Restraining bars buckling by means of FRP wrapping: an analytical approach

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SUMMARY:
Internal transverse steel reinforcements (e.g. stirrups) are the main internal devices that avoid the longitudinal steel bars buckling, but in most of all existing RC structures the quantity and the spacing between steel stirrups are inadequate. In these structures longitudinal bars buckling can be avoided by applying external reinforcement, in particular, by means of Fibre Reinforced Polymer (FRP) wrapping. A novel analytical approach for the study of longitudinal bars buckling in column wrapped with FRP is proposed. longitudinal bars has been considered as axially loaded beam, while the mechanical effect of FRP wrapping on the bars has been modelled by means of springs. The effect of elastic and inelastic behaviour has been taken into account by means of the reduced modulus theory. The well-known relations for steel stirrups has been extended to the case of FRP wrapping to propose an analytical formulation, valid both for circular and noncircular column cross sections, for the evaluation of the FRP thickness needed to avoid the longitudinal bars buckling.

Keywords: Bars buckling, Column, FRP wrapping, Inelastic buckling, Reduced modulus

1. INTRODUCTION

Premature failure modes due to buckling of compressed bars close to beam-column joints or in other locations where steel bars are highly stressed (see Fig. 1.1) could reduce the seismic capacity of existing reinforced concrete structures, and especially typical old-type reinforced columns. Bars buckling may take place at the plastic hinge locations leading to even more critical consequences, thus jeopardizing any capacity design procedure. Buckling failure mode is a premature failure mode related to the instability of compressed slender reinforcing steel bars. Poorly detailed reinforcement presents usually high spacing between stirrups so that the restraint provided by the transverse reinforcement is not sufficient to avoid the instability of slender bars. The main parameter governing this phenomenon has been recognized as the length/diameter ratio; hence recent building codes prescribe strict limitations to the ratio between the stirrups spacing and the diameter of reinforcing steel bars. Hence existing structures may need retrofitting of deficient members by means of external strengthening (e.g. FRP wrappings).

Figure 1.1. Typical bar buckling due to insufficient transverse reinforcement

2. ANTIBUCKLING

Restraining bar buckling by means of internal confinement was originally studied considering steel stirrups since second half of last century. However, the necessity to strengthen existing reinforced concrete (RC) structures lacking of reinforcement detailing arose nowadays. Today deep knowledge on external strengthening of such RC members by means of external FRP wrapping is required. Previous knowledge on steel stirrups as anti-buckling device is reported as a base to improve knowledge on FRP systems. Efficiency of restraining systems depends mainly on the shape of the cross section and, compared to steel, FRP exhibits lower flexural stiffness due to reduced thicknesses. In rectangular cross sections, slender FRP strips result inadequate to restraint buckling phenomena.

2.1. Anti-buckling equation for steel stirrups

Transverse reinforcements (e.g. steel stirrups) in a reinforced concrete column carry out the function of confining device and then they avoid the longitudinal steel bars buckling. A previous experimental program by Prota and Cosenza (2005) investigated the longitudinal steel bars behaviour under axial load. The results of these tests can be used to study the buckling mode of longitudinal bars between two consecutive layers of transverse reinforcement. These tests highlighted that the compressive behaviour of the bars (and then the buckling mode) depends on the l/φ ratio. Where l is the spacing between two consecutive stirrups and φ is the longitudinal bars diameter. In particular varying this ratio the buckling can be elastic or inelastic. Considering the restraint needed to avoid buckling over a critical length involving several stirrups of a longitudinal bar in the strain-hardening range of axial compression, it is possible to evaluate the volumetric ratio, ρ_s, of the transverse confining steel, according to Priestley *et al.* (1996), as:

$$\rho_s = \frac{0.45 n f_s^2}{E_r E_h} \tag{2.1}$$

where f_s is the expected steel stress in longitudinal bars, $E_r = 4\, E_s E_h/(\sqrt{E_s}+\sqrt{E})^2$ is the reduced modulus of the longitudinal reinforcement at f_s, E_h is the transversal reinforcement elastic modulus, E is the longitudinal reinforcement elastic modulus, n is the number of longitudinal bars and E_s is the secant modulus from f_s to f_u, the ultimate stress. Considering typical value for the parameters and lacking of definitive analyses describing the interaction between confinement and buckling restraint the equation (2.1) can be replaced with:

$$\rho_s \geq 0.00013 n \tag{2.2}$$

where the only significant parameter is the number of longitudinal bars, n.

