# Performance of Tuned Liquid Dampers with Different Tank Geometries for Vibration Control of Structures 

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#### Abstract

SUMMARY: TLD is a tank filled with water and serves as a dynamic vibration absorber for passive control of structures. This paper addresses the comparisons of TLD performances corresponding to different tank geometries including rectangular and conical shapes. Due to the importance of water and tank weight as a load on top of a building, volume is considered to be a constant parameter in this study. First, some explanations are provided to elaborate more on theories that are applied to movement and sloshing analysis of liquids in the tanks. The assumptions used for simplification of the problem are presented afterwards and TLD behaviour is discussed by comparing its response under external dynamic excitation. Through a numerical simulation, vibrations with different amplitudes are applied to TLDs. Performance charts are developed and subsequently utilized to present findings from this study. Comparisons are made for the dissipated energy by a TLD with a certain tank shape is also demonstrated.


Keywords: Passive Control, Tuned Liquid Damper (TLD), Tank Geometry, Sloshing, Vibration

## 1. INTRODUCTION

In designing modern sky scrapers, reinforcing the structure against dynamic loads such as wind has been replaced by implementing various structural control approaches [Soong and Dargush, 1997; Housner et al., 1997]. A successful method which has been employed extensively all over the world is to apply energy absorbers. Tuned mass damper (TLD) and tuned liquid damper (TLD) are two devices have been employed as energy absorber in recent decades.

TLDs absorb a part of input energy during the excitation and turn it into liquid sloshing inside the tank and consequently, the amount of entered energy to the structure will be reduced. Since geometric tank shape of TLDs affect the performance of this device substantially, two different tank shapes including conical and cubic are considered.

The natural frequencies of TLDs with each tank shape are assumed to be constant and the comparisons are made for a specific case of frequency. As a result, the dimensions of the conical and cubic tanks are obtained, using the expressed equations, in order to have a constant natural frequency for each case.

In this study, TLD is modelled numerically using ABAQUS software and the amount of dissipated energy, as a key parameter for TLD performance, is investigated for different tank shape.

## 2. FREQUENCY EQUATIONS

Formulation of the natural frequency which used in the models and the assumptions to simplify the problems are discussed in this section.

### 2.1. Conical tank

First, the natural frequency of conical tank (Fig. 1) is considered and spherical coordinate is employed to solve the problem. For an incompressible and inviscid flow, the continuity equation in spherical coordinate will be in the form of

$$
\begin{equation*}
\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial \Phi}{\partial r}\right)+\frac{1}{r^{2} \sin \vartheta} \frac{\partial}{\partial \vartheta}\left(\sin \vartheta \frac{\partial \Phi}{\partial \vartheta}\right)+\frac{1}{r^{2} \sin ^{2} \vartheta} \frac{\partial^{2} \Phi}{\partial^{2} \varphi}=0 \tag{2.1}
\end{equation*}
$$

where $\Phi$ is velocity potential function and $r, \vartheta, \varphi$ are the components of spherical coordinate. The solution for Eqn. 1 can propose in the form of

$$
\begin{equation*}
\Phi(r, \vartheta, \varphi, t)=\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{m n}\left(\frac{r}{a}\right)^{\lambda_{m n}} P_{\lambda_{m n}}^{m}(\cos \vartheta) \cos m \varphi\left(e^{i \omega x}\right) \tag{2.2}
\end{equation*}
$$

In which $t$ is the time, $a$ is the free surface radius, $\omega$ is the angular frequency and $P_{\lambda_{m n}}^{m}(\cos \vartheta)$ are Legendre function of the first kind of degree $\lambda$ and of angular mode $m$ and $a$ is the liquid surface radius. The boundary condition at the free surface can be expressed as

$$
\begin{equation*}
\frac{\partial^{2} \Phi}{\partial t^{2}}+g \frac{\partial \Phi}{\partial r}-\frac{\sigma}{\rho a^{2}}\left[2 \frac{\partial \Phi}{\partial r}+\frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta}\left(\sin \vartheta \frac{\partial^{2} \Phi}{\partial r \partial \vartheta}\right)+\frac{1}{\sin ^{2} \vartheta} \frac{\partial^{3} \Phi}{\partial \phi^{2} \partial r}\right]=0 \tag{2.3}
\end{equation*}
$$

