Vibration Attenuation by Periodic Foundations

Z.B. Cheng, Z.F. Shi & H.J. Xiang
School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, China

15 WCEE
LISBOA 2012

SUMMARY:
Vibration attenuation to horizontal propagating waves by using periodic foundations is investigation. Periodic structure theory is introduced to design a new type of isolation foundations, which has a special frequency band gaps (or '*attenuation zones*') that can barrier waves/vibrations propagation. Materials usually used in civil engineering are used to fabric periodic foundations with the unit cell of 1~2m, whose attenuation zones are obtained by investigating the dispersion relations based on the improved plane wave expansion method. Then, numerical simulation is conducted to verify the efficiency of the periodic foundation. Results show that periodic foundations have a great potential in future application in seismic isolation.

Keywords: Structure isolation; Wave barrier; Periodic structure; Band gap; Attenuation zone

1. INTRODUCTION

Structural isolation is one of the most widely implemented and accepted seismic protection system (Skinner *et al.* 1993; Naeim and Kelly 1999). A considerable progress has been obtained in the last several decades on the base isolation both in technology and theory (Ramallo and Spencer Jr 2002; Kunde and Jangid 2003). Many research works have been conducted to extend base isolation technology to high-rise, even ultrahigh-rise buildings (Ariga *et al.* 2006; Fu *et al.* 2011), and to use base isolation technology for structures subjected to near-field (NF) ground motions (Jangid 2007). Especially, with the development of structure vibration control, some researchers proposed some methods to improve the performance and safety of base isolation structures by using semi-active base isolation systems (Lu and Lin 2009). Large amount of researchers concentrate their attention on improving the properties of isolators with fiber reinforced bearings (Moon *et al.* 2003), and on studying smart/intelligent isolation systems (Johnson *et al.* 1998; Ramallo and Spencer Jr 2002), as well as on investigating the seismic responses of base-isolated structures including the dynamic interaction between soil and structure (Spyrakos *et al.* 2009). However, owing to the tremendous cost of implementing base isolation technique especially for developing countries, applications can only be seen in structures with critical or expensive contents (Tsang 2008). So it is very necessary and meaningful to find new more technically efficient and cost-efficient isolation systems (Tsang 2008; Jia and Shi 2010).

Periodic structures are structures that consist of identical substructures, or cells, connected in an identical manner. Due to the periodicity, these structures have a striking feature, the so-called frequency band gaps (or '*attenuation zones*'), which exhibits unique dynamic characteristics that make them act as mechanical filters for wave propagation (Kushwaha *et al.* 1993; Baz 2001). The special property has many potential interesting applications, such as reducing engine noise in car cabins, controlling the noise in industrial halls, protecting electronic equipment on which sound waves can have a damaging effect and reducing traffic noise along roads. Recently, some researchers are exploring possible application of this new type of structure in engineering. Redondo *et al.* (2011) explored the potential of sonic crystals as efficient sound diffusers for applications in room acoustics. Based on the ideal of locally resonant sonic materials,Yang *et al.* (2010) developed a light-weight, relatively thin acoustic attenuation panel, which can barrier a broad frequency range of 50~1000Hz effectively. Specially, enlightened by the concept of frequency gap existence in phononic crystals, Jia and Shi (2010) proposed a novel method to isolate seismic waves by directly using a two dimensional periodic foundation. Here, *Periodic foundation* is a terminology word, which denotes a type of foundation composited by different materials in a periodic manner. After that, kinds of periodic foundation are proposed and the efficiency of these periodic foundations are examined (Cheng and Shi 2010; Shi *et al.* 2010; Xiang *et al.* 2010).

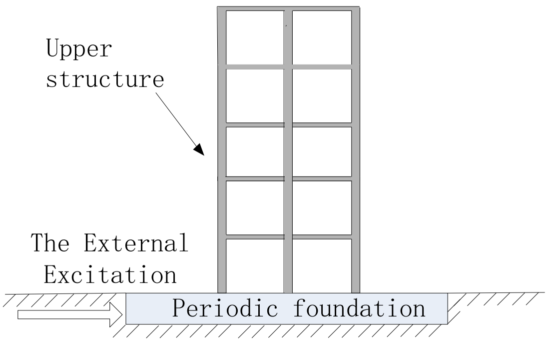


Figure 1. Schematic of a structure with periodic isolation foundation.

