Effects of Spatial Variability of Soil Properties on Natural Frequency of Natural Soil Deposits

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SUMMARY:
Seismic waves filtered as they pass through soil layers, from bedrock to surface, change frequencies and amplitudes and these modifications result in different ground motion characteristics. Therefore, it is important to consider the site effect in the evaluation of earthquake ground motions for the design of structures. As well as, soil properties in a heterogeneous soil layer are affected by a series of uncertainties, such as: inherent spatial variability, measurement error, transformation or model uncertainty. Inherent variability of shear modulus is described and modeled using deterministic and stochastic fields. According to, sources are the bottleneck in the development of seismic methods and spatial heterogeneity of soil; they add complexity to natural frequency and amplification of soils. Therefore this paper highlights the importance of mutual consideration of deterministic and stochastic components of heterogeneity of shear modulus when seeking the natural frequency and amplification of natural alluvial deposits. In this paper, responses of a random soil site subjected to sinusoidal wave on base rock are investigated. Finally the results emphasize that stochastic heterogeneity has significant effects on dynamic properties of natural alluvial deposits.

Keywords: Inherent Variability, Shear Modulus, Natural Frequency, Amplification, Natural Alluvial Deposits

1. INTRODUCTION

Geotechnical earthquake engineering deals with many kinds of uncertainties which should be considered in the computation of site earthquake response. These uncertainties are related mainly to earthquake input which is caused by earthquake source mechanism, transmission path and others, and also to, soil properties which vary from place to place within deposits (Alinejad & Jamshidi, 2011). These uncertainties in field data are increased by the inherent soil heterogeneity, namely spatial variability of soil properties within so-called homogeneous soil layers. Applications of stochastic methods in earthquake engineering have usually been in the form of a random input into a deterministic system. However, the geological configurations and the material properties of soil deposits such as density, elastic moduli and damping coefficients are not always known with sufficient accuracy to justify a deterministic analysis. Consequently, uncertainties in the properties of the medium will result in uncertainties in the predicted responses. In other word, deterministic descriptions of the spatial variability of soil properties are not always feasible, and the sufficiently large degree of disorder exhibited, leads to the use of statistical methods in describing their distribution within a "statistically homogeneous" soil zone (Assimaki et al., 2003).

In this frame, the seismic response is carried out via Monte Carlo simulations combined with the finite difference method. The analysis integrates the influence of, coefficient of variation of the soil properties, the inter-property correlation coefficients, as well as horizontal and vertical correlation lengths. Results of these analyses indicate that heterogeneity highly influences the behavior of the soil profile which induces differential movement at ground surface and makes evident the filtering effect of frequencies, making hence the simulated soil softer (Nour et al., 2003).

The Monte Carlo method consists of performing a set of probabilistic realizations of the medium, used hereunder to predict the transfer function and the extreme acceleration at ground surface via deterministic calculation for each realization, and proceeding thereafter to the statistical treatment of the obtained results.

The scope of this paper is to investigate the effect of inherent variability of soil profile on natural frequency of alluvial deposits. Soil property of interest is shear modulus modelled herein as spatially random fields. Shear modulus was modelled using the lognormal distribution. This choice is motivated by the fact that lognormal distribution enables realization of positive shear moduli with different levels of variability. In this regard, natural frequency and amplification of heterogeneous soil stratum were investigated by developing a 'FISH' code in FLAC. Monte Carlo simulation approach was used in generation of 2D lognormally distributed correlated random fields. Analyses highlights the effect of shear modulus variability introduced as the coefficient of variation and its correlation distance on the natural frequency and amplification behavior of heterogeneous soil stratum in comparison to homogenous condition.