The velocity normal to the tank wall, $\vartheta=\alpha$, can be defined as

$$
\begin{equation*}
\frac{1}{r} \frac{\partial \Phi}{\partial \vartheta}=0 \tag{2.4}
\end{equation*}
$$

which gives [Ibrahim, 2005]

$$
\begin{equation*}
\left.\frac{\partial}{\partial \vartheta}\left[P_{\lambda_{m n}^{m}}^{m}(\cos \vartheta)\right]\right|_{\vartheta=\alpha}=0 \tag{2.5}
\end{equation*}
$$

Applying the boundary conditions and substituting the proposed solution, Eqn. 2, into the free surface boundary condition, Eqn. 3 gives the natural frequency in the form of [Ibrahim, 2005]

$$
\begin{equation*}
\omega_{m n}^{2}=\frac{g \lambda_{m n}}{a}+\frac{\sigma}{\rho a^{3}} \lambda_{m n}\left(\lambda_{m n}-1\right)\left(\lambda_{m n}+2\right) \tag{2.6}
\end{equation*}
$$

### 2.2. Cubic tank

For a cubic tank, the governing equation can be written as [Lamb, 1945]

$$
\begin{equation*}
\frac{\partial^{2} \Phi}{\partial x^{2}}+\frac{\partial^{2} \Phi}{\partial y^{2}}+\frac{\partial^{2} \Phi}{\partial z^{2}}=0 \text { or } \nabla^{2} \Phi=0 \tag{2.7}
\end{equation*}
$$

where $x, y, z$ are components of Cartesian coordinate. Geometry of the cubic tank is illustrated in Fig. 1. The free surface boundary condition can be expressed as

$$
\begin{equation*}
\frac{\partial \Phi(x, y, z, t)}{\partial t}+g \delta(x, y, t)=-\frac{p_{0}}{\rho} \tag{2.8}
\end{equation*}
$$

where $\rho$ is density, $\delta(x, y, t)$ is the small displacement of the free surface above the undisturbed level and $p_{0}$ is the static pressure of the gas above the liquid. The difference between $z=h / 2$ and $z=h / 2+\delta$ turns out to be a higher order term in $\delta$ and so can be neglected. The relation of the surface displacement $\delta$ to the vertical component of the liquid velocity at the surface is

$$
\begin{equation*}
\frac{\partial \delta}{\partial t}=w=\frac{\partial \Phi}{\partial z} \text { at } z=\frac{h}{2} \tag{2.9}
\end{equation*}
$$

Introducing Eqn. 9 into Eqn. 8 gives

$$
\begin{equation*}
\frac{\partial^{2} \Phi}{\partial t^{2}}+g \frac{\partial \Phi}{\partial z}=0 \text { at } z=\frac{h}{2} \tag{2.10}
\end{equation*}
$$

two forms of possible potential functions are [Abramson, 1966]

$$
\begin{align*}
& \Phi_{1}(x, z)=(F) \cos \left[2 n \pi\left(\frac{x}{a}\right)\right]\left\{\cosh \left(2 n \pi \frac{z}{a}\right)+\tanh \left(n \pi \frac{h}{a}\right) \sinh \left[2 n \pi\left(\frac{z}{a}\right)\right]\right\}  \tag{2.11}\\
& \Phi_{2}(x, z)=(G) \cos \left[(2 n-1) \pi\left(\frac{x}{a}\right)\right]\left\{\cosh \left((2 n-1) \pi \frac{z}{a}\right)+\tanh \left((2 n-1) \pi \frac{h}{a}\right) \sinh \left[(2 n-1) \pi\left(\frac{z}{a}\right)\right]\right\}
\end{align*}
$$

Introducing Eqn. 12 into free surface boundary condition, yields [Abramson, 1966; Bauer, 1981, 1984]

$$
\begin{equation*}
\omega^{2}=\pi n \frac{g}{a} \tanh \left(n \pi \frac{h}{a}\right) \tag{2.12}
\end{equation*}
$$



Figure 1. Geometry of tank shapes (a) conical, (b) cubic

## 3. SELECTION OF TANKS DIMENSIONS

Since the purpose is to compare dissipated energy by conical and cubic TLDs, selected TLDs need to have the same frequency. According to the proposed equations for natural frequency of conical and cubic tank, different dimensions are possible in order to have a constant amount of frequency [Ibrahim, 2005; Hosseini et al., 2011]. Since TLDs has been employed in the tall buildings with the natural period of more than 1.5 sec , three amounts of $1.7,2$ and 2.83 are considered for natural frequency to investigate the performance of TLDs. The selected dimensions of each TLD is represented in the following.