2.2. Theory of reduced modulus

The linear elastic analysis is valid for slender bars and the well-known Euler's formula, that describes the critical load for elastic buckling, is only valid for long ideal bars, The ultimate compression strength of the bars material is not geometry-related and it is valid only for short bars. For a bar with intermediate length, the axial buckling occurs after the overcoming of the yield stress but before reaching of the ultimate stress of the bars material. This kind of situation is called inelastic buckling. For accurate analyses different inelastic buckling theories are commonly used (e.g. tangent modulus theory, reduced modulus theory and Shanley (1947) theory. Replacing the elastic modulus, E, in the Euler's formula with a modulus achieved by means of one of those theories the inelastic critical load can be formulated. Using the reduced modulus theory the critical load is obtained by means of the concept of neutral equilibrium. Therefore, the critical load is defined as the axial load at which equilibrium is possible both in the original undeformed position and in an adjacent slightly bent configuration. Starting from this assumption, according to Chajes (1974), the expression for the reduced modulus, regarding a rectangular section bar, is:

$$E_r = \frac{4EE_t}{\left(\sqrt{E}+\sqrt{E_t}\right)^2} \tag{2.3}$$

where E_t is the tangent modulus at the buckling stress value. The expression of the reduced modulus for a generic section bar can be achieved by means of a suitable coefficient, $c = E_{r(\text{circular})}/E_{r(\text{rectangular})}$ that accounts for the different stress-strain response due to the different bar section, Papia *et al.* (1988). For circular sections c is related to E_t/E ratio in Fig. 2.1 and it ranges between 1.17 and 1. As discussed later, on safe side, lowest possible values for E_r should be used, i.e. $E_{r(\text{circular})}/E_{r(\text{rectangular})} = 1$

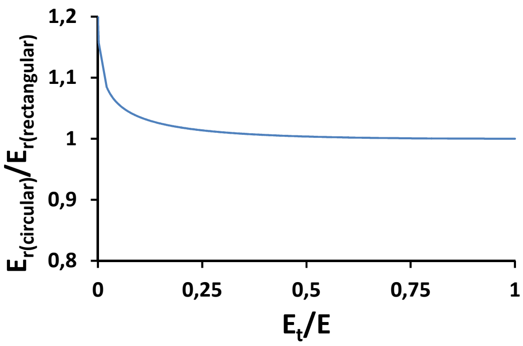

Figure 2.1. Circular bar section corrective coefficient trend

3. ANTIBUCKLING WITH FRP

Longitudinal bars buckling can be avoided by applying further restraint to slender bars in between the restraints provided by the stirrups widely spaced, in particular by means of FRP wrapping. To design a retrofit intervention the main parameter needed is the FRP thickness, t_f, necessary to avoid the buckling. The proposed formulation has been evaluated by means of analytical models based on solid mechanic equations. The analytical model to achieve the critical load is based on a schematization of the longitudinal bars as an axially loaded beam with constant section (see Fig. 3.1). The mechanical effect (lateral pressure) of FRP wrapping on the longitudinal bars has been modelled as springs, thus increasing the critical load for these elements.

