1 \approx MIDR_2 \times H_2 \tag{2.15}$$

or "maximum inter-story drift ratio $\times$ total height" for tall buildings with similar variation pattern for stiffness is mostly dependent on the natural period. Increasing the height of the building model will improve this approximation. The "maximum inter-story drift ratio $\times$ total height" is an upper bound for the roof displacement of the building models. This result will be numerically verified in this work, and then will be used in presentation of the drift spectra.

3. NEAR-FIELD GROUND MOTIONS

As mentioned in section 1, the drift spectrum was originally developed to provide a new measure of induced seismic demands in building models for situations in which the ground motion cannot be appropriately specified by the maximum spectral displacement. The fault-normal component of a near-field ground motion is characterized by a forward directivity pulse (Somerville 1998). The forward directivity-related velocity pulse could impart a large amount of energy to the buildings at the onset of an earthquake episode (Alavi and Krawinkler 2001). Table 1 shows the 10 near-field records used in this study. The forward directivity earthquake records have significant PGV and exhibit long period velocity pulses due to directivity effects. To provide a rational basis for comparison, the ground motion records listed in Table 1 are scaled to 0.5g. All these records were obtained from the PEER website.

Table 1. Details of the near-field ground motions

	Earthquake	M_W	Station	Date	V_{s30} (m/s)	NEHRP Site Class	PGA (g)	Source Dist. Rrupt1(km)	Comp.
1	Cape Mendocino	7.01	Petrolia	92/04/25	712.8	C	0.662	8.2	090
2	Erzincan	6.69	Erzincan	92/03/13	274.5	D	0.496	4.4	EW
3	Imperial Valley	6.53	EC Meloland Overpass	79/10/15	186.2	D	0.296	0.1	270
4	Kobe	6.90	JMA	95/01/17	312.0	D	0.821	1.0	000
5	Kobe	6.90	Takatori	95/01/17	256.0	D	0.611	1.5	000
6	Landers	7.28	Lucerne	92/06/28	684.9	C	0.721	2.2	275
7	Loma Prieta	6.93	LGPC	89/10/18	477.7	C	0.563	3.9	000
8	Northridge	6.69	Sylmar - Olive View	94/01/17	440.5	C	0.843	5.3	360
9	Northridge	6.69	Rinaldi	94/01/17	282.2	D	0.838	6.5	228
10	Tabas	7.35	Tabas	78/09/16	766.8	B	0.836	2.0	LN

4. DRIFT SPECTRUM BASED ON LUMPED MASS NON-UNIFORM BEAM MODEL

It was shown in Section 2 that, if the stiffness variation along the height of a building model is represented by Eqn. 2.2, it is possible to estimate its maximum inter-story drift under earthquake excitation using its fundamental period of vibration. Using the described lumped mass beam model for a given earthquake record, the drift spectrum for N-story building models with specific values of δ and λ and stiffness variation given by Eqn. 2.2, can be computed by simply repeating the procedure for the desired range of fundamental periods. Although a large number of drift spectra could be generated using the above approach, it will be shown that just a few spectra would be sufficient to cover most practical cases. The drift spectra for building models with shear and flexural behavior can also be produced by the proposed method. However, only the results for shear beam models are presented here.

5. THE INFLUENCE OF THE NUMBER OF STORIES ON MAXIMUM DRIFTS

Figs. 2(a) and 2(b) represent the effect of a building's number of stories on its maximum inter-story drift demands for a 0.02 damping ratio for a uniform stiffness beam and a beam with $\delta = 0.35$ and $\lambda = 2$, respectively. Clearly, the number of stories has an important effect on inter-story drift demands. For any given period, any increase in the number of stories has an adverse effect on inter-story drifts. This trend suggests that different presentations of the drift spectra, i.e., the product of inter-story drift and the number of stories, or product of maximum inter-story drift ratio (MIDR) and total height of building (H) versus the fundamental period, as shown in Figs. 3(a) and 3(b), may be more useful.

