Effects of soil-structure interaction on performance-based assessment of masonry buildings

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SUMMARY:

All the modern seismic codes are based on the Performance-Based Assessment (PBA). PBA generally uses pushover analyses and the verification by the non linear static procedures. Usually pushover analyses are performed assuming a fixed-base structure. However for heavy and rigid masonry structures, compliance and geometry of the foundation system, in combination with the non-linear behaviour of the foundation soil, may significantly modify the actual response in terms of both capacity and demand. Regarding the capacity, taking into account soil compliance modifies the pushover curve, leading to more flexible systems. Regarding seismic demand, spectra including soil-foundation-structure interaction differ from those traditionally obtained in case of free-field ground motion. In this paper, firstly, specific impedance functions for flexible masonry foundations are proposed and then parametric non linear analyses on 3D-complex masonry buildings (by using the Tremuri Program) are performed in order to gain insight into the influence of the foundation-soil system compliance on PBA.

Keywords: soil-structure-foundation interaction, masonry, equivalent frame model, non linear static analysis

1. INTRODUCTION

The modern seismic codes for the design of new buildings, as well as the most advanced recommendations for the evaluation and rehabilitation of the existing ones, are based on the performance based assessment (PBA). PBA generally uses the pushover analysis (to obtain the base shear-displacement curve representative of the overall inelastic response of the structure) and the verification by the non linear static procedures (e.g. Coefficient Method, Capacity Spectrum Method, N2 Method). As it is well-known, these procedures are based on the comparison between the displacement capacity of the structure (properly converted in an equivalent SDOF) and the displacement demand of the predicted earthquake (idealized in terms of an elastic response spectrum properly reduced). Usually pushover analyses are performed assuming a fixed-base structure, hyphotesis that in some cases may result quite rough. In fact, the compliance and the geometry of the foundation system in combination with the non-linear behaviour of the foundation soil could significantly modify the actual response in terms of both capacity and demand.

For example, focusing the attention on masonry buildings, historic structures with massive foundation masonry systems are often characterized by significant mass and complex structural systems. Sometimes the foundation may be quite deep. The oscillation of these massive structures certainly interacts with that of the surrounding soil. As a result, the seismic input in the system can be considerably modified by the presence of the building (De Barros and Luco,1995) and the massive masonry foundation. For slender building typologies such as towers, the soil-foundation interaction (SFI) may produce significant rocking effects and associated damping on the structure. The filtering of the signal due to kinematic interaction is modifying the foundation input motion for the structure

(Stewart et al. 1999). For massive high frequency structures the importance of soil-foundation-structure interaction (SFSI) effects may be equally important. In fact, it is well known that for heavy stiff structures resting on soft soil, linear and nonlinear soil-foundation-structure interaction play an important role on the response of the foundation, transferring stress fields from the structure to the foundation, filtering high frequencies and hence modifying the response of the building (Pitilakis D. 2006, Kirtas and Pitilakis K. 2009). Conventional foundation models usually consider the foundation as a non-deformable rigid body. On the contrary, historical masonry buildings have a foundation system that can transfer negligible tensile stress and no bending moment at all. The actual flexibility and geometry of the foundation system, in combination with the non-linear behaviour of the foundation soil, may modify the acceleration and displacement spectra at the foundation level. On the other hand, regarding the result of pushover analyses (that is the capacity curve), taking into account the soil compliance (e.g. by modelling base- restraints through Winkler springs) modifies the pushover curve obtaining more flexible systems.

In this paper, firstly specific impedance functions for flexible masonry embedded foundations are proposed and, then, a set of parametric non linear analyses on 3D-complex masonry buildings (by using the Tremuri Program which works according to the equivalent frame approach) are performed in order evaluate the effects of the foundation-soil system compliance on pushover analyses. In particular, two case studies – representative of an ordinary masonry buiding and a bell tower – are examined.

2. DYNAMIC STIFFNESS OF FLEXIBLE MASONRY FOUNDATIONS

2.1. Description of the methodology

Numerical time history analyses proved that typical monumental systems respond at low frequency range. In the literature (Gazetas 1983, Iguchi & Luco 1981, Gucunski & Peek 1993, Liou & Huang 1994, Chen & Hou 2009) it is assumed that static stiffness is almost equal to the dynamic one ($K_{stat} \approx K_{dyn}$) for low frequency vibrations (or low dimensionless frequency a_0). Therefore, in order to evaluate dynamic stiffness of flexible masonry foundations, a set of elastic static analyses can be performed with properly modified properties material properties.