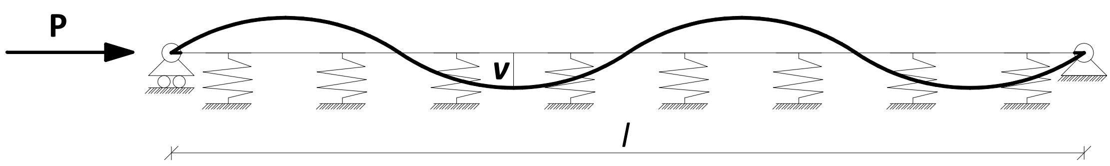

Figure 3.1. Analytical model schematization

The equilibrium differential equation is:

$$EIv^{IV} + Fv^{II} = -kv \tag{3.1}$$

Where E is elastic modulus, I is the moment of inertia for the bar section, k is the additional spring elastic constant and v is the displacement function. The Eqn. 3.1 is a fourth order linear differential equation with constant coefficients. As such, its general solution is:

$$v = A e^{\alpha' z}\cos\alpha'' z + B e^{\alpha' z} sen\,\alpha'' z + C e^{-\alpha' z}\cos\alpha'' z + D e^{-\alpha' z} sen\,\alpha'' z \tag{3.2}$$

The Eqn. 3.2 admit only one solution in which A=B=C=D=0. According with that result, the critical load, F_{cr}, for n bars has been evaluated by means of:

$$F_{cr} = \frac{\pi^2 EIn}{l^2}\left(n_w^{\ 2} + \frac{kl^4}{EIn\pi^4 n_w^2}\right) \tag{3.3}$$

Where n_w = 1,2,3,… is the number of inflection waves. The relationship between F_{cr} and l is represented by a curve whose minimum is obtained for each n_w:

$$F_{cr} = 2\sqrt{kEIn} \tag{3.4}$$

The critical load value achieved by the Eqn. 3.4 is the minimum, and it is independent on beam length. This value is the safest value to be used in design phase. The critical load trend varying the length (for n_w = 1, 2, 3) together with the Euler's curve and the minimum critical load line are shown in Fig. 3.2.

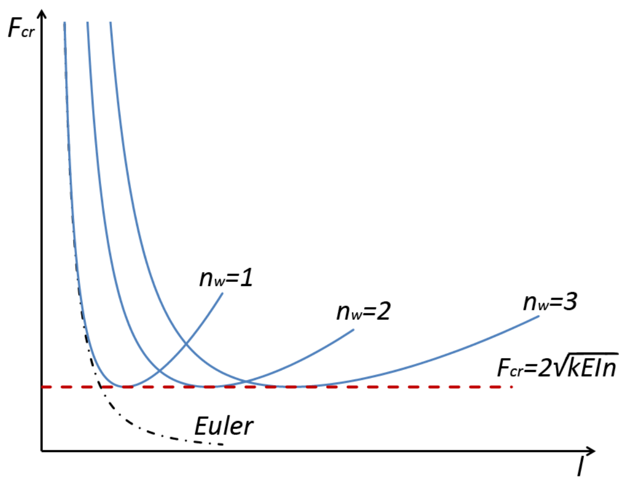

Figure 3.2. Critical load trends varying the length

Starting from the knowledge of the minimum critical load formulation, a companion of Eqn. 2.1 (specific for the stirrups) has been derived for the FRP case. According to the consideration that the FRP wrapping can be considered as smeared stirrups, the FRP thickness to avoid the bars buckling has been derived. For columns with different cross section shapes, the FRP wrapping assumes different stiffness (i.e. different values for spring stiffness). Therefore it is needed to calibrate the equivalent spring stiffness of the analytical model depending on the cross section shape. In the following, specific cases for different column cross sections are shown.

3.1. Circular sections

For circular sections the lateral confining pressure due to FRP wrapping is:

$$f_l = \frac{\Delta r_c E_f t_f}{r_c^{\ 2}} = \Delta r_c k_f \tag{3.5}$$

Where $\Delta r_c = \varepsilon_r\, r_c$, Ef is the FRP elastic modulus, r_c is the radius of the column section, and ε_r is the radial deformation. In the Eqn. 3.5 the ratio $E_f\, t_f/r_c^{\ 2} = k_f$ represent the FRP stiffness. The equivalent spring stiffness has been obtained multiplying k_f for the length over which the FRP is acting, i.e. the column circumferential perimeter: $k = k_f\, 2\pi\, r_c$. Replacing k in the Eqn. 3.6a with $k_f\, 2\pi\, r_c$, the minimum critical stress, σ_{cr}, is simply evaluated as showed in Eqn. 3.6b, taking into account all the n bars distributed inside the cross section:

$$a)\ \sigma_{cr} = \frac{F_{cr}}{nA_b} = 2\frac{\sqrt{kE_r In}}{nA_b} \quad ; \quad b)\ \sigma_{cr} = 2\frac{\sqrt{(E_f t_f 4\pi / d)E_r In}}{nA_b} \tag{3.6}$$

Where d is the column diameter, A_b is the bars area and n is the number of bars. Equating σ_{crit} to the bars yield stress, f_y, the FRP thickness to avoid the buckling is:

$$t_f = \frac{df_y^2 n}{4E_f E_r} \tag{3.7}$$

It is noted that Eqn. 3.6a is general and not only referred to circular cross sections. In Eqn. 3.7, formally comparable to Eqn. 2.1, the steel yielding stress is equated to σ_{crit}, however, the same approach discussed in section 2.1 on the selection of steel stress can be repeated. Eqn. 3.7 clearly highlights that the lower is E_r, the higher is t_f, so that, on safe side, lowest possible values for E_r should be used.

3.2. Noncircular sections

To achieve the equivalent FRP stiffness for noncircular sections it is fundamental to evaluate the force exerted by the longitudinal bar on account of buckling in the direction corresponding to the plane in which this can occur. This force must be evaluated in different modes according to the bar position inside the cross section. In the following, the most two representative cases are shown.

3.2.1. Corner bars

Considering the corner bar (see Fig. 3.3) the force F exerted by the longitudinal bar can assume all the possible directions according to the inclination angle α. A parametric study has been performed to understand the influence of the angle α on the equivalent spring stiffness.

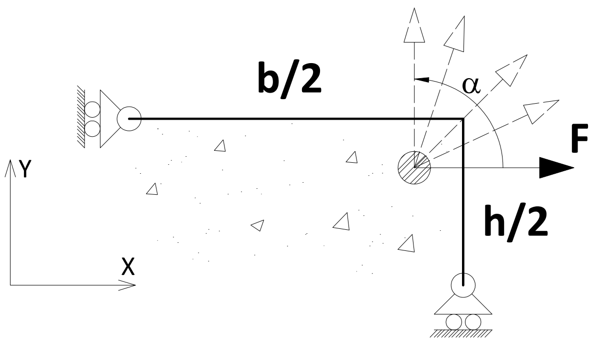

Figure 3.3. Corner bars scheme

According to reference system in Fig. 3.3 (where, due to symmetry, FRP wrap is simply supported in the middle of cross section sides) the total displacement of FRP wrap in the corner due to bars buckling is:

$$\delta = \frac{Fb}{2E_f t_f}\sqrt{\left(\frac{h}{b}\right)^2 sen^2\alpha + \cos^2\alpha} = \xi\frac{Fb}{2E_f t_f} \tag{3.8}$$

where b and h are, respectively, the width and height of the section ($b \geq h$).

Consequently the equivalent spring stiffness is:

$$k = \frac{2E_f t_f}{b\xi} \tag{3.9}$$

According to the function ξ, the trend, varying the force inclination angle α, (see Fig. 3.4) presents a maximum value for $\alpha = 0°$. This means that the lowest spring stiffness is associated to $\alpha = 0°$, i.e. when the force F is oriented in the direction orthogonal to the minimum side of the section (i.e. b).

