Optimal Design of Passive Control Systems Using Stiffness Modification and Pole Assignment Algorithm

M. Ahmadizadeh & A.R. Zare
Department of Civil Engineering, Sharif University of Technology, Tehran, Iran

15 WCEE
LISBOA 2012

SUMMARY:
A methodology is developed for designing optimum passive energy dissipation systems using active control algorithms. A combination of stiffness reduction and increase of damping is utilized to reduce both acceleration and displacement response. In this method, first an active control system is designed using pole assignment control algorithm. Here, the method to assign the new structural poles is modified such that the resulting properties can be achieved by a passive control system using viscous fluid dampers. Next, a passive control system is designed to result in the structural properties close to those extracted from the actively controlled system. It is shown that this method results in control systems that provide structural performances slightly better than, or close to those of ordinarily designed optimum passive systems. Furthermore, by carefully selecting the location of the structural poles, the proposed method provides more versatility in the design of passive control systems.

Keywords: *Active control, Passive control, Softening and supplemental damping, Pole assignment algorithm*

1. INTRODUCTION

It is generally accepted that active control systems provide better structural performance compared to the equivalent passive systems. On the other hand, the control systems used in most actual structures are of passive type, mainly due to the considerable costs associated with the construction and maintenance of active control systems. This study is aimed at the design of passive control systems with performances as close as possible to those of the active systems. For this purpose, it is attempted to utilize active control algorithms to determine the modification of the properties needed to improve the structural performance. Then a passive system is designed, such that along with the necessary modifications in the stiffness, attempts to reproduce the same structural properties to the possible extent.

Most building design codes mainly consider the inter-story drifts as the primary design parameter to minimize the loss of life or structural damage. Inter-story drifts are in fact suitable indicators for structural damage as well as damage to displacement-sensitive non-structural components, such as infill walls and piping. However, recent studies have shown that in addition to the drifts, absolute acceleration response may lead to damage to some non-structural components such as sensitive equipment, furniture and HVAC systems. Accelerations may also increase the human fear during earthquakes (Lavan, Cimellaro et al. 2008). For these reasons, a combination of inter-story drifts and accelerations are used in this study as a performance index that leads to the desired structural response when minimized.

Existing methods for seismic retrofitting of buildings often rely on adding damping and/or stiffness to the structures using passive control devices (Gluck, Reinhorn et al. 1996; Afsharhasani and Ahmadizadeh 2001; Lopez and Soong 2002). These devices usually dissipate energy by means of viscosity, friction, or yielding, and may also stiffen the building, consequently reducing the inter-story drifts (Cimellaro 2009). However, increasing the structural stiffness in most structures may lead to an increase of the acceleration response. Furthermore, the additional forces introduced by control systems may further increase the accelerations (Christopoulos and Filiatrault 2006). For this reason, some research has been put into reducing the accelerations by weakening or softening the structure. The resulting increase of inter-story drifts can then be countered by supplemental damping (Afsharhasani and Ahmadizadeh 2001; Cimellaro 2009). In this method the distribution locations and amounts of stiffness modifications and the distribution of supplemental damping is determined using an active control algorithm. This method is called the redesign approach by (Reinhorn, Lavan et al. 2009).

In this study, a design methodology is proposed for passive control systems using the pole assignment algorithm. Unlike the previous research (Reinhorn, Lavan et al. 2009), the structural mass is kept unchanged as is the case in practice. This is in fact an extension of the work by (Afsharhasani and Ahmadizadeh 2001), where the LQR algorithm was used in the design of passive control system. In the design approach proposed herein, first an active control system is designed for a uniformly softened structure using the pole assignment algorithm. Then, a passive structural control system is designed by using the properties of the actively controlled structure. For these properties to be achievable merely by reduction of stiffness and a passive control system with viscous fluid dampers, the pole assignment algorithm is slightly modified. In order to show the application of the proposed method and its benefits, an example five-story structure is designed using this approach, and its response is compared to those obtained using other design methods.