Impedances proposed in literature concern mainly rigid foundations. Studies for flexible foundations do not propose impedance functions for practical engineering purposes. The aim of this study is to propose impedance functions for flexible masonry foundations. Masonry material properties, geometrical characteristics of the foundations and soil properties considered in the analyses will be summarized and discussed in the following sections.

2.2. Soil-foundation systems for the parametric analyses

Static analyses in 2D plane strain soil – foundation models were performed to calculate the static stiffness for all modes of vibration. For the simulation of the foundation, 4 nodded quadratic plane strain elements are used for the translational degree of freedom and beam elements for the rotational. In both cases the soil is modelled with plane strain elements. The foundation is bonded to the soil through kinematic constrains, assuring solid connection of the two media. In any case, linear elastic behaviour for both soil and foundation is assumed. The required reaction under unit displacement or rotation (spring value) is calculated at the centroid of each foundation.

The depth of the soil model was 10 times the foundation width and the width was 4 times the depth. In all analyses we were checking if the displacements were zero at the boundaries and thus we were sure that the boundaries are far enough. Regarding the estimation of the resisting force, an assumption of uniformly distributed load on the soil-footing interface is made; this is a quite common practice in impedance functions calculation for rigid footings. The schematic representation of the system for the translational degree of freedom is shown in Figure 2.1. The foundation type, geometry and material properties were considered taking into account the data from existing monuments.

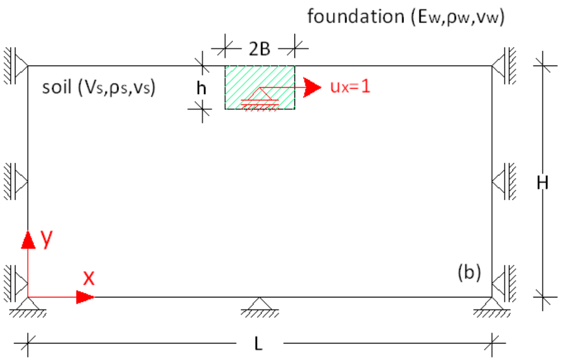


Figure 2.1. Schematic representation of the soil – foundation system for the translational degree of freedom

Material properties have been assumed starting from some reference values proposed in the Italian Code for Structural Design (2008) and its Instruction document (2009) as a function of different masonry types. To this end, rectangular foundations of varying dimensions (height=h and width=2B) and various material properties (in terms of elastic moduli) were assumed for the analyses (Table 2.1 & Table 2.2). Table 2.1 shows the geometry of the foundation systems (12 cases) considered in the analyses. Table 2.2 shows the elastic moduli (E_w) of the foundations and the soil properties adopted in this study. In any case, elastic modulus value in the analysis is reduced by 50% from the initial value, in order to take into consideration the current condition of the masonry materials (cracked, deterioration due to environmental effects, differential settlements etc). The soil properties that were used in these analyses correspond to four different soil classes according to EC8 (Comité Européen de Normalisation, Eurocode 8, part 1) soil classification scheme. A set of more than 2700 parametric analyses were performed. It must be noted that in order to capture both the in plane and the out of plane behaviour of the wall, the adopted dimensions are widely differ.

Table 2. 1. "Foundations" dimensions

	Width (2B)	**Height (h)**
Case 1	0.50	0.5
Case 2		1
Case 3		2
Case 4	1	0.5
Case 5		1
Case 6		2
Case 7	2	0.5
Case 8		1
Case 9		2
Case	10	0.5
Case		1
Case		2

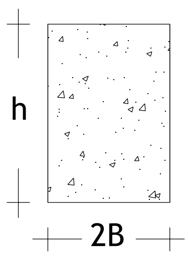


Table 2. 2. Masonry elastic modulus and soil properties adopted in this study

Case	**$E_{w,el.}$ (MPa) – nominal**	**E_w (MPa) – analysis**	**Soil type***	**V_s (m/sec)**	**E_{soil} (MPa)**
1	690	345	**A**	1000	5332
2	1800	900			
3	1980	990	**B**	500	1333
4	2800	1400			
5	2820	1410	**C**	250	333.25
6	3400	1700			
7	4400	2200	**D**	150	119.97

2.3. Impedances for rigid foundations

In order to compare stiffness values that result from the theoretical expressions to the FEM for rigid foundations, we performed a set of static elastic analyses for rigid strip foundations that lay on a homogeneous soil stratum over rigid bedrock, for the horizontal and vertical mode of vibration and for all the 12 foundation geometry cases. In all cases, only the examined degree-of-freedom is permitted in the finite element modelling (i.e. for the horizontal model of vibration, vertical displacement and rotation of nodes are constrained). The resulting values, normalized to the shear modulus multiplied by the half width of the foundation, when using the two approaches (FEM and theoretical expressions) differ to a reasonable percent taking into consideration the differences between the two approaches.