Focused on two-dimensional periodic foundation of three components, this paper aims to investigate the feasibility of a type of isolated foundation, illustrated in Fig. 1. Different from previous works, reinforced concrete (RC) is used. For simplicity, an equivalent orthogonal model is developed to study the orthogonal reinforcement of steel bars to plain concrete. Typical dispersion relations of periodic foundation are obtained by the improved plane wave method and the influence of steel bars on the dispersion curves is investigated. Then, taking a concrete frame structure for example, numerical analyses are conducted to show the efficiency of the new type of isolation foundation. This investigation shows that periodic foundations have a great potential in future application in seismic isolation.

Table 1. Material parameters.

Material	Elastic Modulus E (GPa)	Poisson's ratio ν	Density ρ (kg/m^3)
Rubber	1.37×10^{-4}	0.463	1300
Steel	209	0.275	7890
Concrete	40	0.2	2500

2. BASIC THEORIES

Construction materials are used to fabricate three-component periodic structures in Fig. 2. The material parameters are listed in Table1. In the present analysis, plane strain assumption in *x*-*y* plane will be considered.

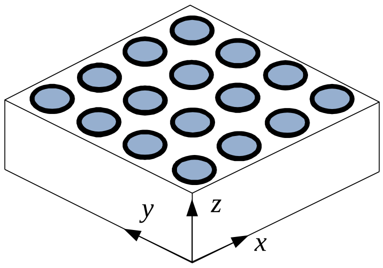


Figure 2. A periodic structure formed by coated cores.

2.1 Plain Concrete

Under the assumption of continuous, isotropic, perfectly elastic and small deformation as well as without consideration of material damping, the governing equation of waves in two dimensional periodic structures can be given as:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (c_{ijkl} u_{k,l})_{,j} \tag{2.1}$$

Where u_i is the displacement component, ρ the density, t the time parameter and c_{ijkl} the material parameter. According to *Bloch*'s theory, solutions of Eqn. 2.1 can be expressed as:

$$u_i(\vec{r},t) = e^{-i\omega t} \sum_{\vec{G}} u_i^{\vec{k}+\vec{G}}(\vec{G}) e^{i(\vec{k}+\vec{G}')\cdot\vec{r}} \tag{2.2}$$

in which $\vec{G}, \vec{G}'$ are the reciprocal vector, $\vec{k}$ the wave vector, ***r*** the coordinate vector, ω the angular frequency. Elastic parameters are also periodic functions, which can also be expanded as:

$$f(\vec{r}) = \sum_{\vec{G}} F(\vec{G}) e^{i\vec{G}\cdot\vec{r}} \tag{2.3}$$

Substituting Eqns. 2.2 and 2.3 into Eqn. 2.1, an eigen-value equation is obtained:

$$\omega^2 \mathbf{MU} = \mathbf{SU} \tag{2.4}$$

Owing to the high symmetry of the typical unit cell, the dispersion curve can be obtained by calculating the eigen-frequencies for wave vector varying just along the boundary of the so-called first irreducible *Brillouin* zone (Kushwaha *et al.* 1993). Once the dispersion curves are obtained, frequency band gaps can be found(Cheng and Shi 2010).

2.2 Reinforced Concrete

To improve the ductility of plain concrete, concrete is always reinforced in civil engineering. In the following analysis, the *x* and *y* direction of the Cartesian Coordinates are set along the two directions of reinforcement. To simplify the analysis, the reinforced concrete will be considered as a homogeneous orthotropic material and the material parameters will be determined according to the equivalent principle of composite mechanics. To do so, five undetermined parameters $E_x, E_y, v_{xy}, v_{yx}, G_{xy}$ need to be expressed by the homogeneous material parameters. Supposing that the steel bars are distributed uniformly in both directions and ignoring the enforcement effect from one direction to another, the elastic parameters E_x, E_y and the Poisson's ratio can be given as:

$$\begin{cases} E_x = (1-\gamma_x)E_m + \gamma_x E_f \\ E_y = (1-\gamma_y)E_m + \gamma_y E_f \end{cases}, \quad \begin{cases} v_{xy} = \gamma_x v_f + v_m(1-\gamma_x) \\ v_{yx} = \gamma_y v_f + v_m(1-\gamma_y) \end{cases} \tag{2.5}$$

In which γ_x and γ_y are the reinforcing ratios in the *x* and *y* directions, respectively, v_{xy} the ratio of strain in *y* direction to the strain in *x* direction with the external tensor force applied in the *x* direction.

