Effect of Spatial Variability of Soil Properties on the Seismic Response of Earth Dams

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SUMMARY:

Variability of soil properties is a major source of uncertainty in assessing the seismic response of geotechnical systems. This study presents a probabilistic methodology to evaluate the seismic response of earth dams. A sensitivity analysis is performed by means of Tornado diagrams. Two-dimensional, anisotropic, cross-correlated random fields are generated based on a specific marginal distribution function, auto-correlation, and cross-correlation coefficient. Nonlinear time-history analyses are then performed using an advanced finite difference software (FLAC 2D). The study is performed using Monte Carlo simulations that allowed to estimate the mean and the standard deviation of the maximum crest settlement. The statistical response is compared with results of a deterministic analysis in which the soil is assumed homogeneous. This research will provide insight into the implementation of stochastic analyses of geotechnical systems, illustrating the importance of considering the spatial variability of soil properties when analyzing earth dams subjected to earthquake loading.

Keywords: Earth Dams, Spatial Variability, Cross Correlated Random Field, Seismic Response.

1. INTRODUCTION

It is well known that soil properties vary in space even within otherwise homogeneous layers. This spatial variability is highly dependent on soil type or the method of soil deposition or geological formation. Nevertheless, many geotechnical analyses adopt a deterministic approach based on a single set of soil parameters applied to each distinct layer. In recent years, the effect of inherent random variation of soil properties has received considerable attention. Griffiths and Fenton (2002), Fenton and Griffiths (2003) and Popescu *et al.* (2005) examined the response of shallow foundations; Haldar and Babu (2007) analyzed the response of a deep foundation under vertical load; Paice *et al.* (1996) studied settlements of foundations on elastic soil; Griffiths and Fenton (2000) studied slope stability; Popescu *et al.* (2005, 1997) and Koutsourelakis *et al.* (2002) studied seismically-induced soil liquefaction whereas Kim *et al.* [2007] reported an update on emergent research related to variability in soil properties. The spatial fluctuation of a parameter cannot be accounted for if the parameter is modelled only using a random variable. Random field theory and/or geostatistics are needed if a more accurate representation of the spatial variability of this parameter is desired in the analysis.

In this study, a numerical procedure for a probabilistic analysis that considers the spatial variability of soil properties is presented in a framework related to the assessment of the seismic response of an earth dam. First of all, a sensitivity analysis by means of Tornado diagrams was performed. Results were used to determine the input parameters that influence the response the most in order to be modelled as random fields. Two-dimensional, anisotropic, cross-correlated random fields are generated based on a specific marginal distribution function, auto-correlation, and cross-correlation coefficient. Subsequently, nonlinear dynamic time-history analyses are performed using an advanced finite difference numerical software, FLAC 2D (Itasca 2000).

The study is performed using Monte Carlo simulations and allowed to estimate the mean and the standard deviation of the maximum crest settlement, which is the selected engineering demand parameter. The statistical response is finally compared with the results of a deterministic analysis in which the soil is assumed homogeneous.

2. SENSITIVITY ANALYSIS

Reduction of the number of uncertain parameters cuts down the computational effort and cost. One way of doing this is to identify those parameters with associated ranges of uncertainty that lead to relatively insignificant variability in the response, and then treating these as deterministic parameters by fixing their values at their best estimate, such as the expected value. In order to rank uncertain parameters according to their sensitivity to the desired response parameters, there are various methods such as Tornado diagram analysis, FOSM analysis and Monte Carlo Simulations (MCS). The latter, which is computationally demanding due to the requirement of a large number of simulations, is not used in this part of the study. Instead, Tornado diagram have been used due to their simplicity and efficiency to assess sensitivity of uncertain parameters.

2.1. Tornado Diagrams

The procedure for constructing a tornado diagram is based on implementing the following steps (Lee and Mosalam, 2005):

- 1. Determine 10%, 50% and 90% fractiles of all Random Variables (RVs);
- 2. Perform dynamic analysis with the set of 7 real-spectrum-compatible records selected for 101, 475 and 2475 years return period (21 analysis) setting all RVs at their median values;
- 3. Select the median Ground Motion (GM) for each Engineering Demand Parameter (EDP);
- 4. Using the median GM of Tr=475 years, run each RV other than ground motion for 10% and 90% fractiles. In this case, seven runs (one for each RV considered) with 10% fractile values plus seven runs with 90% fractile values were performed.