2. SOURCES OF UNCERTAINTY IN GEOTECHNICAL SOIL PARAMETERS

There are three primary sources of uncertainty in geotechnical design parameters: inherent soil variability, measurement error, and transformation uncertainty. Inherent variability is the consequence of natural geologic processes that continually modify the soil mass in situ. Measurement error results from equipment, test-operators, and random test effects during measurements. Transformation uncertainty is introduced when field or laboratory measurements are "transformed" into design soil properties with empirical or other correlation models. Among these three sources of uncertainty, only the inherent soil variability is taken into consideration in this study.

Inherent variability in geotechnical properties can be modelled by Equation 1 in which a depth dependent geotechnical property, ξ is decomposed into the deterministic component, t and the fluctuating component, w that totally represent the inherent soil variability. Figure 1 shows schematically the inherent variability.

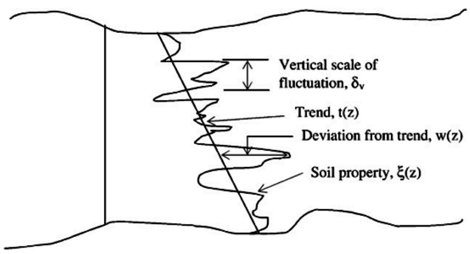

Figure 1. Statistical description of inherent variability (Phoon & Kulhawy,1999)

$$\xi(z) = t(z) + w(z) \quad (1)$$

If $w(z)$ is considered to be statistically homogeneous, the mean and variance of w(z) are independent of depth, and the correlation of w(z) signals at two different depths is a function of their spatial separation rather than their absolute locations (Phoon & Kulhawy,1999).

Quantitative assessment of soil uncertainty modelling requires use of statistics, as well as probabilistic modelling to process data from laboratory or in situ measurements. Probability theory is useful in modelling the observed behavior of a variable parameter if a set of measurements are available. Any quantitative geotechnical variability relies on sets of measured data which are often limited in size and hence, it is referred to sample statistics. The uncertainty in the measured data is expressed in terms of sample mean (μ) and variance (σ^2) evaluated from the following expressions.

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2}$$

$$\sigma^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \mu)^2 \tag{3}$$

Where n is the total number of samples.

A useful dimensionless ratio can be calculated by normalizing σ with respect to the "local" mean soil property, μ obtained from the depth varying trend function, t(z). This ratio is called the coefficient COV.

$$COV = \frac{\sigma}{\mu} \tag{4}$$

The reported COV may be considerably larger than the actual inherent soil variability because of four potential effects (Vanmarcke, 1984): (1) soil data from different geologic units are mixed, (2) equipment and procedural controls are generally insufficient, (3) deterministic trends in the soil data are not properly removed, and (4) soil data are gathered over a long period of time with different testing techniques.

3. INPUT MOTION

The input base acceleration wave is a sinusoidal function with a frequency varying between 1-10 Hz. The amplitude of the input motion is kept constant at 0.05g. Figure 2 shows the acceleration time histories and frequency content of the harmonic input motion. Equation 5 shows the acceleration time history adopted in sweep test to measure the natural frequency and amplification of the soil deposit.

$$\ddot{u}(t) = 0.5t[\sin(2\pi([t]+1)t)] \qquad 0 < t < 10 \tag{5}$$

(a)

(b)