Based on the aforementioned convergence between the analytical and numerical method, we normalized the stiffness for flexible foundations (by FEM) to the stiffness for rigid foundations by FEM (for very large ratio E_w/E_{soil}). Essentially, this is the same as normalizing stiffness values for flexible foundation with the theoretical (analytical) values proposed in the literature for rigid footings. That being said, soil-foundation system flexibility can be accommodated by proper reduction of the foundation system, according to the effective E_w/E_{soil} ratio.

2.4. Impedances for flexible foundations

For embedded flexible foundations the normalized stiffness values are shown in Fig. 2.2 for horizontal (Fig. 2.2a), vertical (Fig. 2.2b) and rotational (Fig. 2.2c) modes of vibration respectively. The average normalized stiffness value for all cases is plotted with the plus and minus one standard deviation. It can be seen that the deviation is larger for the rocking mode, as the foundation geometry is more important in the rotational than in translational modes. Each point in Fig. 2.2 represents a soil-foundation system of specific geometrical and material properties. For embedded foundations, the effect of foundation flexibility is almost the same for horizontal and vertical translational modes of vibration. An important conclusion is that the rotational mode is affected the most by the foundation geometry comparing to the translational ones. The important scattering in stiffness values for the rotational mode is expected due to the fact that the foundation geometry influences mainly the rocking. For the horizontal and vertical modes, foundation geometry does not affect significantly the normalized impedance values.

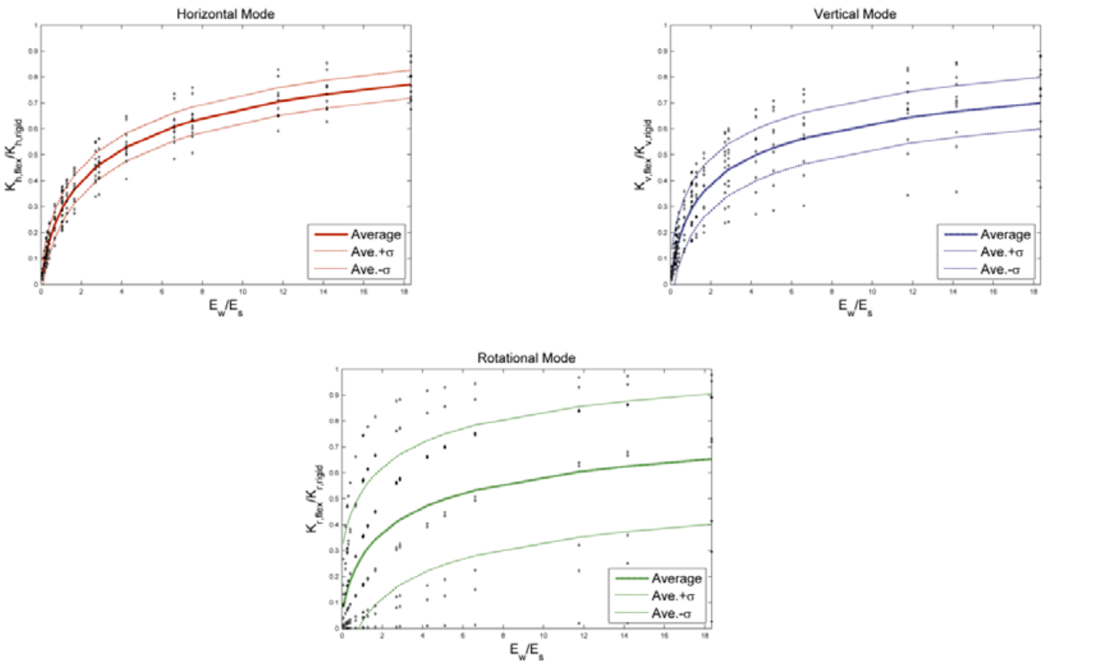


Figure 2.2. Normalized stiffness values of embedded foundations varying in geometry (h=0.5m-2m, B=0.25m-5m) resting on a homogeneous soil for different values relative stiffness between the foundation and the soil medium (Ew/Esoil) for (a) horizontal (b) vertical translational and (c) rocking modes of vibration