On the other hand, Poisson's ratios should also satisfy the Maxwell equation:

$$E_x v_{xy} = v_{yx} E_y \tag{2.6}$$

It is obvious that Eqns. 2.5 and 2.6 cannot be satisfied simultaneously. Considering that Eqn. 2.6 should be satisfied for all homogeneous material, we can add any one of the later of Eqn. 2.5 to determine the Poisson's ratios, which will not result in a notable difference for the Poisson's ratios.

The shear module G_{xy} in *x*-*y* plane can be presented by:

$$G_{xy} = \frac{G_f G_m}{G_f \gamma_f + G_m(1-\gamma_f)}; \quad \gamma_f = \min(\gamma_x, \gamma_y) \tag{2.7}$$

In addition, the equivalent mass density is another important parameter, which affects the dispersion relationship intensively. The equivalent mass density ρ' can be given as:

$$\rho' = \rho_m(1-\gamma_x-\gamma_y) + \rho_f(\gamma_x+\gamma_y) \tag{2.8}$$

ρ_m and ρ_f denote the density of matrix material (concrete) and fiber material (steel), respectively.

3. ATTENUATION ZONES

3.1 Attenuation zones for periodic structures using plain concrete

In the present study, the matrix and coating is always made of concrete and rubber, respectively. When the core is made of concrete too, a periodic structure (Type-CRC) is obtained. When the core is made of steel, another periodic structure (Type-CRS) is obtained. Typical dispersion curve for Type-CRC is shown in Fig. 3 when *R*=0.7m, *d*=0.2m and *A*=2.0m, where *A* is the periodic parameter (or the size of a typical cell), *R* the radius of the core and *d* the thickness of the coating. One can find that there is no eigen-value in the region (7.22, 10.10)Hz, the so-called first attenuation zone (AZ-1). That means that waves in this frequency region cannot propagate in the periodic structure. Besides the first attenuation zone, it is obvious that there are another two attenuation zones below 20Hz.

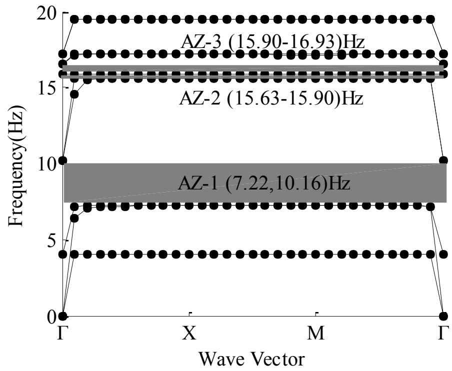
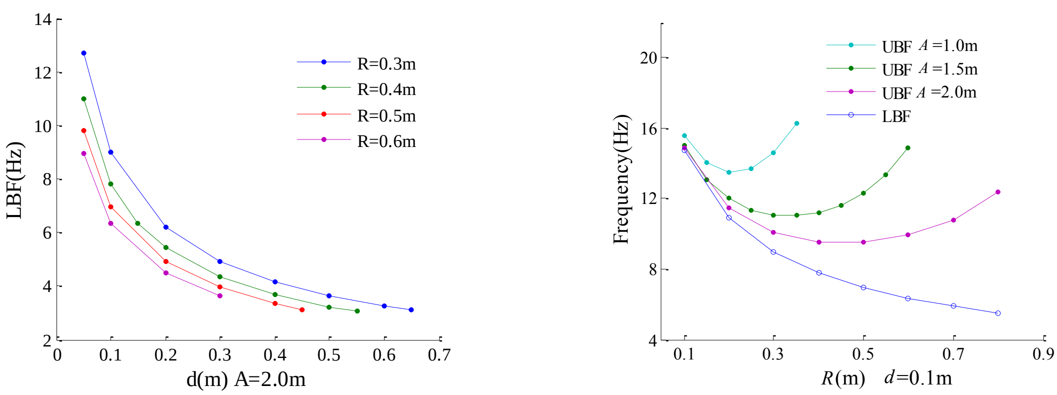


Figure 3. Dispersion curve.