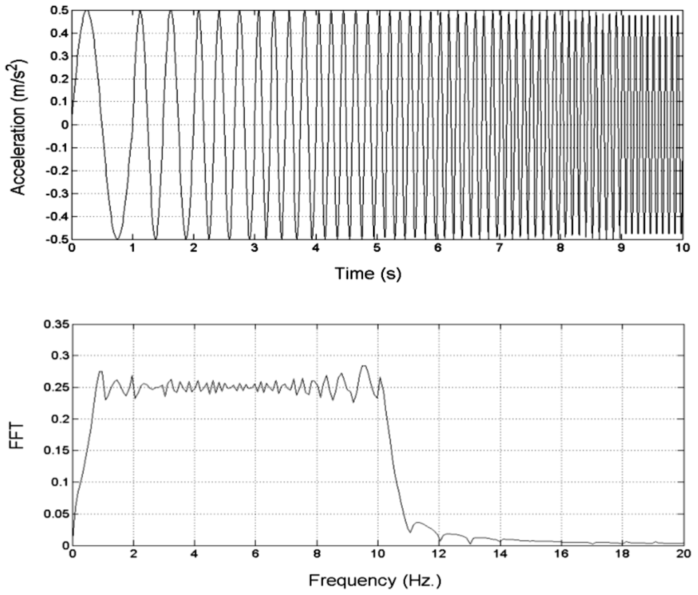

Figure 2. Input motion; a) time domain; b) frequency domain (FFT).

4. SWEEP TEST

A popular method to characterize site amplification has been the use of spectral ratios, introduced by Borcherdt (Borcherdt, 1970). The spectral ratio is calculated by taking the ratio of the Fourier amplitude spectrum (FAS) of a soil-site record to that of a reference-site record. The records should be from the same earthquake. The method is valid only if the distance between the two sites is much smaller than their epicentral distances (i.e. the source and path effects in the records are identical), and, therefore, the differences in the records are solely due to site effects. If y(t) and x(t) denote the discrete-time series of the recorded motions at a soil site and a nearby rock outcrop, respectively, the spectral ratio, R(f), is calculated by the following equation (Phoon & Kulhawy,1999):

$$R(f) = \frac{\sum_{j=1}^{2k+1} W(j)|Y(f-k-1+j)|}{\sum_{j=1}^{2k+1} W(j)|X(f-k-1+j)|} \qquad (6)$$

Where X(f) and Y(f) are the complex Fourier transforms of x(t) and y(t), respectively, W(j) is the symmetric smoothing window with 2k+1 points, and f denotes the cyclic frequency. The smoothing aims to reduce the effects of noise that is always present in the records.

The fundamental frequency of natural soil deposits in one-dimensional medium, f_h^s is inversely proportional to the depth of a soil deposit over bedrock and also directly proportional to the shear wave velocity as inferred from Equation 14. Any change in shear wave velocity will move the alluvial deposits closer to or farther away from the condition of resonance and modify their amplification properties accordingly. Therefore, the shear wave velocity is a dynamic property of soils, crucial for seismic analyses. It is related to the shear modulus, G, by the Equation 8 (Kramer, 1996):

$$\mathrm{V_s} \backsimeq 4\mathrm{H} f_h^s \qquad (7)$$

$$\mathrm{V_s} = \sqrt{\frac{\mathrm{G}}{\rho}} \qquad (8)$$

Where, ρ is the mass density of the soil. Brad and Bouchun (Bard & Bouchon, 1985) showed that the natural frequency of two-dimensional alluvial deposits depends only on two parameters, namely, the one-dimensional natural frequency at the alluvial center and the shape ratio according to equation (9):

$$f_0 = f_h^s \sqrt{1 + (2.9H/0.5L)^2} \qquad (9)$$

Where f_0 is the natural frequency of 2D natural soil deposit and H, L are the depth and length of alluvial respectively. The fundamental frequency of the model under study is found by the numerical sweep test which sweeps a frequency ranging from 1 to 10 Hz and records the response at the control point representing the soil site.

5. AMPLIFICATION DUE TO IMPEDANCE CONTRASTS

It is well known that the amplitudes of seismic waves increase as they propagate through soft soil layers near the surface. The amplification is due to impedance (impedance = density × propagation velocity) contrasts in the soil. We can calculate the amplification for simple cases by using the principle of conservation of energy and neglecting the energy loss due to damping. The seismic energy flux, E(t), along a tube of seismic rays is defined as:

$$\mathrm{E(t)} = \frac{1}{2}(\rho \mathrm{V})\, \dot{\mathrm{u}}(\mathrm{t})^2 \qquad (10)$$