3. EVALUATION OF SSI EFFECTS BY NON LINEAR STATIC ANALYSES ON SOME MONUMENTAL STRUCTURES

3.1. Description of the Examined Case Studies

In order to evaluate the effects of SFSI on masonry structures performance under seismic loading, a set of parametric non linear analyses (pushover analyses) were performed on two types of assets: an Ordinary Building and a Bell Tower (see Fig. 3.1). The soil-foundation system was modeled by means of springs located at the center of foundations. The spring stiffness was computed assuming fully embedded foundations in a homogeneous half space. In the case studies, three constraint's conditions were considered: (*a) Fixed Base condition*, where the structure is modeled as fixed at its base; (*b) Rigid Foundations condition*, where the structure is modeled with rigid foundations embedded in a flexible soil; (*c) Flexible Foundations condition*, where the structure is modeled having flexible foundations embedded in a flexible soil. As described in more detail in §3.2, the structures have been modeled by the Tremuri program, which has been originally developed at the University of Genoa, starting from 2002 (Galasco et al. 2009), and subsequently implemented in the software 3Muri (3Muri, release 4.0.5).

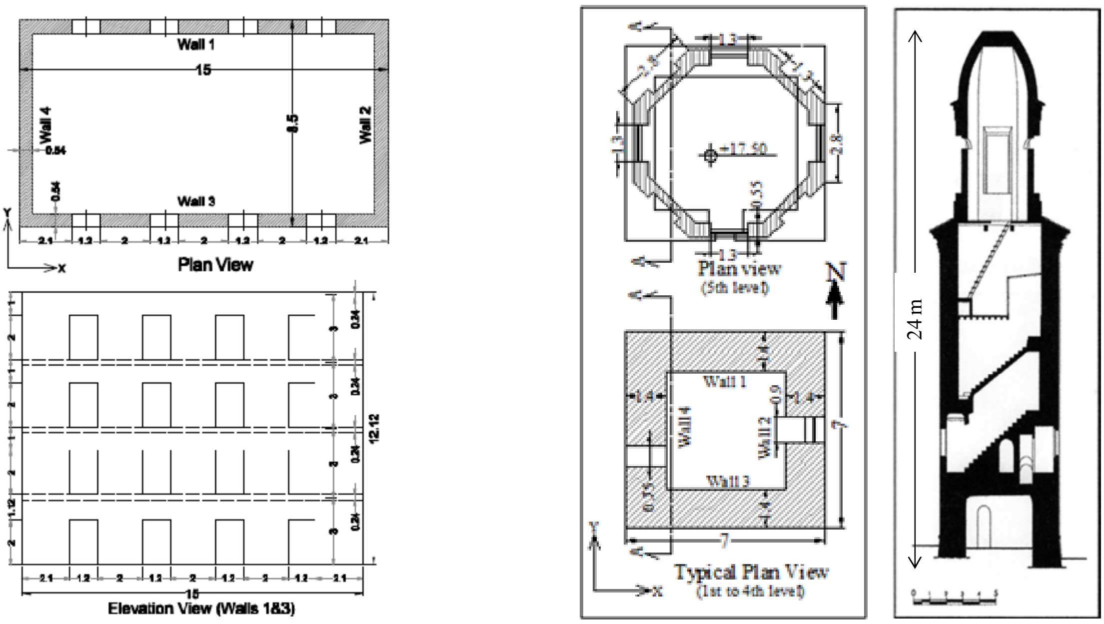


Figure 3.1. Ordinary Building: Plan and Elevation Views (left); Bell Tower (The Gonzaga's Tower– Bagnolo in Piano's Castle (RE) – Italy) - Typical Plan View on the left and Section A-A (right)

The four–storey Ordinary Building sketched in Fig. 3.1 is characterized by the presence of a high number of openings in the two main parallel facades and no openings in the two other ones. The dimensions of the building are 8.5x15.0m in plan and 12.12m in height. This kind of structure was studied with and without the presence of reinforced concrete ring beams (RCB) coupled to spandrel elements. The foundations thickness is assumed to be equal to 0.7m and their embedment depth equal to 1m. The Bell Tower sketched in Fig. 3.1 has a total height of 24 m (4 stories + belfry) and a square base 7x7m. It's a real case study of "the Gonzaga's Tower" in Bagnolo in Piano (RE) – Italy. The foundations thickness is assumed to be equal to 1.4 m, and their embedment depth equal to 1.5m. Analyses were performed considering two types of soil: soil type D having a shear modulus G_s=30MPa, and soil type C having a shear modulus G_s= 50MPa (According to EC8). In case of the Bell Tower, only soil type C has been considered.

