SEISMIC RELIABILITY ASSESSMENT OF 5 AND 10-STORY BUILDINGS. PART III.

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SUMMARY:
A methodology that enables us to obtain efficiently the seismic reliability of buildings for different levels of seismic intensity is presented. The methodology taking into account the uncertainties associated with the properties of the considered structural system and seismic recordings. This methodology is developed by means of Incremental Dynamic Analysis of the Secant Stiffness Degradation Index (IDA-SSDI), whose results enable evaluation of the intensity of system collapse in an easy and efficient manner in order to evaluate the reliability by means of Cornell's β index and a Z safety margin. The methodology is applied to a sample of eight 5-story and eight 10-story buildings generated from Monte Carlo's simulation and 40 accelerograms registered in the Mexico City Valley. The buildings simulated are modeled in the computer program DRAIN 2D for non-linear analysis.

Keywords: Seismic reliability, Cornell's Reliability Index, safety margin.

1. INTRODUCTION AND METHODOLOGY

The evaluation of the reliability of complex structures is not easy, especially it is desired to take into account all of the associated uncertainties. One of the problems is obtaining a collapse system, in order to determine when to establish the collapse, under what criteria, as well as which method to use.

This research uses the Incremental Dynamic Analysis Method (IDAM) proposed by Vamvatsikos and Cornell (2002), which is based on the collapse intensity estimation *(Yc)* with the help of an adequate scale factor that should be applied to the accelerogram. The possibilities of visualizing the evolution of the seismic response amplitudes in the manner in which the intensity grows and of observing the values reached by that intensity before the appearance of the seismic response amplitude outside of its limits, constitutes an important advantage of this method. For these reasons, within this paper, the collapse intensity *(Yc)* was obtained by means of the Incremental Dynamic Analysis Method but applied to the Secant Stiffness Degradation Index (IDAM-SSDI). This index is given through the following equation (Díaz-López and Esteva, 2009):

$$D(y) = 1 - \frac{K_s}{K_0} \tag{1.1}$$

Where y is the intensity of the seismic movement, K_0 is the value that K_S acquires when it is linear, and K_S is the value of the degraded secant stiffness adopted by the system at the moment when the lateral displacement in the roof reaches its maximum value. In this manner, the collapse intensity *(Yc)* is obtained when $D(Yc)=1$.

This $D(y)$ index as an indicator of collapse is very useful since it eliminates the difficulty in defining the collapse condition at the maximum roof displacement point.

Valuable information can be obtained related to the structural system's behavior at ranges between $0.8 \leq d < 1.0$ that are associated to the near collapse limit state through the IDAM-SSDI analysis.

On the other hand, the reliability of a structure is denoted by R and is defined as:

$$R = 1 - P_f \tag{1.2}$$

where P_f is the probability that the structure will fail during the specified reference period.

In this study, interest is focused on the reliability of the collapse; therefore, reliability functions will be referred exclusively to the failure probability, P_f. For simplicity purposes instead of working with the failure probability, the reliability functions will be presented in terms of an approximate indicator, Cornell's β index (1969), expressed in this case for a seismic excitation with known intensity equal to y, but with uncertain detailed characteristics as follows:

$$\beta(y) = \frac{m_z(y)}{\sigma_z(y)} \tag{1.3}$$

Where $m_z(y)$ is the median value of safety margin Z of the building subjected to an earthquake with an intensity equal to (y) occurs, and $\sigma_Z(y)$ is the standard deviation of this safety margin.

In this work, an alternative approximation is presented for the estimation of the index $\beta(y)$, (Díaz and Esteva, 2009), where the safety margin $Z(y)$ is defined as the natural logarithm of the rate of (Yc) and of the intensity of the ground movement (y) that act upon the system.

$$Z(y) = \ln \frac{Y_C}{y} = \ln Y_C - \ln y \tag{1.4}$$

For one y given $E(lny)=lny$ and $\sigma(lny)=0$ therefore $\beta(y)= E(\ln Y_C) - \ln y / \sigma(\ln Y_C)$ where $Z_C=\ln Y_C$.
In this manner, the reliability index is defined as:

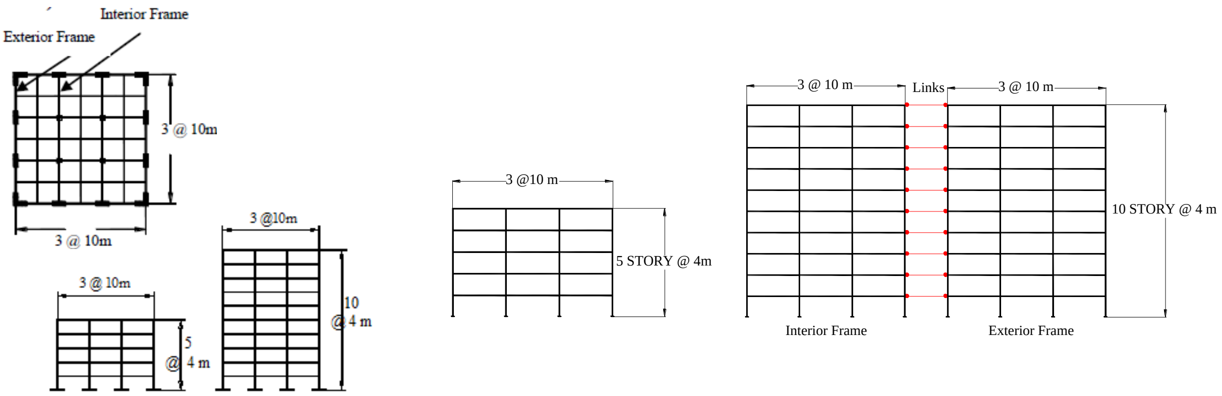
$$\beta(y) = \frac{E(\ln Y_C) - \ln(y)}{\sigma(\ln Y_C)} \tag{1.5}$$

Where, an adequate reliability index $\beta(y)$ should be positive. A negative reliability index is an undesirable index, which will indicate a high probability that the structure will collapse and this occurs when the seismic intensity (y) is greater than the expected value of the natural logarithm of the collapse intensity $E(\ln Y_C)$, which is to say: $y_i \geq E(\ln Y_C)$.

2. ANALYZED STRUCTURES

The aforementioned methodology is applied to a sample of eight 5-story and eight 10-story buildings from the Monte Carlo simulation, taking into account the uncertainties associated with the concrete strength $(f'c)$, the steel yield (fy), the live loads $(Wvmax)$ and 40 accelerograms registered in the Mexico City Valley. The geometric properties of the original buildings are shown on Figure 2.1. The buildings were designed in accordance with the Mexico City Seismic Design Code (2004) for official use. The fundamental periods of the buildings are $To = 0.67s$ and $To = 1.17s$ and have yield strength coefficients of $Cy=0.34$ and $Cy=0.4$, for 5-story and 10-story buildings, respectively (Montiel, 2006).

The building simulations were modeled with DRAIN 2D program, modified by Campos and Esteva (1997), for a non-linear analysis. Two models were considered in the study: the first model is a 5 story building and the second one is a 10 story. The 10 story building has one external and one internal frame which are connected by a rigid diaphragm. The coupling between the frames was taken into account by means of a two-dimensional structural model connected by hinged links (see figure 2.1c). For the 5 story model only one two-dimensional simple frame was considered (see figure 2.1b). The frames are formed by columns and beams in flexure. The relation moment-rotation for each element was calculated using the model for confined concrete proposed by Kent and Park (1971) and modified by Park et al, (1982). The axial stress-strength relation used on the steel rods was modeled using the Mander´s model (Mander, 1984). The hysteretic structural behavior was considered as bilinear, hence the post-fluency relation has a 3% of initial stiffness.



a) b) c)

Figure 2.1. a) Original building plan and elevation and two-dimensional model used for non-linear analysis of the b) 5-story building and c) 10-story building.