2. CONCEPT OF SOFTENING AND DAMPING

The simultaneous use of weakening or softening and damping allows for control of both drifts and accelerations. To illustrate, consider a single-degree-of-freedom as shown in Figure 2.1 which is excited by a ground motion.

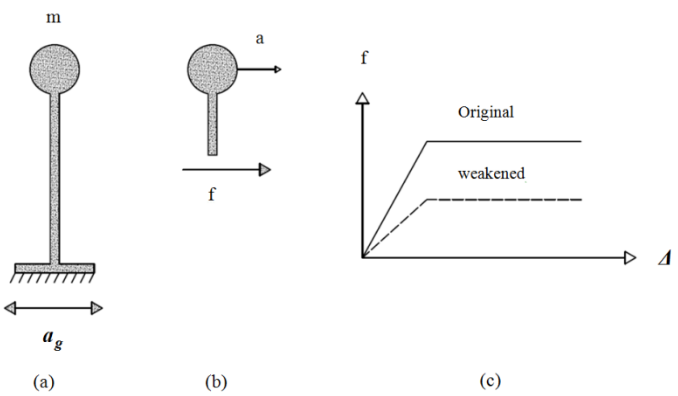

Figure 2.1. Weakening and softening in a single-degree-of-freedom system.

Assuming linear behavior for the columns which support the mass *m*. according to Newton's second law the acceleration of mass *m* is defined as:

$$a_{elastic} = \frac{f}{m} \tag{2.1}$$

where f is the force developed in column due to the excitation. Since the structural mass m is generally constant, one can reduce f by reducing the column stiffness for decreasing the acceleration when the column behaves elastically. Alternatively, when the column is to behave nonlinearly, reducing the column strength can lead to a reduced column force, thus reducing the accelerations. In Figure 2.1c it can be seen that simultaneous softening and weakening of the column have occurred, to reduce and limit the acceleration response of the mass. This in turn leads to an increase in the displacement response, which can be countered by supplemental damping. However, the design procedure is more complicated in multi-degree-of-freedom structures, since the proper locations and amounts of stiffness and damping modification are not known. To address this problem, active control algorithms can be used to determine the desired structural properties, thus providing the necessary information on the required modifications in the stiffness and damping of the structure and their locations.

3. ACTIVE CONTROL USING POLE ASSIGNMENT ALGORITHM

As the first step in the proposed design procedure, pole assignment algorithm is used to design an active control system for the structure. The equation of motion for a linear multi-degree-of-freedom subjected to an external excitation (such as an earthquake) is given by:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\,\mathbf{x}(t)=\mathbf{D}\mathbf{u}(t)+\mathbf{E}\mathbf{f}(t) \tag{3.1}$$

where $\mathbf{x}(t)$ is the displacement vector, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{u}(t)$ is the control force vector, $\mathbf{f}(t)$ is the excitation matrix, and $\mathbf{D}$ is the control force location matrix. For lumped-mass shear buildings considered in this study, matrices $\mathbf{M}$ and $\mathbf{K}$ are diagonal and tridiagonal, respectively. The damping matrix is also tridiagonal in classically-damped passively-controlled systems. In the above equation $\mathbf{E}$ is excitation location matrix. Eqn (3.1) in the state-space form can be written as:

$$\dot{\mathbf{z}}(t)=\mathbf{A}\,\mathbf{z}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{H}\mathbf{f}(t) \tag{3.2}$$

in which $\mathbf{z}(t)=\{\mathbf{x}(t) \quad \dot{\mathbf{x}}(t)\}^T$, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{H}$ are state variables, system matrix, control location and excitation location matrices, respectively. These matrices are defined as:

$$\mathbf{A}=\begin{bmatrix} \mathbf{0}_N & \mathbf{I}_N \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \tag{3.3}$$

$$\mathbf{B}=\begin{bmatrix} \mathbf{0}_N \\ \mathbf{M}^{-1}\mathbf{D} \end{bmatrix} \tag{3.4}$$

$$\mathbf{H}=\begin{bmatrix} \mathbf{0}_N \\ -\mathbf{1}_N \end{bmatrix};\ \mathbf{1}_N=\begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{Bmatrix}_N \tag{3.5}$$

where N is the number of degrees of freedom of the system or the number of stories in shear buildings.