Taking Type CRS for example, the influences of geometric parameters *R* and *d* on the AZs are investigated and is given in Fig. 4. It is found that the Low Bound Frequency (LBF) decreases with the increase of coating thickness for a given *A* and *R*. For a given *R* and *d*, the Upper Bound Frequency (UBF) decreases with the increase of periodic parameter *A*.



(**a**) LBF versus the geometric parameters. (**b**) UBF versus the geometric parameters.

Figure 4. Influences of geometric parameters on the boundary frequencies.

3.2 Effects of the steel reinforcement on the attenuation zones

Reinforcing ratio is governing parameter in calculation. In engineering design, the reinforcing ratio depends on the type of structure members and external loadings. For example, for the thick plate lying on ground, the reinforcing ratio should be in the region 0.15% to 5% according to the structural concrete code (ACI-Committee-318 2008). In order to take the extreme orthotropic into account, the reinforcing ratio in the x and y directions will be assumed as 0.15% and 5% in the present analysis, respectively. The equivalent material parameters for reinforced concrete are listed in Table 2.

Two models will be considered. Model-1means that the matrix concrete is reinforced but the core is plain concrete. Model-2 means that both matrix and core concrete are reinforced. The unit cell is selected as A=2.0m, R=0.7m and d=0.2m for comparison. The first three attenuation zones for both models are given in Table 3. It is found that the effect of reinforcement on the attenuation zones can be ignored, which is very convenient for engineering applications.

Table 2. Material parameters for the equivalent orthotropic material.

reinforcing ratio	E_x (GPa)	E_y (GPa)	v_{xy}	v_{yx}	G_{xy} (GPa)	ρ (kg/m^3)
γ_x = 0.15% γ_y = 5%	40.2535	48.45	0.2038	0.1693	16.69	2778

It can be concluded that the dispersion curves of reinforced concreted periodic structures can be approximated by those of plain concrete periodic structures with the equivalent density and the elastic parameters. In other word, the reinforcement of steel bars can strengthen the ductility and crack resistance ability for the fragile plain concrete, and has little influences on attenuation zones, which is very beneficial for engineering application.

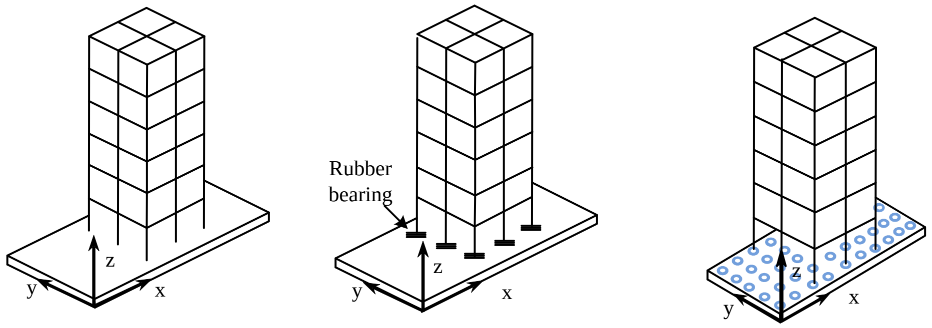
Table 3. Comparison of attenuation zones between two cases.