2.2. Definition of seismic input and distribution of input parameters

Suites of real ground motion records selected for the 101, 475 and 2475 years return period (39%, 10%, and 2% probability of exceedance in 50 years) were used respectively as lower, mean and upper bound, to carry out a series of dynamic analyses of the dam with the soil properties fixed at their best estimate (mean values). Further details regarding the seismic hazard at the site and the selection of the records can be found in Sanchez (2011). Fig. 2.1 shows the set of 7 real selected records that were used for the 475 years return period.

Once a lower and upper bound has been selected for the Intensity Measure (IM), in this case the Peak Ground Acceleration (PGA), the variables to be considered as randomly distributed can be perturbated for their use in sensitivity analyses. Different fractiles values are chosen as input for Tornado analyses as summarized in Table 2.1 and Table 2.2.



Figure 2.1 Suite of 7 selected real accelerograms for the 475 years return period

Realistic probability distributions were assumed for the soil strength parameters such as friction angle $[\phi']$, cohesion [c'], and small-strain shear modulus [G] of the embankment and foundation soil. A normal probability distribution was used for the friction angle and cohesion while a lognormal probability distribution was assumed for the small-strain shear modulus of the embankment and of the foundation material. Such distributions were used based on similar studies found in the literature (Na *et al.*, 2008), from which the Coefficient of Variations (CoVs) were also determined.

Table 2.1. Distributions adopted for the strength random variables and fractile values used in sensitivity analysis (Table 1 of 2).

	φ' embankment [°]	¢' core [°]	¢' foundation [°]	Cohesion embankment [Pa]	Cohesion core [Pa]	Cohesion foundation [Pa]
Distrib.	Normal	Normal	Normal	Normal	Normal	Normal
Χμ	24	30	35	30000	40000	50000
CoV %	9	9	9	40	40	40
Χμ-σ	21.84	27.3	31.85	18000	24000	30000
Χμ+σ	26.16	32.7	38.18	42000	56000	70000
X10%	21.23	26.54	30.96	14620	19500	24370
X90%	26.77	33.46	39.04	45380	60500	75630

Table 2.2. Distributions adopted for the stiffness random variables and fractile values used in sensitivity analysis (Table 2 of 2).

	G embankment [MPa]	G foundation [MPa]
Distrib.	Lognormal	Lognormal
Χμ	86	441
CoV %	12	12
Χμ-σ	75.6	375
Χμ+σ	96.3	510
X10%	73.2	381
X90%	99.5	494

2.3. Results of Sensitivity Analysis

The Tornado diagram corresponding to the residual vertical displacement at the top of the crest of the dam is shown in Fig.2.2. In this chart, the vertical line (also called "reference line") represents the response of the dam computed exclusively with the mean values. The swing corresponding to the intensity measure (PGA) is generated by plotting the EDP obtained with the median input of each set.

It can be seen that PGA dominates the variability. Cohesion of the embankment, together with ground motion, induces an important swing, meaning that these three variables represent the highest source of uncertainty. The high variability of the response due to the embankment cohesion can be attributed to the large CoV used for this parameter (40%). All other random variables have a very small influence. For the example study at hand, it turns out that the effective friction angle and the cohesion of the various parts of the embankment are the most significant geotechnical parameters in terms of their effect on the computed response. Therefore, they were modelled as random variables.



Figure 2.2 Tornado diagram of the vertical residual displacement (crest settlement) at the top of dam