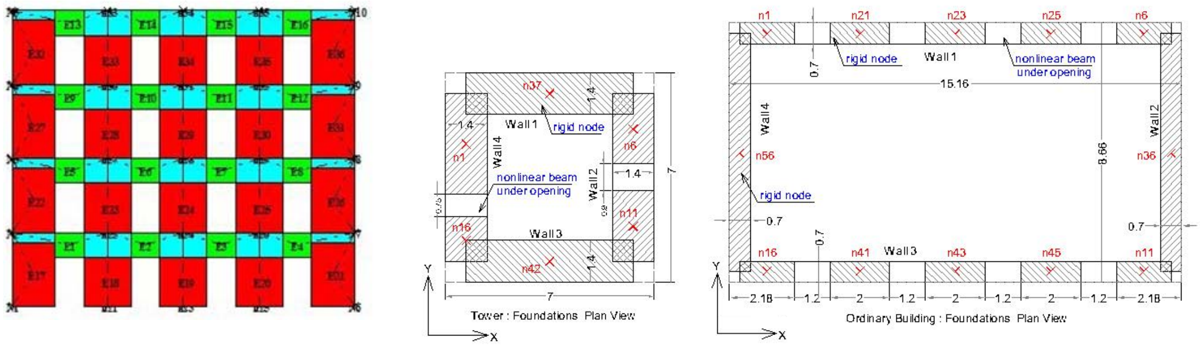
Mechanical parameters have been assumed according to values proposed in the Italian Code for

Structural Design (2008) and its Instruction Document (2009) for different masonry types. In the tower cases, an "un-cut stone masonry with facing walls of limited thickness and infill core" in levels 1 to 4 and "brick masonry with lime mortar" in the 5th level have been assumed respectively. The modulus of elasticity (E) corresponding to these two materials is 1230 and 1500MPa, respectively; the shear modulus (G) is 410 and 500MPa, respectively; the shear strength (τ_0) is 3.58 and 6.33N/cm^2, respectively. In the ordinary building cases, a "masonry in bricks and lime mortar" with good mortar quality has been assumed (E= 2250MPa; G= 750MPa; N/mm^2; τ_0=9.49N/cm^2). Values of E and G refer to elastic condition.

3.2. Modelling by the equivalent frame approach

The several case studies were modeled by Tremuri program (Galasco et al. 2009) based on the equivalent frame approach. According to this modeling strategy, each wall is discretized by a set of masonry panels (*piers* and *spandrels*), in which the non-linear response is concentrated, connected by a rigid area (*nodes*). For example, Fig.3.2a illustrates in case of Ordinary building the frame idealization of Wall 3. Masonry panels are modeled by a non-linear beam idealization: thus the response is directly faced in terms of stiffness, strength and ultimate displacement capacity by assuming a proper shear-drift relationship. For further details on the hypotheses of Tremuri program see also Galasco et al. (2004) and Lagomarsino and Cattari (2009).

In models considering SFSI, the foundations, assumed to be rectangular and totally embedded in the soil, are modeled as rigid nodes under the piers elements, with stiffness simulating the soil-foundation system (computed assuming rigid then flexible foundation) applied at the centre of the foundation, while they are modeled as nonlinear beams connecting the adjacent rigid nodes under openings. The spring stiffness values were computed according to literature (Gazetas 1991) when the foundations were modeled as rigid, and according to the expressions introduced in section 2.2 when they were modeled as flexible. As an example, in case of soil C, for the ordinary building cases, stiffness values are around 4.5x10^{8}N/m, 6x10^{8}N/m and 5.5x10^{8}N.m for horizontal, vertical and rotational stiffness, respectively, in the case of rigid foundations, and 1x10^{8}N/m, 3x10^{8}N/m and 3x10^{8}N.m, in the case of flexible ones; while, in the tower cases, they vary from 7.2x10^{8}N/m, 7.1x10^{8}N/m and 2.1x10^{9}N.m in the case of rigid foundations to 2.2x10^{8}N/m, 4.7x10^{8}N/m and 3x10^{8}N.m in the case of flexible ones.



(a) (b)

Figure 3.2. (a) Equivalent frame idealization of Wall 3 of Ordinary building (in red piers, green spandrels and cian nodes, respectively); (b) Nodes ID at the foundations level generated by Tremuri in case of the Tower and the Ordinary Building

Pushover analyses have been performed parametrically as a function of: two load patterns (proportional to masses and to the height - masses product, quoted as "pomas" and "pomaz", respectively); different soil configurations (soil C and D classes as proposed in Eurocode 8, corresponding to the adoption of G modulus equal to 30 and 50MPa, respectively); both rigid and flexible embedded foundations (quoted as "rig" and "flex", respectively); different configurations of models. In particular, in case of the Ordinary building, reinforced concrete beams have been modeled or not coupled to spandrel elements (quoted as “Bldg with RCB” and “Bldg without RCB”, respectively).