Poles of the uncontrolled system are in fact the eigenvalues of the system matrix $\mathbf{A}$ defined in Eqn (3.3), which determine the modal frequencies and damping ratios of the original structure. In the pole assignment algorithm, the control forces are selected such that the poles of the structure are moved to new positions, usually to increase the damping, and move the natural frequencies away from expected excitation frequencies. Assuming that the control forces are given by:

$$\mathbf{u}(t)=\mathbf{G}\,\mathbf{z}(t) \tag{3.6}$$

in which **G** is control gain matrix, Eqn (3.2) can be rewritten as:

$$\mathbf{z}(t)=(\mathbf{A}+\mathbf{BG})\mathbf{z}(t)+\mathbf{Hf}(t) \tag{3.7}$$

That is, the control forces result in a change in system matrix from **A** to **A+BG**. By comparing these two matrices, the modifications in the structural properties resulting from the control forces can be determined. Hence, the problem of design of the passive control system using the proposed procedure consists of two major steps; first the designer needs to determine the suitable locations of new structural poles in the complex plane, and determine the required control logic for the system to have the prescribed eigenvalues. Second, the resulting modifications of structural properties should be determined, and applied by a passive control system that can only modify the structural stiffness and damping. That is, the modifications in the structural stiffness and damping should result in a new system matrix $\mathbf{A_{new}}=\mathbf{A}+\mathbf{BG}$.

A simple method to move the structural poles to the desired locations proposed by (Soong 1990) is as follows. For the new system to possess the prescribed eigenvalues $pole_j$, the following set of determinant equations should be satisfied:

$$\left|pole_j \times \mathbf{I}_{2N} - \mathbf{A} - \mathbf{BG}\right| = 0 \tag{3.8}$$

which can be rewritten as:

$$(\left|pole_j \times \mathbf{I}_{2N} - \mathbf{A}\right|)(\left|\mathbf{I}_{2N} - \psi(pole_j)\mathbf{G}\right|) = 0 \tag{3.9}$$

where:

$$\psi(pole_j) = (pole_j \times \mathbf{I}_{2N} - \mathbf{A})^{-1}\ \mathbf{B} \tag{3.10}$$

Since $pole_j$ are not generally the same as the eigenvalues of the original open-loop system:

$$\left|pole_j \times \mathbf{I}_{2N} - \mathbf{A}\right| \neq 0 \tag{3.11}$$

Hence, Eqn(3.9) leads to:

$$\left|\mathbf{I}_{2N} - \psi(pole_j)\mathbf{G}\right| = 0 \tag{3.12}$$

Again, this equation can be rewritten as:

$$\left|\Delta(pole_j)\right| = \left|\mathbf{I}_{2N} - \psi(pole_j)\mathbf{G}\right| = \left|\mathbf{I}_N - \mathbf{G}\psi(pole_j)\right| = 0 \tag{3.13}$$

For the j^{th} eigenvalue $pole_j$, one way to satisfy Eqn (3.13) is to entirely make the elements of a column or a row of $\Delta(pole_j)$ zero. This yields the necessary number of equations governing the control gains that move any of the $2N$ poles of the system to desired locations (Soong 1990).

Using this method, all unknown parameters in matrix **G** will be found, but the resulting gain matrix is not unique, since it depends upon the choice of columns of $\mathbf{\Delta}(pole_j)$. This simple method, however, does not necessarily result in modifications in the structural stiffness and damping that can be directly applied using passive control systems. Ideally, the required stiffness and damping matrices of a passively-controlled shear building (extracted from $\mathbf{A_{new}}$) should be tridiagonal.