Where ρV is the impedance of the medium with ρ and V denoting the mass density and propagation velocity respectively, and $\dot{\mathrm{u}}(\mathrm{t})$, is the velocity of seismic shaking. Since the energy must remain constant during the propagation, any reduction in the impedance is compensated by an increase in the shaking velocity. If the impedance changes gradually from ρV_1 to ρV_2, the equality of energy flux gives the following for site amplification (A):

$$A = \left[\frac{\dot{u}_2(t)_{max}}{\dot{u}_1(t)_{max}}\right] = \sqrt{\frac{(\rho V)_1}{(\rho V)_2}} \qquad (11)$$

6. DYNAMIC MODELLING

Spatial variability of mechanical soil properties is modeled in two dimensional plane strain condition. Time and frequency-domain analyses were performed on the soil as nonlinear elastic material by adapting hysteretic damping behavior. As shown in Figure 3, the finite difference model used in the dynamic analysis statistically variable soil profile of 60 m horizontally and 30 m vertically, overlying flexible bedrock where the input seismic motion is prescribed. The lateral extent of the soil is truncated by a transmitting boundary which eliminates spurious reflective waves and simulates the missing part of soil extending to infinity (Wass,1972). For the problem under study, 1800 quadrilateral finite difference elements are used. The mesh was fixed at the base with quiet boundary, and restrained only vertically at the sides to model the free field conditions.

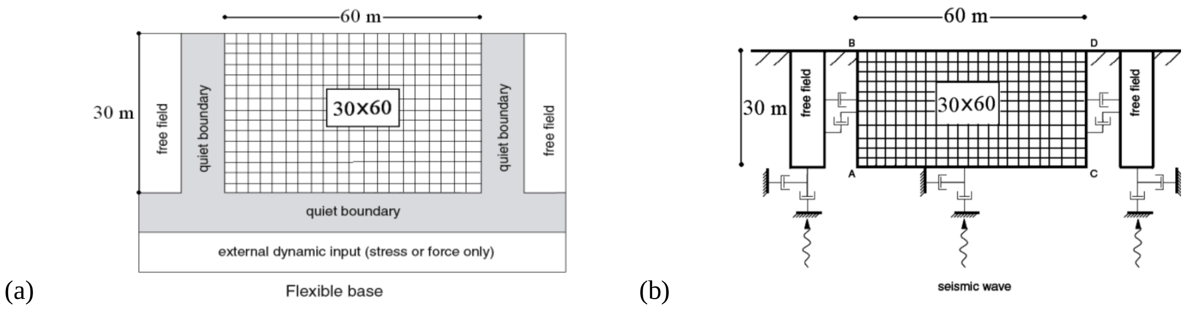

Figure 3. Dynamic model and boundaries; a) quiet boundaries and b) free field (Itasca,2005)

The probability distribution of the natural frequency is studied by depicting acceleration time histories at the surface midpoint of the finite difference mesh for different realizations of the same stationary random field. For the stochastic analyses, different COV of shear modulus are selected for comparison. The values of COVs considered are 20,40,60,80 and 100%. Properties of the soil in deterministic and stochastic phases are indicated in Table 1.

Table1. Material properties in stochastic analyses

Properties	Symbol	Soft soil	Medium soil	Stiff soil
Density	ρ (kg/m^3)	1800	1900	2000
Poisson's ratio	v	0.35	0.35	0.35
Shear modulus	G (MPa)	32.41	46.3	74.08
Coefficient of variation of shear modulus	COV_G (%)	0,20,40,60,80,100	0,20,40,60,80,100	0,20,40,60,80,100

It should be noted that laboratory tests such as resonant column or cyclic triaxial cell cannot reproduce the in-situ fabrics of the grains skeleton. Furthermore, the shear stiffness measured in the laboratory is not influenced by aging or preloading effects. These effects may lead to an increased in situ G_{max} (Alpan, 1970). Thus, laboratory tests are usually deemed to under-estimate the in-situ values of G_{max}. The shear modulus data presented in Table 1 are therefore increased according to a correlation diagram given in Figure 4. It shows that the ratio of small strain ("dynamic") and large-strain ("static") stiffness moduli is a function of the static values. The selected ranges for shear modulus in table 1, fits with soil type IV according to the Iranian code of practice for seismic resistant design of buildings (standard No. 2800-05).