3.3. Discussion of results

Results of pushover analyses performed on the several case studies, as previously mentioned, are shown in Figures 3.3 and 3.4 For example, pushover curves leading to the more cautionary results are represented; they correspond to the application of load patterns proportional to masses – in case of the Tower – and proportional to mass-height product – in case of the Ordinary Building.

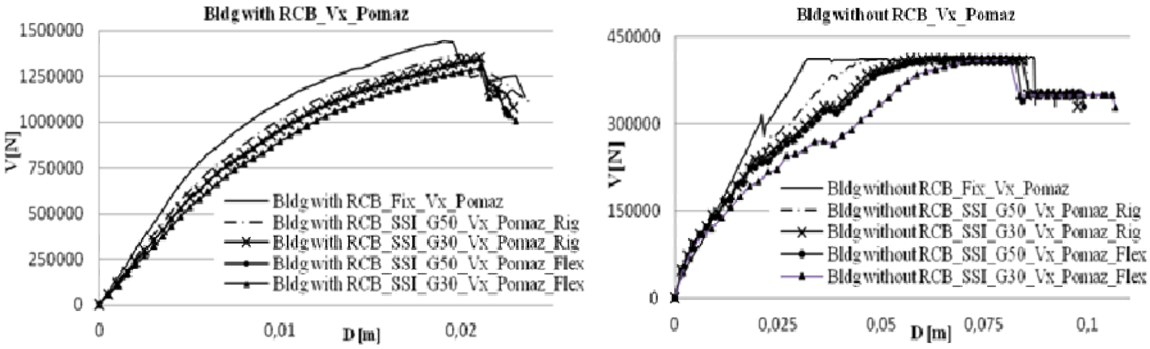


Figure 3.3. Pushover curves in the case of Ordinary buildings with RCB (at left) and without RCB (at right) for the several end conditions analyzed with applied horizontal force in the x direction

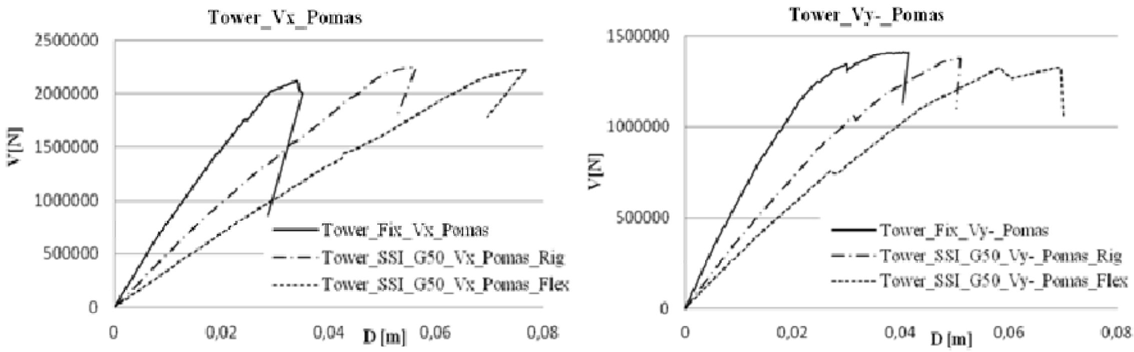


Figure 3.4. Pushover curves in the case of the Tower for the several end conditions analyzed with applied horizontal force in the x direction (at left) and in the y direction (at right)

Since pushover curves differ in terms of strength, stiffness and ductility - all three aspects they play a fundamental role in the seismic assessment – results are compared in the following by referring to non-linear static procedures. To this aim, pushover curves representative of the original MDOF have been converted into those of the equivalent SDOF system. Among the different approaches proposed in the literature, the criteria adopted in both Eurocode 8 and Italian Code for Structural Design (2008) are assumed as reference; they basically refer to the N2 Method originally proposed in Fajfar (2000) and, as known, based on the use of inelastic spectra. Fig. 3.5 shows some resulting capacity curves.