In the proposed design method, the applicability of the structural property modifications by a passive control system is ensured by selecting a control gain matrix **G** is of the following form:

$$\mathbf{G}=\begin{bmatrix}\mathbf{G_1} & \mathbf{G_2}\end{bmatrix} \tag{3.14}$$

$$\mathbf{G_1}=\begin{bmatrix} -g_1 & 0 & 0 & \cdots & 0 \\ g_2 & -g_2 & 0 & \cdots & 0 \\ 0 & g_3 & -g_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & g_N & -g_N \end{bmatrix};\mathbf{G}_2=\begin{bmatrix} -g'_1 & 0 & 0 & \cdots & 0 \\ g'_2 & -g'_2 & 0 & \cdots & 0 \\ 0 & g'_3 & -g'_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & g'_N & -g'_N \end{bmatrix} \tag{3.15}$$

The above gain matrix leads to:

$$\mathbf{BG}=\begin{bmatrix} \mathbf{0}_N & \mathbf{0}_N \\ \mathbf{M}^{-1}\mathbf{DG_1} & \mathbf{M}^{-1}\mathbf{DG_2} \end{bmatrix}=\begin{bmatrix} \mathbf{0}_N & \mathbf{0}_N \\ \mathbf{M}^{-1}(\mathbf{-K_{added}}) & \mathbf{M}^{-1}(\mathbf{-C_{added}}) \end{bmatrix} \tag{3.16}$$

Here, it is assumed that control devices are installed in all stories as shown in Figure 3.1 for a three-story structure. The matrix **D** can then be written as:

$$\mathbf{D}=\begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{3.17}$$

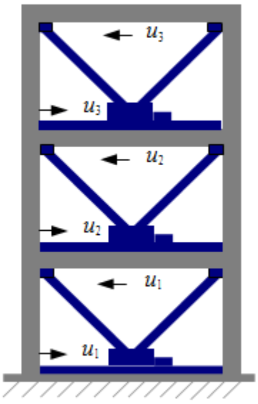

Figure 3.1. Distribution of the actuators in the three-story structure

Therefore, the modifications of the damping and stiffness properties of the structure are given by:

$$\mathbf{C_{added}} = -\mathbf{DG_2} = \begin{bmatrix} g_1' + g_2' & -g_2' & 0 & \cdots & 0 \\ -g_2' & g_2' + g_3' & -g_3' & \cdots & 0 \\ 0 & -g_3' & g_3' + g_4' & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & -g_N' & g_N' \end{bmatrix} \tag{3.18}$$

$$\mathbf{K_{added}} = -\mathbf{DG_1} = \begin{bmatrix} g_1 + g_2 & -g_2 & 0 & \cdots & 0 \\ -g_2 & g_2 + g_3 & -g_3 & \cdots & 0 \\ 0 & -g_3 & g_3 + g_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & -g_N & g_N \end{bmatrix} \tag{3.19}$$

It is seen that the matrices $\mathbf{K_{added}}$ and $\mathbf{C_{added}}$ are tridiagonal and symmetric, and hence of a form that can be applied by only modifying the stiffness and damping of the stories. In addition, the above selection of gain matrices produces the same number of unknown parameters as the poles of the system, yielding a unique set of gains. That is, all unknown parameters can be found by substituting each pole of the controlled system into the Eqn(3.13) .

As mentioned earlier, the location of the poles of the controlled structure is central to the resulting performance. One way of selecting the poles for the controlled structure is simply using the site design spectrum, which helps determine the required amount of damping and suitable frequencies for the structure. The poles can then be fine-tuned to meet the design requirements. On the other hand, since the optimum structural performance is sought in this study, the new locations of poles are determined using a performance index. For this purpose a performance index is defined as a weighted combination of maximum drift and maximum absolute acceleration response of the structure:

$$\text{PI} = \alpha \frac{\text{maximum drift of controlled structure}}{\text{maximum drift of uncontrolled structure}} + \beta \frac{\text{maximum accelaration of controlled structure}}{\text{maximum accelaration of uncontrolled structure}} \tag{3.20}$$

where weighting constants α and β show the relative importance of acceleration and displacement response of the controlled structure and are chosen accordingly.