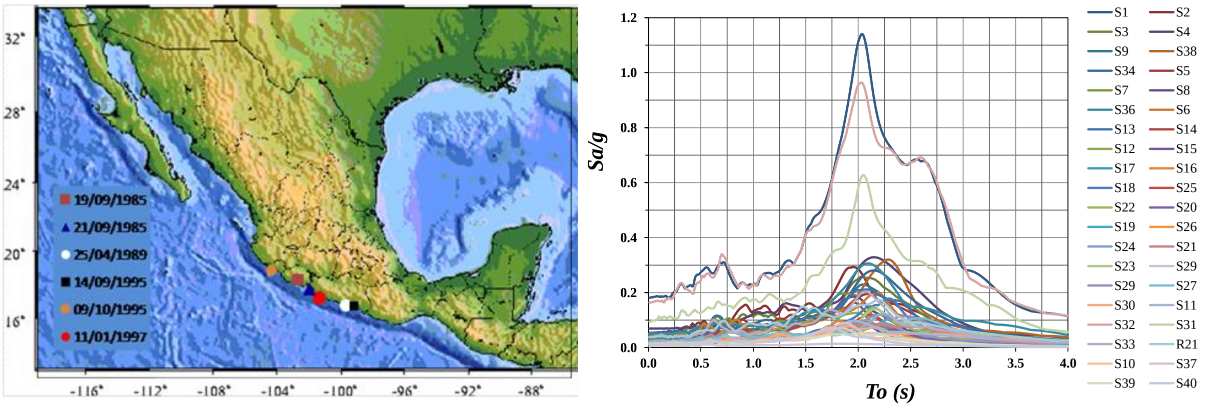
The Table 2.1 shows the eight models considered for each building, these models take into account the uncertainties in the mechanical properties of the materials (*f'c* and *fy*) and the maximum live loads (*Wmax*), furthermore it's, such shows the nominal (MN) and mean (MM) values for each one. This table applies the (M1-M8) nomenclature to denote the different uncertainties in the mechanical properties of the materials and the maximum live loads acting in each model, (M9) it is for the model with mean values and (M10) with nominal values. The uncertain values of each parameter were obtained through simulations, Rangel et al (2005).

Table 2.1. *f´c ,fy* y *Wvmáx* values used in frame analysis

Frame	f'_c (kg/cm^2)	f_y (kg/cm^2)	$W_{máx}$ (kg/m^2)
M1	308	5090	78
M2	246	4445	134
M3	287	3970	40
M4	234	4816	89
M5	250	4055	57
M6	199	5553	99
M7	330	5974	104
M8	304	4407	67
M9=MM	268	4680	75
M10=MN	250	4200	180

3. SEISMIC MOVEMENTS USED FOR THE ANALYSIS

The analysis of seismic reliability requires the use of registers that can adequately reflect the dynamic characteristics and the energy content of the seismic movements expected to occur at the construction site. For this analysis, forty narrow-band earthquakes registered in the Mexico City lake region were used. All the seismic movements were recorded during subduction events with epicenters located on the Pacific coast of Mexico, see Figure 3.1a, which present magnitudes *(Mw)* from 6.0 to 8.1 grades. The corresponding elastic strength spectra of pseudo-acceleration are shown on Figure 3.1b, for a critical damping percentage (ξ) of 5%. Notice that the seismic movement spectra have dominant periods between 1.5 and 2.2s.



a) b)

Figure 3.1. a) Subduction earthquake epicenters used for the analyses and, b) the pseudo-acceleration corresponding elastic spectra for 5% damping.

4. INCREMENTAL DYNAMIC ANALYSIS OF THE SECANT STIFFNESS DEGRADATION INDEX *(IDA-SSDI).*

The *I*ncremental *D*ynamic *A*nalysis *M*ethod of the *S*ecant *S*tiffness *D*egradation *I*ndex *(IDAM-SSDI)* was used to obtain the probabilistic estimations of the seismic intensity values that lead to system collapse in order to evaluate the structural reliability. The analysis results are shown on figure 4.1a and 4.1b, the 40 seismic movements (S1-S40) from the building of 5 and 10 story's and their respective 8 models of each one (M1-M8).

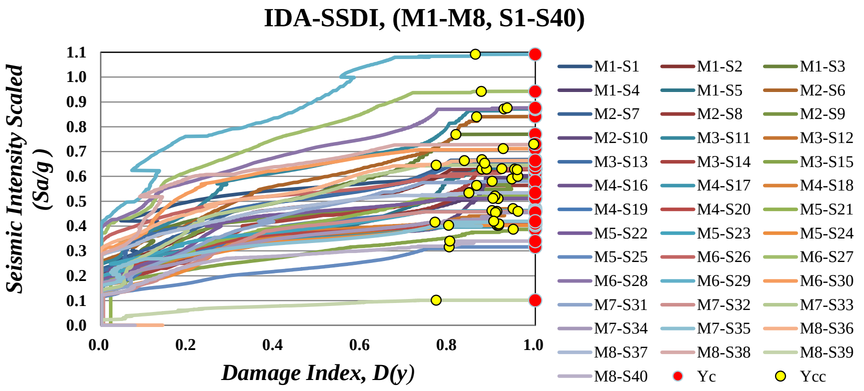
In these figures the vertical axis shows the level of seismic intensity measured with pseudo-acceleration spectra associated to the fundamental period of the structure with a 5% of critic damping, where the pseudo-acceleration is divided by the gravity acceleration *(g)*. The seismic intensity has been affected by a proper scaling factor to lead the structure into a collapsing state.