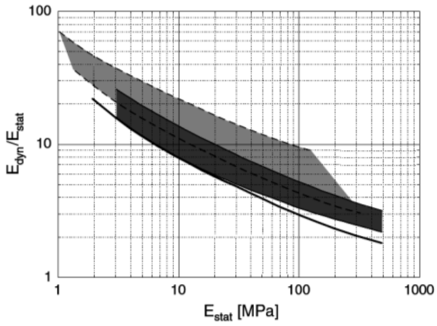

Figure 4. Comparison of the correlation E_{dyn}/E_{stat} by Alpan (Alpan,1970).

7. RANDOM FIELD MODELLING

The in-situ soil property variation is represented by the mean value, the coefficient of variation and the correlation distance, the most representative parameters for stochastic design. In present study, shear modulus, G is considered as a random variable and assumed to be a correlation log-normally distributed parameter introduced by mean, μ_G standard deviation, σ_G and correlation distance. The use of lognormal distribution is appropriate in geotechnics as the soil properties are strictly non-negative and the distribution also has a simple relationship with normal distribution.

The total computation process is conducted by developing a ‘FISH’ code in FLAC 2D. Monte Carlo simulation approach is used in generation of correlated 2D realizations of random field. In present study, values of COV_G lie in the range of 20–100% (Table 1). Mean shear modulus, μ_G bears the value of 32.41, 46.3 and 74.08 MPa and are kept constant throughout the domain. The study, indeed aims at investigating the effect of uncertainties embedded in shear modulus estimation on dynamic behavior of alluvial deposits. For each set of statistical parameters given in Table 1, Monte Carlo simulation is adopted by performing 500 realizations of shear modulus random field. The sufficiency of the number of stochastic analyses in Monte Carlo simulation scheme was examined by monitoring the mean natural frequency estimation error and COV of the natural frequency for the worth case, i.e. at COV_G=100% against the number of realization as illustrated in Figure 5. Results indicate that compromising on 500 realizations for different input parameter G makes sense in both views of accuracy sought and the analysis cost which has direct relation to the number of realizations.

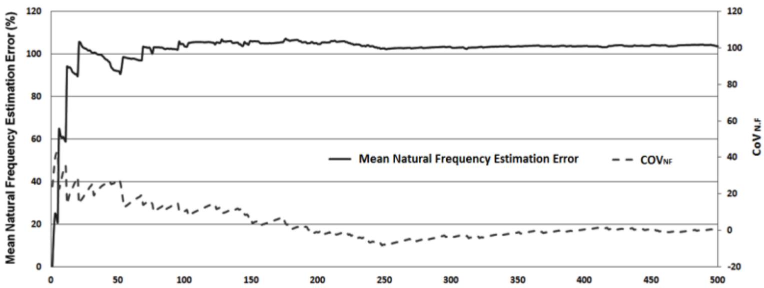

Figure 5. Validity test on the number of realizations ($\mu_G = 1\ MPa$ COV_G=100%, L_V=1m and $\frac{L_H}{L_V} = 1$)

8. RESULTS AND DISCUSSION

This study considers the stochastic property of shear modulus, and investigates its effects on the extreme ground surface acceleration statistics (time domain), as well as on the mean transfer function (frequency domain). In-situ and laboratory determination of the coefficient of variation of shear modulus is troublesome and a large number of samples are tested very rarely. In order to cover a wide number of variability considers, different coefficient of variation of modulus varying between 0 to 100%. Variability in shear modulus influences natural frequency and amplification statistics of alluvial deposits.

8.1. Influence of Soil Spatial Variability on the Natural Frequency of Site

To this aim the transfer function is calculated and plotted against frequency and the natural frequency statistics are obtained by extremizing the transfer functions for different realization.