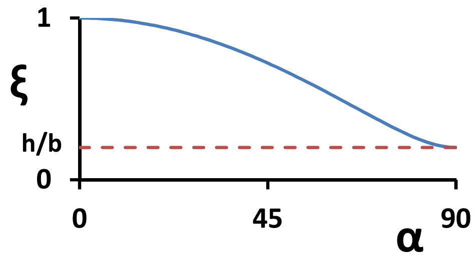

Figure 3.4. ξ/α function

Therefore the minimum value of the equivalent spring stiffness, recalling that $b \geq h$, is:

$$k = \frac{2E_f t_f}{\max\{b;h\}} \longrightarrow k = \frac{2E_f t_f}{b} \tag{3.10}$$

Equating σ_{crit} to the bars yield stress, f_y, and replacing the equivalent spring stiffness, k, achieved for the corner bar of the noncircular section in the Eqn. 3.6a, the FRP thickness to avoid the buckling is:

$$t_f = \frac{b f_y^2 n\pi}{2E_f E_r} \tag{3.11}$$

This equation is still comparable to previous Eqns. 3.7 and 2.1. Same comments can be repeated on the expected steel stress in longitudinal bars.

3.2.2. Central bars

Considering the central bar (see Fig. 3.5) the force F exerted by the longitudinal bar can only assume the direction normal to the considered side (because it cannot enter concrete, i.e. enter into the core of the section).

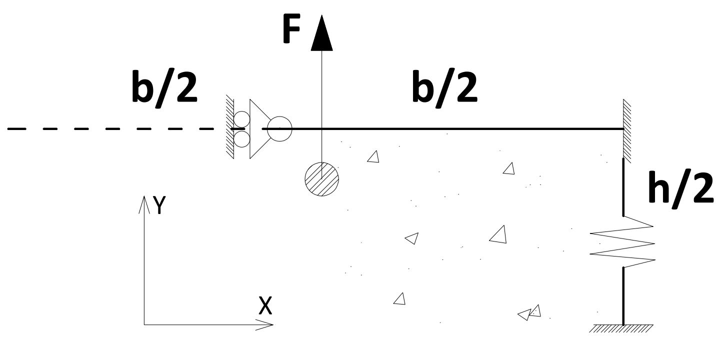

Figure 3.5. Central bars scheme

According to reference system in Fig. 3.5 the FRP the total displacement due to bars buckling is:

$$\delta = \frac{Fb}{2E_f t_f} + \frac{Fb^3}{16E_f t_f} \tag{3.12}$$

This value is much higher than the previous provided by Eqn. 3.8 mainly because it is well-known that FRP thickness is very low and flexural stiffness of FRP wraps is commonly negligible. Consequently the equivalent spring stiffness is:

$$k = \frac{16E_f t_f^3}{\max\left\{8ht_f^2 + b^3; 8bt_f^2 + h^3\right\}} \longrightarrow k = \frac{16E_f t_f^3}{8ht_f^2 + b^3} \tag{3.13}$$

It is noted that even if $b \geq h$, it is not possible to select a priori the maximum value in the denominator of Eqn. 3.13, from an analytical point of view; however, analysing the two terms, it can be derived that if $t_f > 1.63h$, obvious in practical applications, the former term is always larger than the latter.
In this case a closed form solution in terms of t_f is not straightforward; however the problem can be solved numerically, as discussed in section 4.2.2.

4. APPLICATIONS OF PROPOSED MODEL

In technical practice it is useful to have a closed form solution. Introducing the numerical constant μ the proposed model is herein modified in the format:

$$t_f = \mu \frac{f_y^2 nd'}{E_f E_r} \tag{4.1}$$

Or, for more simplicity, introducing the constant μ^* and assuming a value for f_y and E_r, in the format:

$$t_f = \mu^* \frac{nd'}{E_f} \tag{4.2}$$

Where *d'* represents a characteristic dimension of the cross section (the diameter of circular cross sections, or the maximum dimension in noncircular cross sections). The new parameter, μ^*, introduced in Eqn. 4.2, can be easily related to the former μ, $\mu^* = \mu f_y^2 / E_r$.
Considering, for the steel constitutive model (see Fig. 4.1a), the well-known Ramberg-Osgood (1943) relationship (Eqn. 4.3) the trends shown in figs. 4.1b, c, d respectively refer to previously discussed secant, tangent and reduced modulus.