In order to minimize the above performance index, the following procedure is used. First assume that the poles of the uncontrolled structure are:

$$pole_j = a_j \pm b_j \times i = -\xi_j \omega_j \pm i \times \omega_j \sqrt{1-\xi_j^2} \;\; ; \; j = 1:N \tag{3.21}$$

with ξ_j and ω_j being the damping ratio and natural frequency of the j^{th} mode. The frequencies and damping ratios of the controlled structure are defined as:

$$\xi_j' = n \times n_j \times \xi_j \; ; \omega_j' = m_j \times \omega_j \tag{3.22}$$

in which the index *j* denotes the mode number, and n , n_j and m_j are constant parameters that govern the new dynamic properties of the structure. Hence, for the controlled structure the poles are defined as:

$$pole_j' = (n \times n_j \times m_j) \, a_j \pm \left(\frac{b_j \times m_j \times i}{\sqrt{1-(\xi_j)^2}} \times \sqrt{1-(n \times n_j \times \xi_j)^2} \right) \; ; \; j = 1:N \tag{3.23}$$

that shows the effects of the parameters n and m on the poles. In order to optimize the performance and minimize the index given by Eqn (3.20), first, the parameter n in Eqn (3.23) is modified iteratively until finding a minimum *PI*. In each iteration, poles are recalculated using the trial n, and matrix **G** is determined by solving the corresponding system of $2N$ equations and $2N$ unknowns, using Newton-Raphson algorithm (Neumaier 2001). The control force vector is then computed according to Eqn (3.6) and is applied to the structure subjected to the design earthquake(s) to determine the controlled performance and the corresponding *PI*. Next, Parameters n_j and m_j are chosen to further reduce the *PI* to its least possible value. This way, it is ensured that the poles are moved to their optimum locations in the complex plane.

4. DESIGN OF THE EQUIVALENT PASSIVE CONTROL SYSTEM

Using pole assignment control algorithm explained in the previous section, the new system matrix can be expressed as:

$$\mathbf{A_{new}} = \mathbf{A} + \mathbf{BG} = \begin{bmatrix} \mathbf{0}_N & \mathbf{I}_N \\ -\mathbf{M}^{-1}\mathbf{K_{new}} & -\mathbf{M}^{-1}(\mathbf{C_{device}} + \mathbf{C}) \end{bmatrix} \tag{1.1}$$

where matrix **G** is obtained using the procedure describe in the preceding section. By pre-multiplying the mass matrix to the second row of the above matrix, the modified stiffness and damping matrices of the passively-controlled structure can be obtained. It is expected that using these structural properties, the passive control system will result in a performance close to that obtained by the actively control system. However, the flexibility of the braces that are used to connect the dampers will reduce their improving effect on the structural performance (Ahmadizadeh 2007). In this study, it is assumed that the braces are designed for the nominal damper force capacity.

5. CASE STUDY– MDOF 5-STORY SHEAR BUILDING

In this section, five-story shear building is designed with the proposed method and the results are presented. The building structure is 15.0 meters high, and its plan dimensions are 25.0 meters (5 bays) by 18.0 meters (3 bays). The lateral force resisting system in both directions consists of moment resisting frames with pinned connections of the columns at the foundation level. Live and dead loads of each story are assumed to be 250.0 Kg/m^2 and 500.0 Kg/m^2 respectively. According to the Iranian Code of Practice for Seismic Resistant Design of Buildings (BHRC 2005), effective seismic mass for all stories are calculated to be 250.0 tons. The structure is assumed to be built on soil type II according to this code. The inherent damping of the structure is selected to be 5% of critical in the first two modes. The primary structure is designed according to the AISC-LRFD manual (AISC-LRFD 2001) this study, the design earthquake is selected to be 1940 El Centro earthquake (Mw=6.9, PGA=0.319g) with acceleration amplitude normalized to 0.4g.

In the design of the active control system, the actuator forces are limited to 1500 KN and the performance index *PI* is used with weighting coefficients of $\alpha=0.7$ and $\beta=0.3$ .The initial uniform reduction in the structural stiffness is selected to be 30% as suggested in (Afsharhasani and Ahmadizadeh 2001). The control systems studied and their corresponding abbreviations are listed in Table 5.1. It should be noted that the optimal design of passive control system is performed by directly minimizing the performance index through several trial values of damping coefficients (Soong and Dargush 1997).