The horizontal axis shows the structural performance measure by the Secant Stiffness Degradation Index *D(y)*. This index is a proposed damage indicator that the structure suffered, caused by a seismic movement while the intensity raised. The seismic effects de-crease the capacity of the building leaving the structural system in a state of greater vulnerability and eventually leading to collapse.

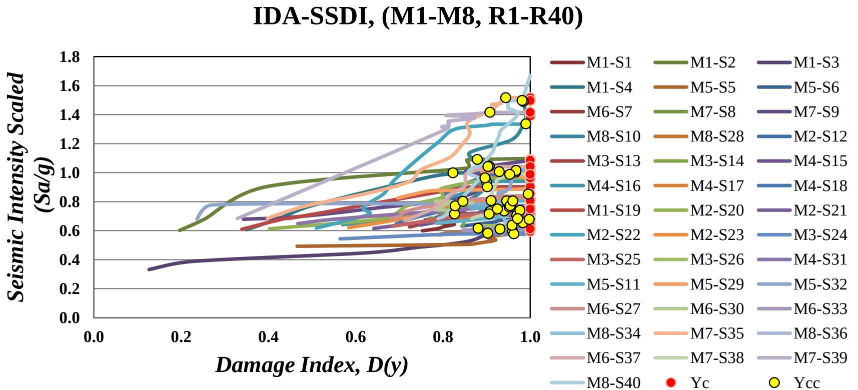
For the 5-story building a computer program is used to attain the IDAs, for which each curve contains approximately 1,000 points. One important aspect to consider for the analyses is the increase in the scale factor that varies from earthquake to earthquake, affecting the duration.

In the case of the 10-story building, (because it was the first building analyzed, the curves were done by hand, without fortran computer program), which implies a smaller number of points in each curve (between 30 and 50). That is why the curves were initiated from high damage indices. In this case, the analyses are much more laborious and took up much more time.

The curve forms on Figure 4.1a and 4.1b vary depending on the earthquake, the frame, the number of structures levels and of the non-linear behavior structure. These curves are obtained by scaling the seismic intensity until a state of collapse occurred. This is indicated in the figures by means of red circles *(Y_C,)*, which occur when the secant stiffness degradation value adopted by the system at the moment when the lateral displacement in the roof reaches its maximum value, that is zero, *($K_s=0$)*. That is to say, when the Secant Stiffness Degradation Index is equal to the unit *($D(Yc)=1$)*, which corresponds to a base shear in terms of very low or zero capacity. One moment before the state of collapse occurs, the seismic intensities nearing collapse are obtained, which are indicated in the figure with yellow circles (Y_{Cc}), varying from $0.769\leq D(y)<1.0$. Using *D(y)* as an index of collapse is very useful because it eliminates the difficulty of defining the collapse state from the maximum displacement of the roof.



a) 5-Story Building



b) 10-Story Building

Figure 4.1 Incremental Dynamic Analysis of the Secant Stiffness Degradation Index (*IDA-SSDI*) for 40 earthquakes. Yellow circles indicate the intensity approaching collapse (Y_{CC}) and red circles indicate the collapse intensity (Y_C) when *D(y)=1*.

The statistical values corresponding to the parameters of mean and standard deviation from the collapse limit state and the near collapse associated to the intensity *(y)* and the Damage Index *D(y)* are shown in tables 4.1 and 4.2.

As seen in the tables there is not significant difference between the Y_C and Y_{CC} intensities on the collapse limit state and the near collapse to evaluate the reliability of the system. On the other hand the *$D(Yc_c)$* and *D(Yc)* values vary depend on the structure sensibility near to the collapsing zone.

Table 4.1. Statistical parameters of the limit state near collapse and at collapse. 5-story building.