These figures show for the shear buildings with more than 20 degrees of freedom (DOFs), the value of maximum inter-story drift ratio multiplied by the total height is not very sensitive to the number of stories. To better quantify the difference between these spectra, Figs. 4(a) and 4(b) show the same spectra normalized by the spectrum of 50-DOFs. These figures show that, except for periods less than

0.5 sec (which are not feasible for high-rise buildings), the differences between the drift spectra for shear buildings with more than 20 degrees of freedom are less than 2 percent. Therefore, for tall buildings with specific values of λ and δ, a single (maximum inter-story drift ratio × total height) spectrum can be used to accurately describe their behaviour. This result is supported by Eqn. 2.18. One can conclude that it is acceptable to consider a 50-DOFs lumped mass shear beam as an approximation for a continuous shear beam. In the rest of the work, the term "drift spectra" will mean "maximum inter-story drift ratio × total height of building".

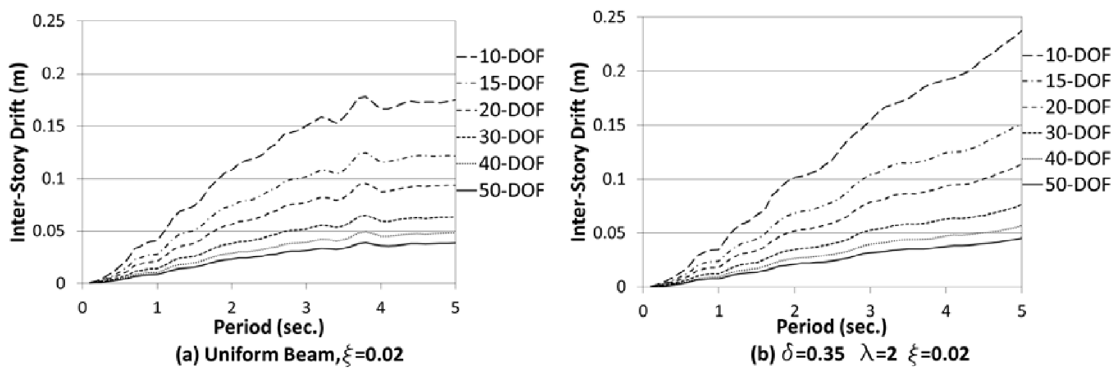

Figure 2. The effect of number of stories on the maximum inter-story drifts demands.

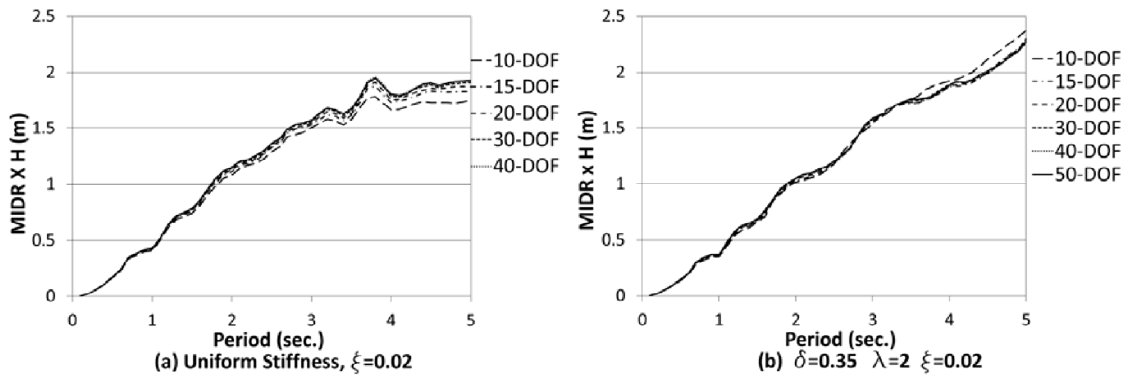

Figure 3. Alternative representation of the effect of number of stories on the maximum drifts demands.

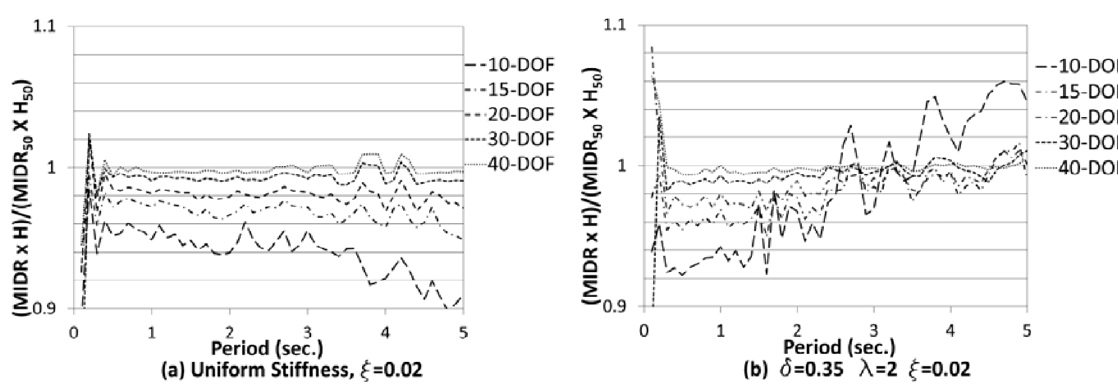

Figure 4. The effect of number of stories on the maximum inter-story drifts demands.