3. RANDOM FIELD MODEL

A random field $H(x,\theta)$ is a collection of random variables associated with a continuous index $x \in \Omega \subset \mathbb{R}^n$, where $\theta \in \Theta$ is the coordinate in the outcome space. The field is completely defined by its mean $\mu(x)$, variance $\sigma^2(x)$ and autocorrelation function $\rho(x,x')$. A random field H is distributed according to $H \sim N(x, \Sigma)$ where Σ is the variance-covariance matrix. The matrix Σ is generated with the autocorrelation matrix, for this study an exponential autocorrelation function is used and different autocorrelation distances in the vertical and horizontal directions are used as follows:

$$\rho(\mathbf{x}, \mathbf{y}) = \exp\left(-\frac{|\mathbf{x} - \mathbf{x}'|}{\theta h} - \frac{|\mathbf{y} - \mathbf{y}'|}{\theta \mathbf{v}}\right)$$
(3.1)

where θ h and θ v are autocorrelation distances in the horizontal and vertical direction respectively. The correlation distance θ h and θ v are used to prepare the correlation matrix, whereas the CoV is used to determine the standard deviation of the input variables. The value of lag distance |x| and |y| is the center-to-center distance of two consecutive grid zones (from the FLAC model). Once Σ is established, it is decomposed using Cholesky decomposition technique. The correlated standard normal random field is obtained using Eq.3.2 where G(x) is the multiplication of the decomposed

correlation matrix and a sequence of independent standard normal random variables (with zero mean and unit standard deviation).

$$H(x,\theta) = \mu(x) + \sigma(x) \cdot G(x)$$
(3.2)

Typically, more than one random property is involved in geotechnical problems. In this study, friction angle and cohesion are considered as random geotechnical parameters. Cross-correlation between cohesion and friction angle is known, therefore, the framework presented by Vořechovský (2008) was adopted to generate Gaussian cross-correlated random fields with a specific marginal distribution function, autocorrelation function, and cross-correlation coefficients.

3.1. Gaussian cross-correlated random field

For this study, the Karhunen-Loève (KL) expansion method is adopted to discretize anisotropic random fields of soil properties in a two-dimensional space. The KL expansion of a random field $H(x,\theta)$ is based on the spectral decomposition of its autocorrelation function $\rho(x,x')$. According to Vořechovský (2008), the set of deterministic functions over which any realization of the field $H(x,\theta)$ is expanded is defined by the eigenvalue problem as

$$\int_{\Omega} \rho(\mathbf{x}, \mathbf{x}') \varphi_{i}(\mathbf{x}') d\Omega \mathbf{x}' = \lambda_{i} \varphi_{i}(\mathbf{x})$$
(3.3)

in which ϕ_i and λ_i denote respectively the eigenfunctions and eigenvalues of the autocorrelation function. The series of the deterministic set forms the expansion of $H(x,\theta)$:

$$H(x,\theta) = \mu + \sum_{i=1}^{\infty} \sigma \sqrt{\lambda_i} \phi_i \xi_i(\theta), \quad x \in \Omega$$
(3.4)

where $\xi(\theta)$ is a set of orthogonal random coefficients (uncorrelated random variables with zero mean and unit variance).

Each field of cohesion and friction angle is expanded using a set of independent random variables, and these sets are then correlated with respect to the assumed cross-correlation matrix between two expanded random fields according to the framework presented by Cho and Park (2009) and Vořechovský (2008), where the Gaussian random field is obtained using Eq.3.5, with \hat{H} equal to $\sqrt{\lambda_j}\varphi_j(x)\chi_{i,j}^D(\theta)$. More details regarding the methodology and the definition of X^D can be found in Vořechovský (2008).

$$H(x,\theta) = \mu i(x) + \sigma_i . \dot{H}$$
(3.5)

for (i=c, ϕ)

3.2. Simulations

The question as to how cohesion and friction angle are correlated is still not clearly defined in the literature, and certainly depends very much on the soil being studied. Cherubini (2000) quotes values of cross-correlation coefficient r ranging from $-0.7 \le r \le -0.24$, as does Wolff (1985), Yuceman *et al.*

(1973) reported values in a range of $-0.49 \le r \le -0.24$, while Lumb (1970) noted values of $-0.7 \le r \le -0.37$.

A negative correlation implies that low values of cohesion are associated with high values of friction angle and vice versa. In other words, a negative correlation between the cohesion and the friction angle means that the uncertainty in the calculated shear strength is smaller than the combined uncertainty in the two parameters values used to model the shear strength.

Fig.3.1 and Fig.3.2 show in a 3D plot the random realizations (\hat{H}) to simulate two-variate Gaussian fields for cohesion and friction angle. X^D was computed for a hypothetical field of 20x40 elements where r ranges from -0.7≤r≤0.4. The CoV of each variable and the autocorrelation distances are not changed from one simulation to another. In this way, the influence of the cross-correlation coefficient can be addressed.