	Y_{Cc} Sa/g	Y_C Sa/g	$D(Y_{Cc})$	$D(Y_C)$
Average	0.5728	0.5743	0.8899	1.00
Standard deviation	0.1895	0.1891	0.0573	0.00

Table 4.2. Statistical parameters of the limit state near collapse and at collapse. 10-story building.

	Y_{Cc} Sa/g	Y_C Sa/g	$D(Y_{Cc})$	$D(Y_C)$
Average	0.8832	0.8898	0.9308	1.00
Standard deviation	0.2610	0.2620	0.0470	0.00

In Figure 4.2 the hysteresis cycles can be observed when the structure is submitted to different seismic intensity levels. There is a secant stiffness for each hysteretic semi-cycle. It is possible to obtain the secant stiffness (K_i) for each semi-cycle by means of the following equation:

$$K_i = \frac{V_i}{D_i} \tag{4.1}$$

where, D_i is the roof displacement of the cycle *i*. V_i is the base shear force corresponding to that displacement. The interest of this study lies in the cycle with the maximum roof displacement (D_{max}), from where is obtained the most degraded secant stiffness (K_s):

$$K_s = \frac{V_b}{D_{\max}} \tag{4.2}$$

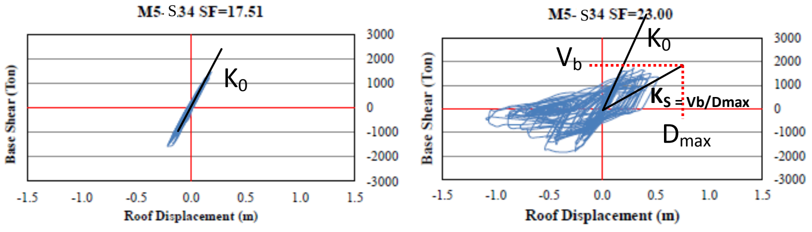
where V_b is the shear in the base associated with D_{max}.

With this stiffness (K_s) and the original stiffness of the system (K_0), the degradation index of the secant stiffness *D(y)* is obtained, corresponding to the equation 1.1.

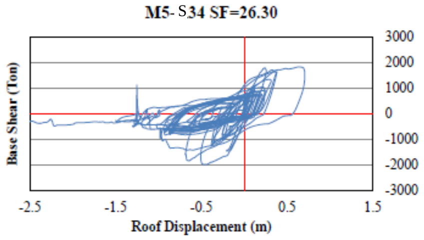
For example: in Figure 4.2a, it can be observed that when the earthquake is scaled by a factor of *SF=17.51*, there is a behavior which is practically linear for this earthquake and frame (S34, M5), for which the secant stiffness $K_s=7196.92$ tons/m is in the order of the elastic stiffness: $K_0=9882.86$ tons/m, which in the aforementioned damage index is low: *D(y)=0.272*. That is to say, a structure that is practically elastic.

In Figure 4.2b, for *SF*=23, the nonlinear behavior is significant, therefore the secant stiffness is low: $K_s=384.40$ *tons/m*, resulting in a damage index close to one: *D(y)=0.9611*.

In Figure 4.2c, for a *SF*=26.30, as can be observed, the structure has collapsed, and displacements is considerable, giving a value of $K_s=0$; and therefore, a damage index of *D(y)*=1. Certainly, in the previous cases, there were intermediate *SFs*, which made possible the observation of the evolution of the damage index *D(y)* as can be seen in Montiel and Díaz de León, 2011, for different levels of seismic intensity.



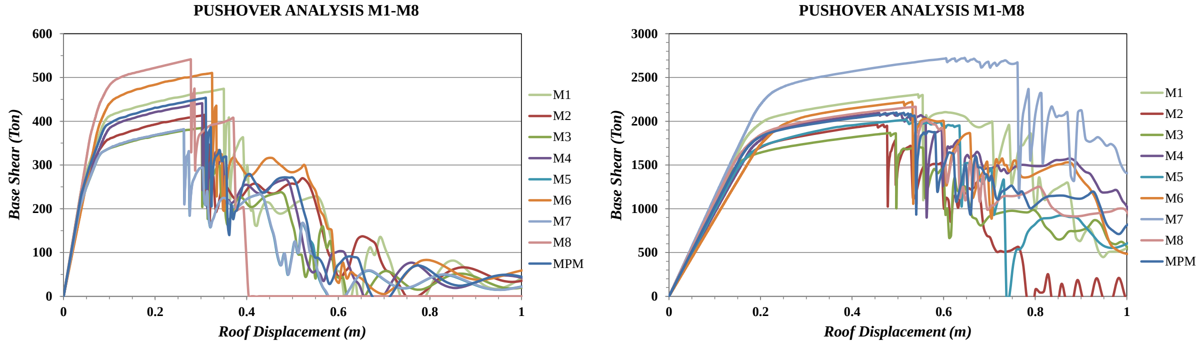
a) Practically elastic structure, *D(y)=0.272.*   b) Near collapse structure, *D(y)=0.9611.*