Models	AZ-1(Hz)			AZ-2(Hz)			AZ-3(Hz)		
	LBF	UBF	WAZ	LBF	UBF	WAZ	LBF	UBF	WAZ
1	7.22	9.96	2.74	15.63	15.90	0.27	15.90	16.42	0.52
2	6.87	9.70	2.82	15.63	15.82	0.19	15.82	16.42	0.60

4. NUMERICAL SIMULATIONS

To verify the effect of this new isolation method, both the harmonic and transient responses of a six-story frame structure with different foundations are studied by using the ANSYS finite element software. Three different foundations are considered for comparison purposes: the first is a concrete foundation, the second is a foundation with traditional isolation rubber bearings between the upper structure and the foundation, the third is a periodic foundation. Fig. 5 illustrates the frame structure with different foundations.

The dimension of the upper structure is 8m×8m×3.3m, the cross section of columns and beams are 0.5m × 0.5m and 0.5m × 0.3m, respectively. The thickness of floor is 0.1m. The second characteristic frequency of the upper structure is 7.32Hz.



(**a**) No isolation. (**b**) Traditional base isolation. (**c**) Periodic foundation isolation.

Figure 5. Schematic of a frame structure with different foundations.

For the traditional isolation foundation, the laminated rubber bearing GZP-500V4A is adopted in the present simulation. The bearings are added at the bottom of each column and nine bearings are used. The height and horizontal stiffness of each GZP-500V4A is 194mm and 0.91KN/mm (Dang *et al*. 2007), respectively. The third characteristic frequency of the structure with isolation bearings is 10.10Hz.

The unit cell of the periodic foundation is chosen as A=2.0m, R=0.7m and d=0.2m, whose AZ-1 (7.22-10.16)Hz covers the second or third fundamental frequency of the structure with no isolation or the structure with isolation bearings, respectively. The size of the periodic foundation is 10m×22m×0.5m.

4.1 Harmonic responses

A horizontal dynamic displacement with unit amplitude and a single frequency is inputted on the nodes at the surface x=0m. The boundary conditions are considered as: all the displacements in the z direction for every node at z=0m and the displacements in both y- and z-directions for every node at x=0m are fixed.

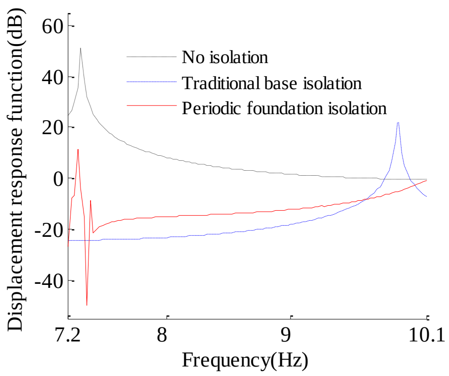


Figure 6. Displacement response function versus the incident frequency in AZ-1.

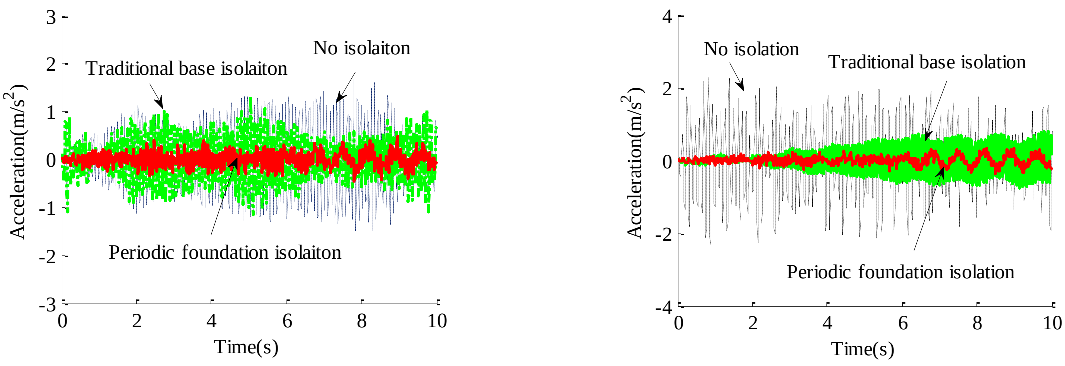
Fig. 6 gives the maximum steady-state displacement response ($20\log(u/u_0)$) in x-direction, in which u is the amplitude of displacement response at the roof of the structure and u_0 is the amplitude of displacement inputted at the foundations. Due to the horizontal incident vibration is attenuated dramatically by the periodic isolation foundation, the displacement response at the roof of the structure is reduced seriously especially in the AZ-1 region.