Table 5.1. Cases of control systems for the five-story structure

Uncontrolled	Uncontrolled + Softening	Active + Softening	Active Without Softening	Equivalent passive + Softening	Equivalent passive without Softening	Optimal passive
U	**US**	**AS**	**A**	**PS**	**P**	**O**

Table 5.1 represents the maximum inter-story drifts and maximum absolute accelerations in all stories in each of the control cases. As shown, softening causes inter-story drifts to increase and absolute accelerations to decrease (comparing **U** with **US**). The best performance among all control cases is seen to be that of **AS**. In this case the inter-story drifts and absolute accelerations are observed to be the smallest. The next best performance can be seen in the case of **PS**, which is the equivalent passive system for the **AS** system. In this case, the maximum inter-story drift that occurs in the first floor is seen to be smallest after the actively controlled system **AS**. As shown, some performance degradation in the equivalent passive system can be recognized compared to the active system, which is partly attributed to the finite stiffness of the braces used for the installation of dampers. The brace flexibility, on the other hand, does not have a significant effect on the performance of the active control system.

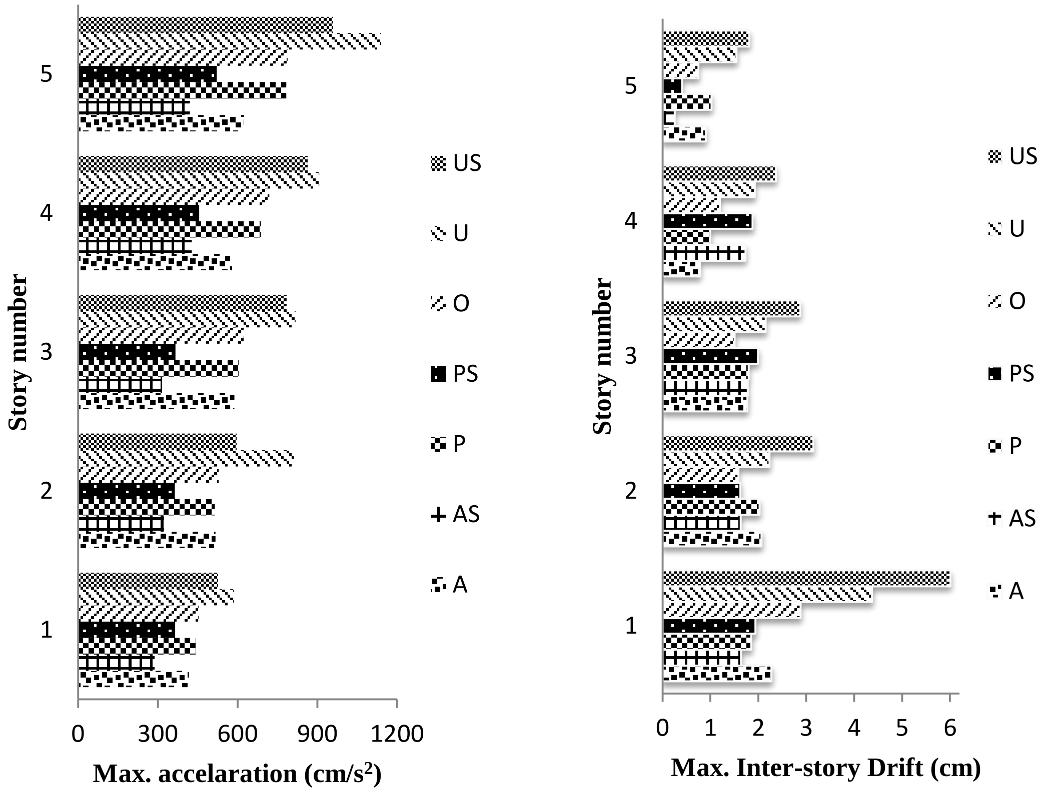

Figure 5.1. The maximum absolute acceleration and inter-story drift in the five-story structure