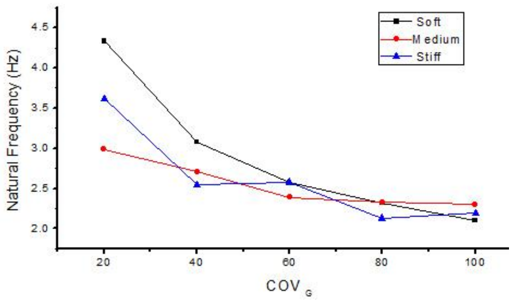

Figure 8. Mean natural frequency variation with the shear modulus spatial variability for different types of clay

Results of stochastic analyses for different sets of statistical parameters for shear modulus, introduced earlier are provided in Figure 8, in terms of mean natural frequency, μ_{NF} against the coefficient of variation of the shear modulus. This figure shows the influence of the spatial variability of shear modulus on mean natural frequency of alluvial deposits of different soil conditions and anisotropic correlation structure.

Variation of natural frequency along with their spectral ratios are depicted in Figure 9. For different soil conditions and the coefficients of variability of the shear modulus. Super imposed on the same graph is the theoretically calculated transfer function in solid line exteremize at modal frequencies. The mean shear modului were adopted in equation 17 to calculate the theoretical transfer function assuming zero damping.

$$|F(\omega, \xi = 0)| = \frac{1}{\sqrt{cos^2\left(\frac{\omega H}{V_{ss}}\right) + a_z^2 sin^2\left(\frac{\omega H}{V_{ss}}\right)}} \qquad (17)$$

Where:

$$a_z = \frac{\rho_s V_{ss}}{\rho_r V_{sr}}$$

And, F is transfer function, ρ_s is mass density of soil layer, ρ_r is mass density of elastic bedrock, V_{ss} is shear wave velocity of soil, V_{sr} is shear wave velocity of bedrock and ω is frequency.

It is observed that for low coefficient of variability of shear modulus the natural frequency of soil stratum is estimated with less uncertainty and the results are concentrated around the calculated modal frequencies.

For COV_G more than 60% which are less probable to occur for natural alluvial deposits the uncertainty in natural frequency estimation becomes more highlighted and the frequency data is observed to be more sparse in comparison to low variability cases. This means that the coefficient of variation of natural frequency (COV_{NF}) increases as the COV_G increases (See Figure 10).