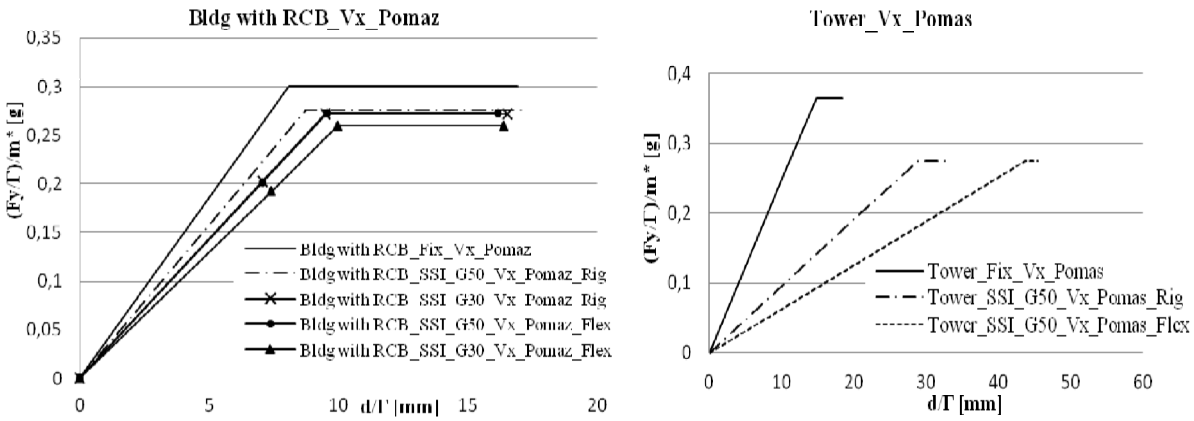


Figure 3.5. Capacity curves in the case of Ordinary buildings with RCB(at left) and Tower (at right) for fixed base, rigid foundations and flexible foundations approximations.

In the following, in order to evaluate the response of the structure for the several case studies, a comparison of the maximum acceleration that can support the structure before failure ($a_{g\ max}$) was done. It represents a synthetic parameter that allows including at the same time the response of the structure - in terms of strength, stiffness and ductility- and the comparison with the seismic demand. The value of $a_{g\ max}$ has been obtained by imposing the target displacement of the structure (computed according to expressions proposed in Fajfar (2000) and adopted in Eurocode 8) with the ultimate displacement capacity of each configuration (assumed corresponding to 20% decay of the maximum base shear reached). Table 3.4 summarizes these results (for example in case of Soil C). Fig. 3.6 shows the comparison of different examined configurations in terms of $a_{g\ max}$ ratio between the SSI and fixed conditions.

Table 1.4. Summary of results in terms of data of equivalent SDOFs and **a**g max (in case of soil C)

					Soil Type C	
Configuration	Case:	T* [s]	Ay=Fy/Γ/m*[g]	du* [cm]	$a_{g\ max}$ [m/s²]	q*
Bldg with RCB	Fix_Vx_Pomaz	0.33	0.30	1.69	1.64	1.60
	SSI_G50_Vx_Pomaz_Rig	0.36	0.28	1.73	1.49	1.58
	SSI_G50_Vx_Pomaz_Flex	0.38	0.27	1.61	1.33	1.43
Bldg without RCB	Fix_Vx_Pomaz	0.91	0.10	6.29	1.51	3.15
	SSI_G50_Vx_Pomaz_Rig	1.05	0.09	6.64	1.45	2.56
	SSI_G50_Vx_Pomaz_Flex	1.17	0.09	7.14	1.40	2.29
Tower	Fix_Vx_Pomas	0.40	0.36	1.84	1.45	1.16
	SSI_G50_Vx_Pomas_Rig	0.65	0.27	3.28	1.15	1.13
	SSI_G50_Vx_Pomas_Flex	0.80	0.27	4.55	1.30	1.04
	Fix_Vy_Pomas	0.49	0.22	2.26	1.18	1.55
	SSI_G50_Vy-_Pomas_Rig	0.77	0.16	3.08	0.91	1.33
	SSI_G50_Vy-_Pomas_Flex	0.91	0.15	4.26	1.07	1.36
Where: T* : Period of the Equivalent SDOF; m* : mass of the Equivalent SDOF;du*: maximum displacement of the performance point for the Equivalent SDOF; q* : the ratio between the acceleration in the structure with unlimited elastic behavior Se (T*) and in the structure with limited strength Fy* / m*.						

It may be noted as differences in the initial period, strength and ductility affect in different way the seismic response. For example, in case of tower, it may be observed that going from the fixed case ("Fix") to the structure modeled with flexible foundations ("SSI-Flex"), the period of the structure has increased, while the acceleration demand spectrum decreased continuously; moreover, the capacity resisting force of the structure in case of structure modeled with rigid foundations ("SSI-Rig") becomes lower than that in the fixed case but almost equal to that in case of flexible foundations. The combination of all these factors imply that the case of rigid foundations approximation is the most critical (in fact: $a_{g\ max\_SSI\_Rig} < a_{g\ max\_\ SSI\_FLex} < a_{g\ max\_Fix}$). Analogous considerations shoud be discussed for other cases.