By comparing the control cases **O** and **PS**, it can be observed that a better performance is achieved using the **PS** system compared to case **O**, showing the optimal performance of control system designed using the proposed method. Some of the above-mentioned improvement stems from the reduction of structural stiffness as dictated by the active control algorithm. However, it should be noted that a trial-and-error process may eventually reach the results of any design method, and the advantage of the proposed method may not be clear in this regard. On the other hand, by using the pole assignment algorithm and modifying the structural stiffness, the proposed procedure provides a more versatile means for the educated modification of the structural properties towards the desired performance.

Table 5.2 lists the values and the distribution of the softening and added damping in the structure in different control cases. By comparing cases **P** and **PS**, it is evident that the larger stiffness of most stories in the case **P** has resulted in larger absolute accelerations of this case, shown in Figure 5.1. As another observation, the values of supplemental damping in the structure in the case **O** are generally larger than those of **PS** case. This can be partially attributed to the increased level of forces in the structure due to higher stiffness that consequently would require larger forces for control.

Table 5.2. Damping and stiffness values of control systems

	U		US		P		PS		O
Floor NO.	C (KN.s/m)	K (KN/m)	C (KN.s/m)	K (KN/m)	Cdevice (KN.s/m)	K total (KN/m)	Cdevice (KN.s/m)	K total (KN/m)	Cdevice (KN.s/m)
1	762.6	203636.0	638.0	142545.2	10452.1	285000.5	10713.9	150209.0	5906.4
2	1004.7	343846.0	840.6	240692.2	8529.4	244224.8	7199.8	178873.6	8520.0
3	891.6	305135.0	746.0	213594.5	9667.4	223772.9	6996.7	113954.1	7866.1
4	774.7	265135.0	648.2	185594.5	7039.5	335062.1	4031.3	106124.8	9360.2
5	713.4	186812.0	596.9	130768.4	3842.3	179362.5	5614.4	289443.8	12960.0

Figure 5.2 and Figure 5.3 represents the device force histories of the first floor in the two cases of **AS** and **PS**. It can be seen that the actuator force reaches its predetermined maximum value of 1500 kN at some points in both cases. However, this occurs less frequently in the passive system, where the control forces seem to be generally smaller. This can be explained in that the active control forces serve for both stiffness and damping modifications of the structure. However, in the passive system, the strain-dependent restoring forces are directly included in the structure itself by modifying its stiffness, and the device forces are only due to supplemental damping. In other words, the portion of control forces that modify the story stiffness is omitted in the passive control system.

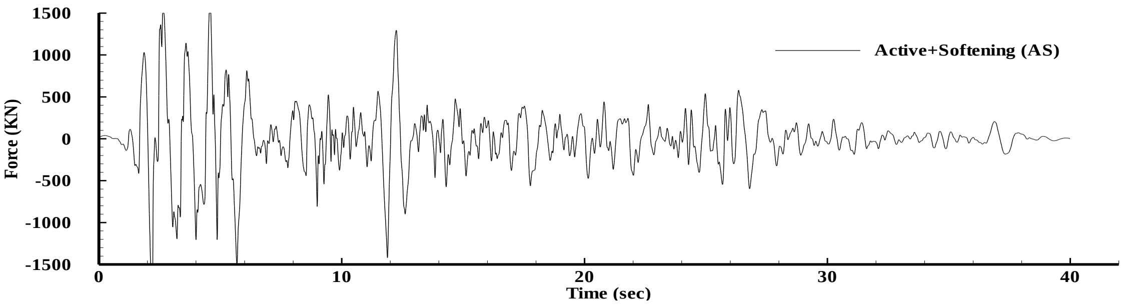

Figure 5.2. Device force in the **AS** case (first floor)