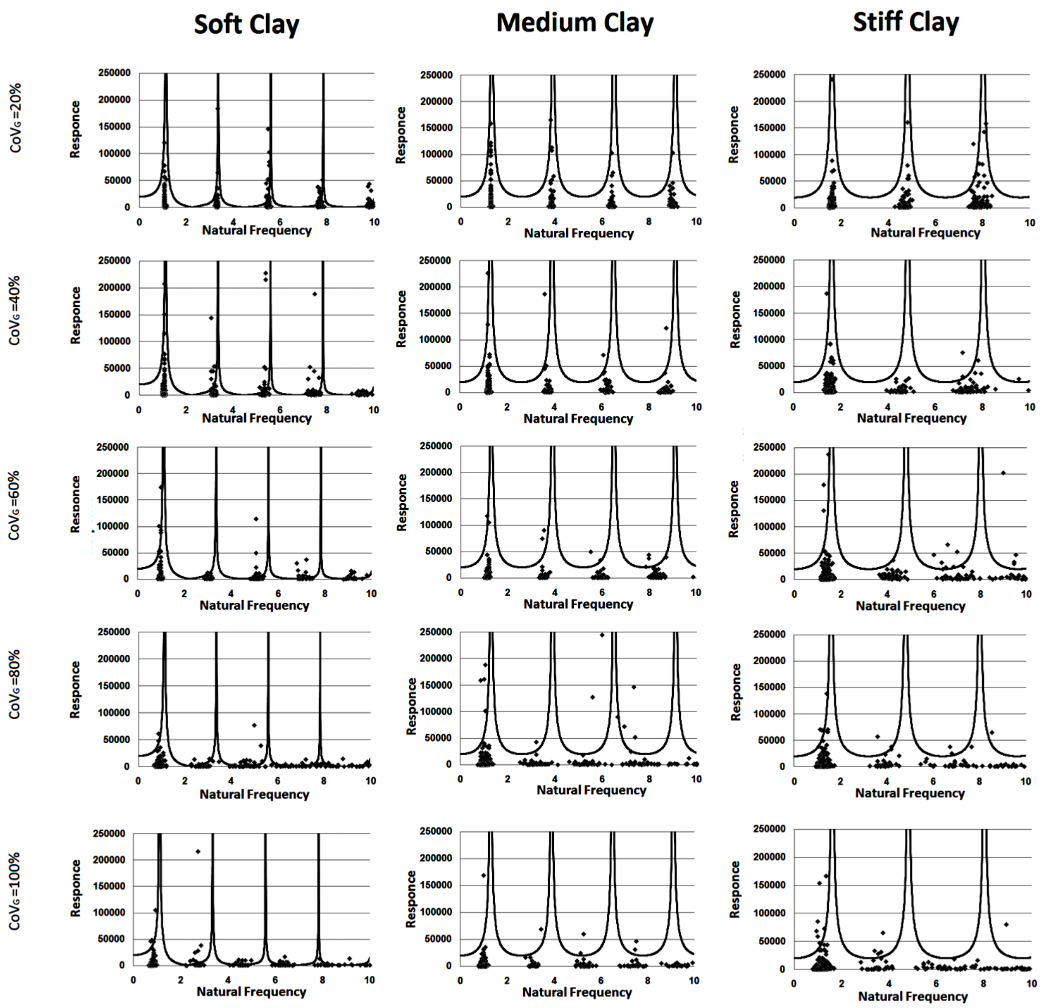

Figure 9. Results of natural frequencies for stochastic analyses superimposed by deterministic curves

Another observation from Figure 9 is that the overall concentration of the natural frequency points moves toward left when the coefficient of variation of shear modulus increases. This implies that the mean natural frequency decreases with the increase of COV_G and it is easily confirmed in Figure 8.

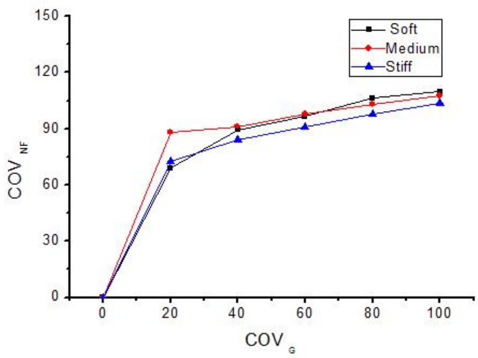

Figure 10. Coefficient of variation of natural frequency variation with CoV_G for different soil types

8.2. Influence of Soil Spatial Variability on the Amplification of Site

COV_G has physical significance as they reflect the nature (erratic or homogeneous) of a random field. The effect of COV_G can be observed in Figure 11a–c, which shows the gray scale representation of possible realizations of the shear modulus. Darker shades denote stronger zones having higher shear modulus. Realizations of random field generated for COV_G = 20%, 60% and 100% are presented in Figure 11a-c. The results indicate that increase in COV_G contributes to the erratic nature of soil and hence more number of weak zones are present. A higher range of shear modulus is generally observed for a field of high COV_G (say 100%) compared to a field of low COV_G values (say 20%) and this aspect is clearly evident from Figure 5a and c. Results of the mean amplification values are presented in Figure 12. Ensemble average of amplification values for stochastic analyses is taken as the mean amplification for spatially variable soil. It is evident that variation of shear modulus has an incremental effect on the amplification behaviour of heterogeneous soil stratum and the homogenous model used in deterministic analysis renders the highest amplification.