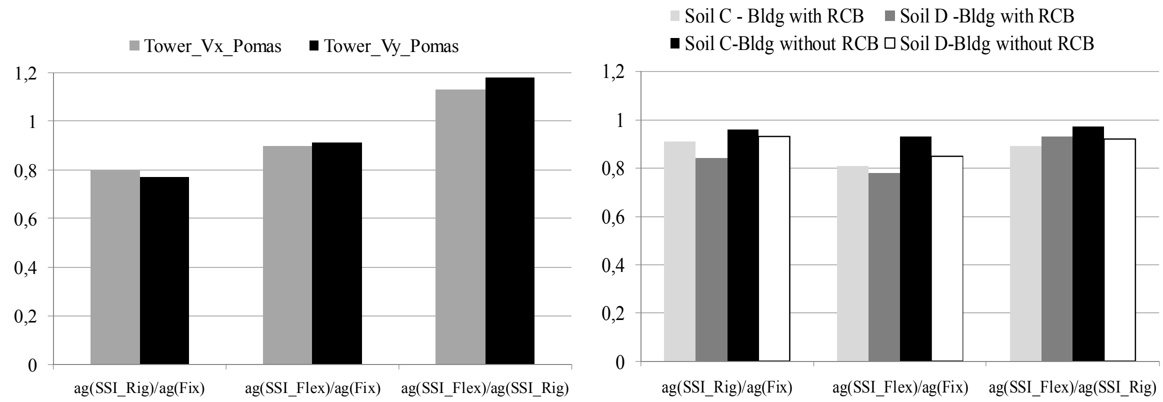


Figure 3.6. Comparison of different configurations in terms of $a_{g\ max}$ ratio between the SSI and fixed conditions: tower (left); ordinary building (right)

In general, the following conclusions can be drawn:

- The effects of SFSI are usually more significant in the case of slender structures (Tower case). In fact, the maximum PGA that the structure can support before failure is reduced to more than 20% in the case of Towers when the SFSI was considered in the analyses, while the reduction in the case of ordinary buildings may vary from 5 to 15%. But in any case, simplifying the analyses by neglecting the SFSI effects is not always on the safe side.
- The influence of flexible foundations, with respect to rigid ones, is more significant in the case of the Ordinary building. In that case, the foundation compliance reduced the acceleration demand of the structure by additionally 10%, with respect to the case of rigid foundation. On the contrary, in the case of the slender Tower, it may be sufficient to consider the approximation of rigid foundation since it leads to precautionary results.

4. FINAL REMARKS.

In the present study we propose dynamic stiffness values for flexible soil-foundation systems, for wall-to-soil modulus ratio E_w/E_{soil} up to 18. This range refers to monumental masonry structures. Dynamic stiffness for flexible foundations is provided for embedded, for horizontal, vertical and rotational vibration modes. The dynamic stiffness is a function of the foundation geometry and the wall-to-soil ratio of elastic moduli. It was shown that deviation from the average stiffness is larger for the rocking mode, as footing geometry is more important than it is for the translational modes. Finally, the values that result from numerical analyses (FEM) for rigid foundations were compared with the results from analytical solutions, to strengthen their validity. The comparison is found to be adequate, as their difference does not exceed ±15% (average value). Consequently, for larger values of E_w/E_{soil}, it seems appropriate to use the impedance functions for rigid footings proposed in literature.

Pushover analyses performed on two case studies examined – to be representative of an ordinary building and a tower – proved that the effects of SFSI are more significant in the case of slender structures. Comparisons, in terms of seismic assessment by the evaluation of the maximum PGA that the structure can support before failure, stressed out that neglecting SFSI effects is rather simplifying and not always on the safe side (with an higher vulnerability increase in the case of tower than ordinary building). Concerning the influence of SFSI and flexible foundations on system response, with respect to rigid ones, it seems to be more significant in the case of the ordinary building. On the contrary, in the case of the slender tower it may be sufficient to consider the approximation of rigid foundation since it leads to precautionary results.

Although these results cannot be considered exhaustive of the SFSI effects for all the wide variety of monumental masonry types, they reveal the significance of further developements on this topic and how the rough approximation to neglect them could lead in some cases to not conservative design.

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