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K_r}\right)^{\frac{1}{n_r}} \tag{4.3}$$

Where ε is the strain, σ is the stress and K_r and n_r are constants that depend on the material being considered.
Assuming f_y = 450MPa, according to Ramberg-Osgood relationship, E_r = 31.14 GPa hence $\mu^* = 6.5\ \mu$.

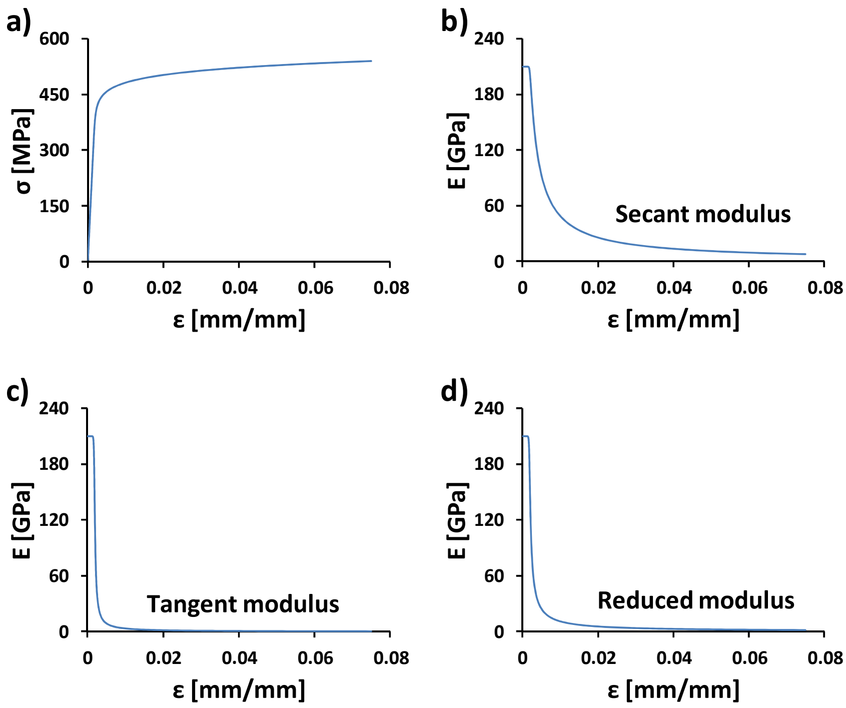

Figure 4.1. a) Ramberg-Osgood model. b, c, d) Secant, tangent and reduced modulus trends

4.1. Circular sections

For circular cross sections, according to Eqn. 3.7 (and given f_y and E_r according to section 4), maximum values are $\mu = 0.25$ and $\mu^* = 1.625$. However in reality (see Fig. 4.2) the parameters depend on l/φ ratios, and clearly for values of l/φ lower than 6.5 there is no need of FRP wrapping because Euler critical stress, σ_{crit}, is already higher than f_y (see Fig. 3.2).

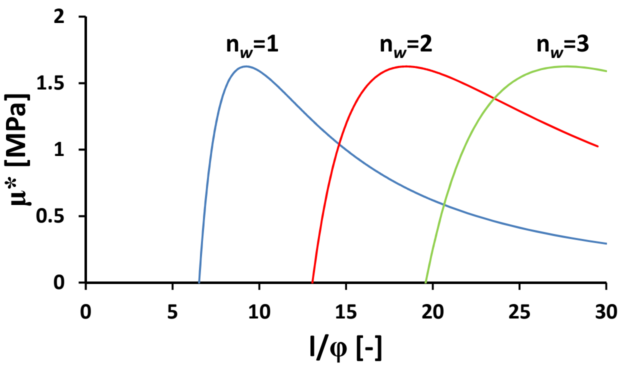

Figure 4.2. $\mu^*/l/\varphi$ trend for circular cross sections

4.2. Rectangular sections

4.2.1. Corner bars

For corner bars in rectangular section, according to Eqn. 3.11, assuming $d' = \min\{b;h\}$ (where b°and h are, respectively, the width and height of the section), and given f_y and E_r as written in section 4, maximum value are $\mu = \pi/2$ and $\mu^* = 6.5\ \pi/2$ but as the circular case it depends on l/φ ratio, and clearly for values of l/φ lower than 6.5 there is no need of FRP wrapping because Euler critical stress, σ_{crit}, is already higher than f_y as shown in Fig. 4.3