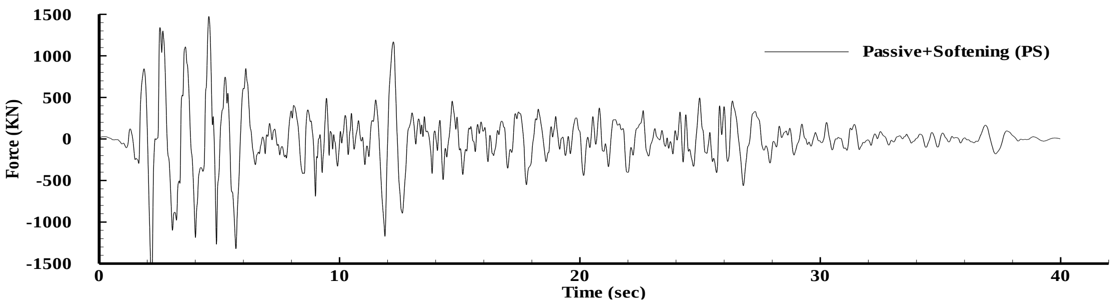

Figure 5.3. Device force in the **PS** case (first floor)

6. CONCLUSIONS

An alternative methodology was proposed for the design of optimum passive control systems using stiffness modification and viscous fluid dampers to reduce both the acceleration and displacement response. In this method, after uniformly reducing the stiffness, an active control system is designed for the structure using the pole assignment algorithm. The resulting structural properties are then used to determine the desired properties of the passive control system that takes advantage of stiffness reduction and supplemental damping. In this design method, the poles of the structure can be relocated to appropriate positions to minimize the structural response (or a performance index), or simply by considering the required properties of the controlled structure according to the design goal. Although the design results are shown to be better than or close to those of optimum passive systems designed using the usual trial-and-error method, some performance degradation is expected in the process of replacement of the active control system by its passive equivalent. This is partly due to the finite stiffness of the braces that significantly affect the performance of passive dampers. The effectiveness of the proposed design approach is demonstrated in the performance comparisons of a five-story structure controlled using various methods. The method is shown to provide a practical means to modify the properties of the structure in the desired manner using passive control devices to achieve the design goals.

REFERENCES

Afsharhasani, R. and Ahmadizadeh, M. (2001). Design of Optimal Passive Energy Dissipation Systems Using Active Control Theory *Sixth International Conference of Seismology and Earthquake Engineering*.

Ahmadizadeh, M. (2007). On equivalent passive structural control systems for semi-active control using viscous fluid dampers. *Structural Control and Health Monitoring* **14**(**6**): 858-875.

AISC-LRFD (2001). Manual of steel construction - load and resistance factor design. Chicago (IL, USA) American Institute of Steel Construction.

BHRC (2005). *Iranian code of practice for seismic resistant design of buildings* (standard No.2800-05), Building and Housing Research Center.

Christopoulos, C. and Filiatrault, A. (2006). Principles of Passive Supplemental Damping and Seismic Isolation, University of Pavia.

Cimellaro, G.P. (2009). Optimal weakening and damping using polynomial control for seismically excited nonlinear structures. *Earthquake Engineering and Engineering Vibration* **8**(**4**): 607-616.

Gluck, N., Reinhorn, A.M. , et al. (1996). Design of supplemental dampers for control of structures. *Journal of Structural Engineering-Asce* **122**(**12**): 1394-1399.

Lavan, O. , Cimellaro, G.P., et al. (2008). Noniterative optimization procedure for seismic weakening and damping of inelastic structures. *Journal of Structural Engineering-Asce* **134**(**10**): 1638-1648.

Lopez, G.D. and Soong, T.T. (2002). Efficiency of a simple approach to damper allocation in MDOF structures. *Journal of Structural Control* **9**(**1**): 19-30.

Neumaier, A. (2001). Frontmatter Introduction to Numerical Analysis, Cambridge University Press.

Reinhorn, A., Lavan, O. , et al. (2009). Design of controlled elastic and inelastic structures. *Earthquake Engineering and Engineering Vibration* **8**(**4**): 469-479.

Soong, T.T. (1990). Active Structural Control: Theory and Practice. New York, Longman.

Soong, T.T. and Dargush, G.F. (1997). Passive energy dissipation systems in structural engineering. New York, Wiley.