## 4. NUMERICAL SIMULATION

TLDs are numerically modelled using ABAQUS software and the models consist of two parts. The first part is employed to model water and the TLD tank is modelled by the second part. CPE3 and R2D2 elements are used to model the water and the rigid tank, respectively [ABAQUS 6.9].

Gravity loading is defined for the water and the water is considered incompressible and inviscid. Frictionless contact is defined between water and tank.

Furthermore, the linear equation of state is employed with a wave speed of $45 \mathrm{~m} / \mathrm{s}$ and density of $983.2 \mathrm{~kg} / \mathrm{m}^{3}$. Also, adaptive meshing option of ABAQUS is used in this model. A section of each conical and cubic TLD is selected and modeled. The obtained results can be extended to the whole tank.

A sample section of cubic tank under harmonic excitation is shown in Fig. 2 in several time increments.

### 4.1. Verification of the numerical models

To verify the numerical model, a rectangular tank under pitching motion is simulated and the obtained results are compared to a numerically obtained solution given by Nakayama and Washizu (1980).

Furthermore, another method is used to verify the numerical models. An initial dynamic condition is applied to the models and the duration of a free surface cycle is measured and compared to the one obtained using the proposed equations.


Figure 2. Numerical model of cubic TLD under harmonic excitation (a) $t=0$, (b) $t=0.9$, (c) $t=1.8$, (d)

$$
t=2.7 \text {, (e) } t=3.6 \text {, (f) } t=4.5, \text { (g) } t=5.4, \text { (h) } t=6
$$

Also, numerical model of conical TLD under harmonic excitation is demonstrated in several time increments [Fig. 3].

A cubic tank with the width of 8.5 and height of 3.2 and a conical tank with the free surface radius of 5 and vertex angle of $60^{\circ}$ are considered and modelled numerically with a natural frequency equal to 1.7. The criterion which is used to compare the performance of each TLD is the dissipated energy
of a unit mass. To reach on this purpose the total dissipated energy is divided by the volume of the tank. The dissipated energy of each tank shape versus the time is demonstrated in Fig. 4.


Figure 3. Numerical model of conical TLD under harmonic excitation (a) $t=0$, (b) $t=0.3$, (c) $t=0.6$, (d) $t=0.9$, (e) $t=1.5$, (f) $t=1.8$, (g) $t=4.2$, (h) $t=5.1$

As can be seen in Fig. 4, conical tank has a better performance compared to the cubic one. Both TLDs are under a similar harmonic excitation with the frequency of 1.7 . Furthermore, the results are obtained for other amount of excitation frequencies and were similar to Fig. 4.


Figure 4. Dissipated energy for tanks with $\omega^{2}=3$

For the case of $\omega^{2}=4$, a cubic tank with the width of 7 and height of 3.5 and a conical tank with the free surface radius of 4 and vertex angle of $50^{\circ}$ are considered. The dissipated energy of each tank shape versus the time is demonstrated in Fig. 5.


Figure 5. Dissipated energy for tanks with $\omega^{2}=4$
For the case of $\omega^{2}=8$, a cubic tank with the width of 3.5 and height of 1.7 and a conical tank with the free surface radius of 4 and vertex angle of $60^{\circ}$ are considered. The dissipated energy of each tank shape versus the time is demonstrated in Fig. 6.


Figure 6. Dissipated energy for tanks with $\omega^{2}=8$

In this section, the cubic tank is compared to the same tank with a conical and invert conical wall. Dimensions of the tanks are shown in Fig. 7.


Figure 7. Dimension of different tanks


Figure 8. Energy dissipation of conical, rectangular and inverse conical tanks

The amount of dissipated energy of each tank is demonstrated in Fig. 8 and the tank with conical walls is dissipated more energy and the tank with inverted conical wall is less efficient. It should be mention that the natural frequencies of the tanks are not equal.

## 5. CONCLUSION

There has been always a general trend to have more efficient devices. As a result, an investigation on the performance of conical and cubic TLDs leads us to

- The conical TLD is more efficient than a cubic TLD if the free surface diameter size of conical tank is chosen around the measure of cubic tank width.
- In general, absorbed energy of a cubic tank is less than the absorbed energy of the same tank with conical walls and inverted conical tank has the least amount of energy absorption.
- The conical TLD has a bigger effective mass and larger free surface area compared to cylindrical and cubic TLDs. It should be mentioned that TLDs with deep liquid depth are investigated in this study.


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