c) Collapsed structure, *D(y)=1.*

Figure 4.2 Global response hysteresis cycles for different Scale Factors *(SF).* 10-Story Building.

The elastic stiffness *(K_0)* of the simulated frames (M1-M8) were obtained from the Pushover analyses, which are shown in Figure 4.3



a) 5-Story Building   b) 10-Story Building

Figure 4.3. Pushover analyses for eight frames (M1-M8). MPM corresponds to the frame with mean mechanical properties.

5. SEISMIC STRUCTURAL RELIABILITY: *β (y) vs Z(y).*

With the collapse intensities *(Y_C)* obtained from the *IDA's*, the seismic reliability of buildings in terms of the safety margin Z in Figure 5.1 is obtained. The straight lines of the figure are obtained using Cornell's index *β(y)* (vertical axis) corresponding to the eight 5- and eight 10- story frames and 40 earthquakes used for the analyses and how they relate to the safety margin for *Z(y)* (horizontal axis), from the state of system collapse when $Z(y=Y_C)=0$, to values of $Z(y<<Y_C)=1$, when the intensity of the earthquake *(y)* is much lower than that of collapse *(Y_C)*, when the structure is elastic. As can be seen, their relationship is linear, so a greater safety margin *Z(y)* in the system corresponds to a higher reliability index *β(y)* in the structure.

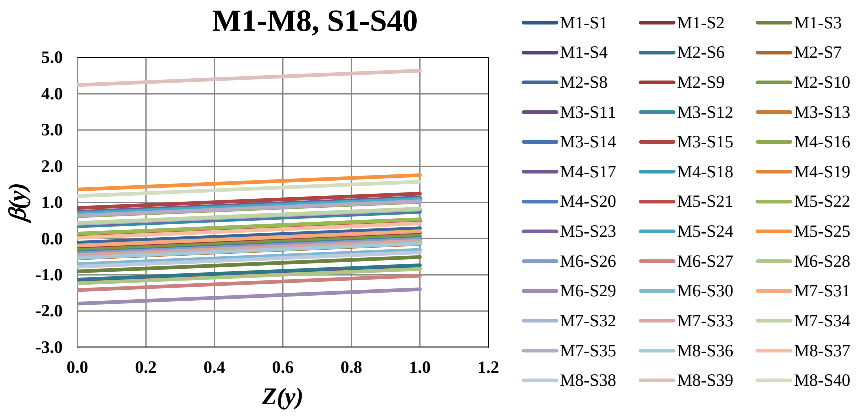
With lower resistance in the frame and higher intensity of the earthquake, a straight line with a lower reliability index is obtained on Figure 5.1. For the same *Z(y)*, different reliability levels are obtained depending on the level of uncertainties in the frame and the earthquake in question.

For purposes of design or evaluation, a suitable reliability index $\beta(y)$ must be positive, reaching values of up to 5 as shown in Figure 5.1b for a safety margin $Z(y)=1$, indicating a structure with a completely elastic behavior. A negative reliability index is an undesirable index, which indicates a high probability of collapse and this happens when the expected value of the natural logarithm of collapse intensity is reached and/or surpassed by the intensity of the earthquake; in other words: $yi \geqslant E\ (lnY_C)$. In this way, a safety margin *Z(y)* would be recommended as long as it is high enough for a determined limit state for a positive reliability index. For example, for the straight lines in the lowest part of the figure 5.1b, the minimum *Z(y)* required should be greater than 0.6 to obtain a positive index.