6. INFLUENCE OF THE STIFFNESS REDUCTION PATTERN ON MAXIMUM INTER-STORY DRIFT DEMANDS

Fig. 5(a) shows the effect of the stiffness reduction pattern λ on maximum inter-story drifts for 20-DOFs beam models with $\delta = 0.35$ and damping ratio $\xi = 0.02$. The sensitivity of the maximum inter-story drift demands to changes in λ is small, thus making it possible to assign it a fixed value. Since, for most of the building models, λ lies between 1 and 3, for the remainder of this work, this parameter is assumed to have a value of $\lambda = 2$. Fig. 5(b) shows the same spectra normalized by the spectrum of $\lambda = 2$. It is seen that the error due to ignoring λ is in general less than 10%, while for a wide range of periods, it is less than 5%.

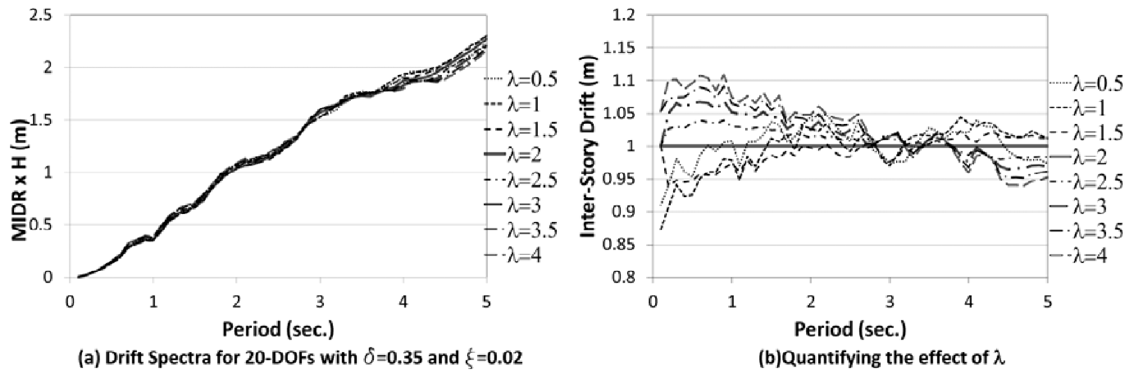

Fig. 5. The sensitivity of the maximum inter-story drift demands to the stiffness reduction pattern.

7. INFLUENCE OF THE STIFFNESS RATIO OF THE TOP STORY TO THAT OF THE FIRST STORY ON MAXIMUM INTER-STORY DRIFT

Fig. 6(a) shows the effect of the lateral stiffness ratio of the top story to that of the first story, δ, on maximum inter-story drift demands for 20-DOFs models with $\lambda = 2$ and $\xi = 0.02$. Fig. 6(b) shows the same spectra normalized by the spectrum for $\delta = 1$. These figures clearly demonstrate that the effect of δ on maximum inter-story drifts for values of $\delta < 0.5$ is not small. Reducing the lateral stiffness ratio causes inter-story drifts to increase significantly.