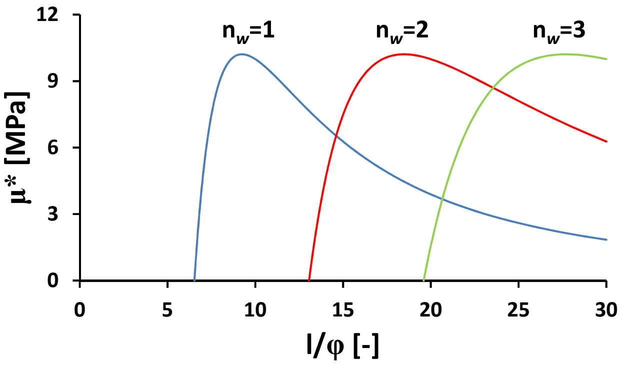

Figure 4.3. $\mu^*/l/\varphi$ trend for rectangular cross sections (corner bars)

4.2.2. Central bars

For central bars in rectangular section a closed form, even for maximum values of μ and μ^* is not simple, but the problem can be solved numerically. Equating σ_{crit} to the bars yield stress, f_y, and replacing the equivalent spring stiffness, k, achieved for the central bar of the noncircular section in the Eqn. 3.6a, the FRP thickness to avoid the buckling is not provided in closed form (since Eqn. 3.13 is not linear in t_f). However inverting Eqn. 4.2 the trend for μ^* is found (see Fig. 4.4). It is highlighted that preliminary results show that dependency on b, h/b and n is negligible and $\mu^* \approx 318$.

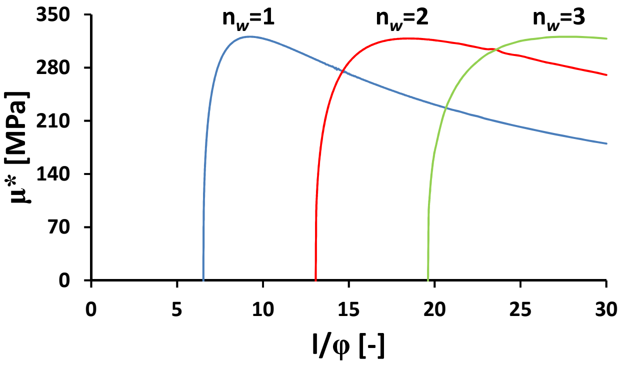

Figure 4.4. $\mu^*/l/\varphi$ trend for rectangular cross sections (central bars)

The increase of FRP thickness required to avoid buckling of a central bar is much higher than the thickness required to avoid buckling of a corner bar.

5. CONCLUSIONS

An analytical approach to study the buckling of longitudinal bars was proposed, when an elastic device is used to confine and restrain this phenomenon. Solid mechanics equations were the basis of the proposed model, accounting for longitudinal bars as Euler beams restrained by elastic springs along their length. Previous formulations to assess minimum ratio for steel stirrups to avoid longitudinal bars buckling were discussed and compared to present case of FRP wrapping. These formulations account also for inelastic buckling, by means of reduced modulus theory.

Beside analytical refined formulations, simplified equations, practitioners oriented, were proposed based on few parameters. They were also analytically derived with the only exception of central bars in noncircular cross sections. These formulations has shown that the effect of FRP wrapping is negligible for l/φ ratios less than about 6.5 for both circular and rectangular cross sections. For rectangular cross sections the FRP is not able to avoid the central bars buckling, mainly because of reduced thickness of FRP wraps (hence because of reduced flexural stiffness). Conversely it is effective for corner bars in noncircular sections and always in circular sections.

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