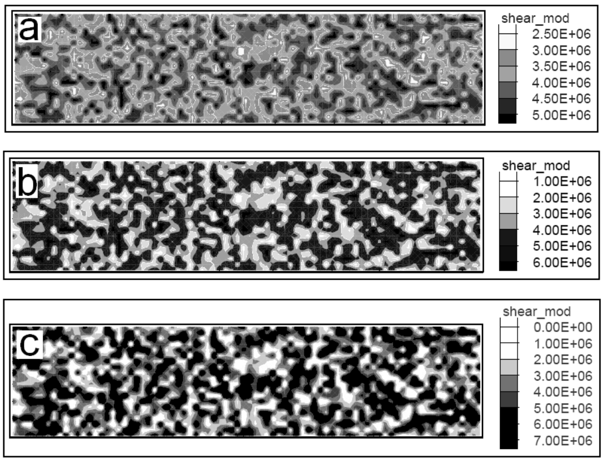

Figure 11. One realization of random field, (a) CoVG=20%, (b) CoVG=60%, (c) CoVG=100%

Also provided in Figure 13 is the result of the coefficient of variation of the amplification ratio with variation of the shear modulus. It is deduced that the increase of the COVG has a incremental effect on the COVA. High variation of shear modulus introduces higher contrast in amplification behaviour of different zones and wider range of variation of it which in effect induces a larger variation of the amplification results.

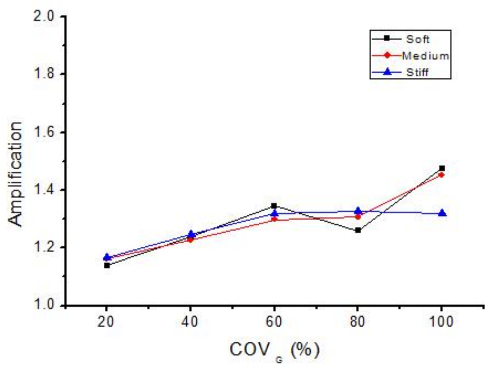

Figure 12. Variation of the mean amplification

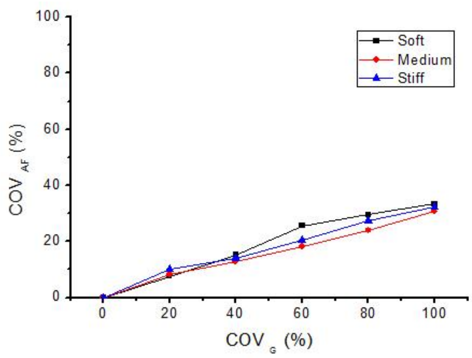

Figure 13. Variation of COV_A with COV_G with COV_G

9. CONCLUSIONS

The major contribution of the present study is to investigate the effect of uncertainty in soil shear modulus estimation on resonance behavior of naturally occurred alluvial deposits. It is observed that there is a significant variation in natural frequency and amplification due to the variation of shear modulus. Variation of natural frequency of soil deposits is stated in term of mean natural frequency and the coefficient of variation of natural frequency and amplification. From the results of numerical dynamic analyses it is concluded that:

1- The mean natural frequency of spatially variable natural alluvial deposits decreases due to an increase in variability of shear modulus or the shear wave velocity in other words. This behavior is justified by formation of weak zones introduced by high variability in natural deposits.
2- The results of the current study emphasizes that ignoring the spatial variation of shear modulus of natural alluvial deposits will in effect lead to overestimation of the natural frequency of soil deposits.
3- The amplification of natural alluvial deposits increases with the increase in variability of the shear modulus.
4- The uncertainty involved in natural frequency and amplification estimation increases with the increase in uncertainty embedded in shear modulus or shear wave velocity estimation.
5- Monte Carlo simulation technique combined with numerical analysis is a very useful tool to analyses the stochastic behavior of natural alluviums.

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