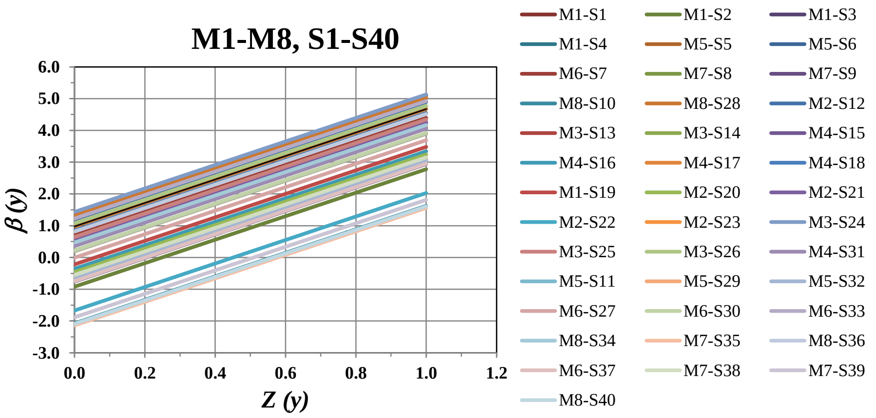
The forty straight lines corresponding to each building and earthquake (*M-S*, in Figure 5.1) are all parallel to each other, consequently, they share the same slope *m*, showing the differences between each one of these straight lines in the corresponding values for $\beta(Y_C)$ when $Z(Y_C)=0$. Due to the linear relationship of the variables, in general terms, the lines in the figure can be expressed as follows:

$$\beta(y)=\beta_{Z=0}\,(Y_C)+m\bullet Z(y) \tag{5.1}$$

Where: $\beta_{Z=0}(Y_C)$ is the ordinate at $x=0$, which is to say that the value of $\beta(y)$ when collapse occurs: $Z(y)=0$.



a) 5-Story Building



b) 10-Story Building

Figure 5.1. Cornell's Reliability Index *β(y)* for different values of the Safety Margin *Z(y)* for 40 earthquakes used in the analyses.

Moreover, a particular expression can be found for obtaining the reliability index, or the safety margin that we wish to assign it to the structure in question considering that $\beta_{Z=0}(Y_C)$ is known or easily obtained from the results already obtained, in this case for the 5-story and 10-story buildings:

$$\beta(y)=\beta_{Z=0}(Y_C)+0.3969\ Z(y) \ \ldots \text{ 5-story building} \tag{5.2}$$

$$\beta(y)=\beta_{Z=0}(Y_C)+3.698\ Z(y) \ \ldots \text{ 10-story building} \tag{5.3}$$

where: $\beta_{Z=0}(Y_C)$ indicates the level of uncertainties associated with the earthquake, the mechanical properties and the live loads of the building considered in this study. For the particular case in which the ordinate is zero: $\beta_{Z=0}(Y_C)=0$, the index $\beta(y)$ will have a relationship of 0.4 and 3.7 times the safety margin $Z(y)$ for the 5-story and 10-story buildings respectively:

$$\beta(y)= 0.4\ Z(y) \ \ldots \text{ 5-story building} \tag{5.4}$$

$$\beta(y)= 3.7\ Z(y) \ \ldots \text{ 10-story building} \tag{5.5}$$

Which implies that for an equal value of $Z(y)$, the 10-story building has a greater reliability, which can be seen on figure 5.1b where lines have higher slope than figure 5.1a.

6. COMMENTS AND CONCLUSIONS

From the analysis of the buildings the following conclusions were obtained:

- IDA curves were obtained in order to acquire the intensity of the collapse system, which allows evaluation of the reliability index $\beta(y)$ and the safety margin $Z(y)$.

- Vulnerability functions were obtained for $\beta(y)$ vs $Z(y)$ whose curves take into account the uncertainties of the mechanical properties of the materials, the live loads and the uncertainties associated with earthquakes. From these curves it can be observed that the 10-story building attains a greater level of reliability compared with the 5-story building.

- Expressions, see Equations 5.2 and 5.3, that allows us to directly and simply evaluate the reliability of a 5-story and 10-story building from a determined safety margin were obtained. These expression can be applied to the evaluation and/or design of buildings for a determined limit state.

- Adequate limits can be established from Figure 5.1 for purposes of design and/or evaluation of $\beta(y)$ from $0.0\text{-}2.0$ and $Z(y)$ from $0.2\text{-}1.0$, for a 5-story building and $\beta(y)$ from $0.0\text{-}5.0$ and $Z(y)$ from $0.6\text{-}1.0$, for a 10-story building, depending on the limit state to be evaluated.

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