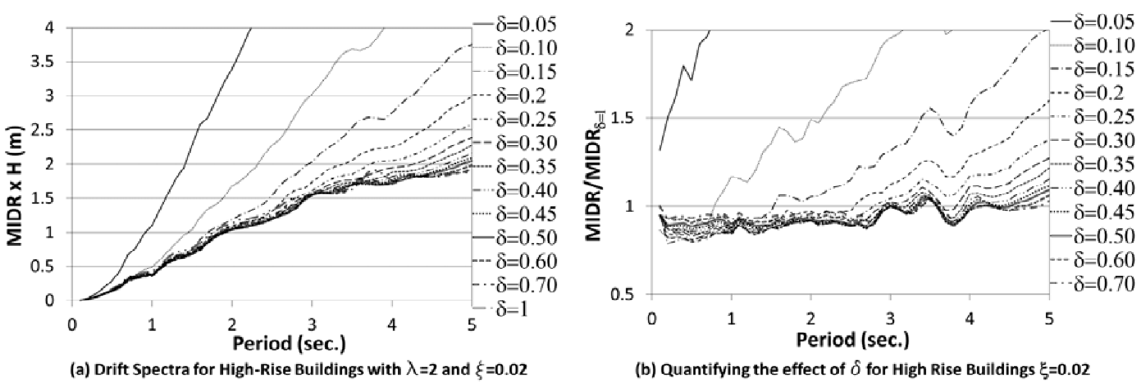

Fig. 6. The sensitivity of the maximum inter-story drift demands to the parameter, δ.

This effect for values of δ greater than 0.5 is insignificant. A value of $\delta = 0.5$ can be used for values of $\delta \geq 0.5$ with an error below 10%, while for values of $\delta < 0.5$, different spectra should be generated. These figures show that the effect of lateral stiffness ratio of the top stories with respect to the first ones is considerable. For further investigation, the distribution of inter-story drift ratio demands over height of two shear buildings with 30 stories and fundamental periods of 3 and 4 seconds are illustrated in Figs. 7(a) and 7(b), respectively. In these figures, demands are computed for various values of δ and with $\lambda = 2$ and $\xi = 0.02$. The results show that the effect of lateral stiffness ratio of the top story to that of the first story not only affect the value of maximum inter-story drift demand, but also influences the distribution of drift demands.

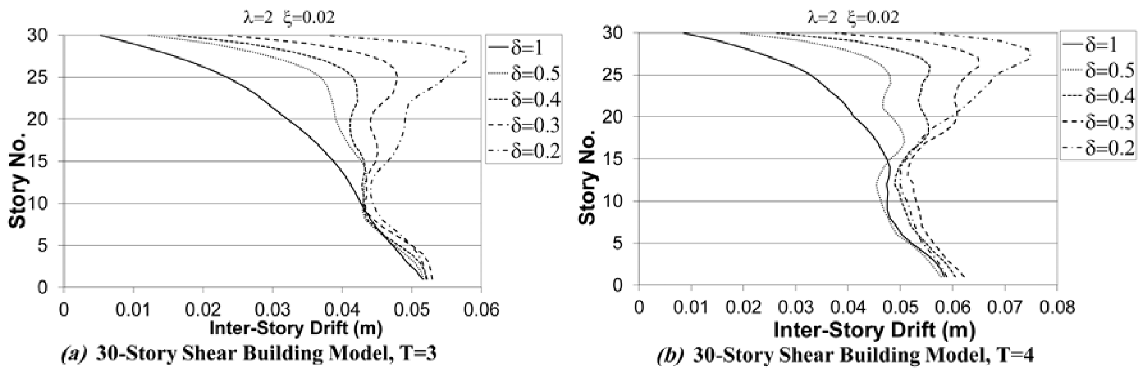

Fig. 7. The effect of height-wise distribution of stiffness on height-wise distribution of maximum inter-story drift demands.

8. CONCLUSIONS

A new method based on a lumped mass shear beam model for estimating the inter-story drift demands of buildings is proposed. The proposed method is simple because it uses well-established lumped mass beam concept rather than complicated continuous shear-flexural beam representation. No code-based relationship is used in this approach to estimate the height of the structural models from their fundamental period. The drift spectra for shear buildings with different number of stories and non-uniform distribution of stiffness along the height are presented. It is observed from the parametric studies that the maximum inter-story drift is not strongly dependent on the stiffness variation pattern, and this parameter can therefore be ignored. For shear buildings in which the lateral stiffness ratio of top to base stories is greater than 0.5, a single drift spectrum can be used, but for ratios less than 0.5, different spectra would be needed. Moreover, it is deduced that the product of maximum inter-story drift ratio and total height for tall buildings with similar variation pattern for stiffness are mostly dependent on the natural period. Based on this finding, for any lateral stiffness ratio of top floor to the base story, for buildings taller than 20 stories, a single spectrum is presented. The proposed drift spectra are powerful tools for structures with regular height-wise stiffness distribution. They can be used to study the effect of various parameters on inter-story drift demands. Finally the proposed approach can be employed for preliminary assessment of drift demands in structures, providing guidelines to optimum design of structures.

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