Figure 3.1 a)-b)Random realization \hat{H} of simulated two-variate Gaussian Random field for cohesion and friction angle for an assumed cross-correlated coefficient equal to -0.7. c)-d) Random realization \hat{H} of simulated twovariate Gaussian Random field for cohesion and friction angle for an assumed cross-correlated coefficient equal to -0.5.



Figure 3.2 a)-b)Random realization Ĥ of simulated two-variate Gaussian Random field for cohesion and friction angle for an assumed cross-correlated coefficient equal to 0 (independent random field) c)-d) Random realization Ĥ of simulated two-variate Gaussian Random field for cohesion and friction angle for an assumed cross-correlated coefficient equal to 0.4.

Fig. 3.3 shows one realization of cross-correlated random fields of friction angle. The CoV used was 9% (see Table 3.1), the cross correlation coefficient between friction and cohesion was set equal to 0.5, the correlation distance assumed in the vertical direction was 4 m while in the horizontal direction it was assumed correlated along all the length.

For this study, the computational process was conducted by purposely-developing a MATLAB function that generates 2D normal random fields for each of the input variables considered. The value assigned to each element of the FLAC mesh to be used for the nonlinear analysis was obtained by mapping the element centroids to the field obtained from the random field generator. For each set of statistical properties given in Table 3.1, MCS is performed and n realizations are generated for each value of PGA. Details regarding the numerical modelling of the embankment with FLAC 2D can be found in Sanchez (2011).



Figure 3.3 Realization of cross-correlated random field of friction angle for the example dam.

Property	Mean	CoV (%)	Distribution
c' embankment (Pa)	30000	40	Gaussian
¢' embankment (°)	24	9	Gaussian
c' foundation (Pa)	50000	40	Gaussian
φ' foundation (°)	35	9	Gaussian
c' core (Pa)	40000	40	Gaussian
φ' core (°)	30	9	Gaussian
G foundation (MPa)	441	12	Non-Gaussian
¢' layer (°)	24	9	Gaussian

Table 3.1. Uncertainty assumed for the input parameters

4. RESULTS

At the purpose of comparing deterministic and stochastic analyses, 100 realizations of the numerical model were generated and analyzed using only one record (input 4 of set Tr=475 years). The same input motion was used when analyzing the deterministic model (mean values). In this way the results reflect only the influence of the uncertainty of geotechnical input parameters and can be compared with the deterministic case. A total of 700 computer hours were needed to obtain Fig.4.1. The 100 y-displacement time histories obtained after assuming spatial variability of the soil are plotted together with the mean of such realizations and the y-displacement time history of the deterministic analysis in which the soil was assumed as homogeneous.

One thing that becomes evident is the variability of the response when the uncertainty of input parameters is accounted for. The value of the maximum crest settlement obtained with 100 analyses ranged from zero displacement to almost 12 cm. Furthermore, it is observed that the y-displacement time history of the analysis carried considering the soil as homogeneous is considerably lower that the mean displacement time history obtained with the 100 analyses.



Figure 4.1 Comparison of y-displacement time history obtained with the assumption of homogeneous soil and the mean y-displacement obtained from 100 realizations assuming spatially varying soil.

5. CONCLUSIONS

A numerical procedure for a probabilistic analysis that considers the spatial variability of soil properties is presented. The methodology was implemented to study the seismic response of earth dams. A sensitivity analysis was performed by means of Tornado diagrams. Two-dimensional cross-correlated non-Gaussian random fields were generated and mapped into the FLAC 2D model. A comparison was made between deterministic and stochastic analyses. The value of crest settlement obtained with the analysis carried out considering the soil as homogeneous is considerably lower that the mean crest settlement obtained with 100 analyses. Therefore, for this study the random modelling of soil properties increases the seismic demand hazard. Accounting for the uncertainty of soil parameters was found to be a significant factor in the prediction of the response of the earth dam. The obtained results offer insight regarding the stochastic analysis in the field of geotechnical engineering and demonstrate the importance of the spatial variability of soil properties in the outcomes of a probabilistic assessment of geotechnical system subjected to earthquake loading.

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