4.2 Transient responses

In order to study the transient response of the structure with different foundations, an acceleration time history process will be considered. The acceleration process is a superposition of several harmonic acceleration loads whose frequencies fall in AZ-1 and can be expressed as follows:

$$a(t) = \frac{1}{\gamma_{amp}} \sum_{m=1}^{M} \gamma_o^m \cos(\varphi_o^m + 2\pi f_m t) \tag{4.1}$$

In which φ_o^m is the initial phase of the component with frequency $f_m \in$ AZ-1, γ_o^m the weight coefficient for f_m, γ_{amp} a constant used to normalize the acceleration amplitude and M the total number of components of the acceleration process. In the following analysis, we will take $f_m = 7.2 + 0.1(m-1)$, $m \in (1, 28)$, $t \in (0s, 10s)$.



(**a**) At the top surface of the foundation. (**b**) At the second floor.

Figure 7. Acceleration responses of a structure with different foundations

The time history of acceleration $a(t)$ is added in the x-direction at every node at x=0m by Big-Mass Method. The boundary constraints are the same as given in the horizontal analysis. The acceleration responses at the top surface of the foundation and at the second floor of the upper structure are shown in Fig. 7. It is easily found that transient acceleration response of the structure with a periodic foundation is much lower than those with other two types of foundations.

In addition, the maximum relative displacement response of the upper structure is given in Fig. 8. It is observed that the maximum relative displacements of the structure without isolation system or with traditional isolation system are much larger than that of a structure with a periodic foundation.

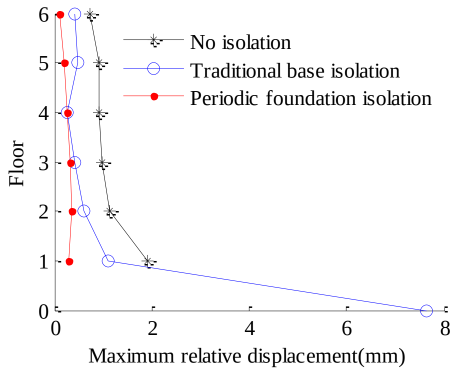


Figure 8. Relative displacement responses of a structure with different foundations.

5. CONCLUSIONS

The dispersion curve of a three-component periodic structure is studied and the attenuation zones are obtained. The influences of the geometrical parameters on the attenuation zones are discussed. The possibility of the periodic foundation to seismic isolation is numerical simulated and verified. Some conclusions can be drawn:

1. Periodic foundations made of materials usually used in civil engineering can enjoy the attenuation zones below 20Hz.

2. Effect of reinforcement on the attenuation zones can be ignored. Attenuation zones of periodic foundations with reinforced matrix can be approximated by using the equivalent mass density and the elastic parameters of plain concrete.

3. Both harmonic analysis and transient analysis show that the response of the structure with periodic foundation can be dramatically attenuated, especially when the incident frequency falls in the attenuation zones.

ACKNOWLEDGEMENT

This work is supported by the National Natural Science Foundation of China (51178036) and the Fundamental Research Funds for the Central Universities (2011YJS258).

REFERENCES

ACI-Committee-318 (2008). Building code requirements for structural concrete(ACI 318-08) and commentary, American Concrete Institute, American Concrete Institute.

Ariga, T., Kanno, Y., et al. (2006). Resonant behaviour of base-isolated high-rise buildings under long-period ground motions. *The Structural Design of Tall and Special Buildings*. **15**:**3**, 325-338.

Baz, A. (2001). Active control of periodic structures. *Journal of Vibration and Acoustic*. **123**, 472-479.

Cheng, Z. and Shi, Z. (2010). Application of Periodic Structures: A New Kind of 2D-Periodic Isolation Foundation. *The 8th National Conference of Earthquake Engineering*. Chongqing, China.

Dang, Y., Du, Y.F., et al. (2007). Base isolated structure design and construction guidelines, China Water Power Press/Intellectual Property Publishing House, Beijing,China.

Fu, W.Q., Chen, F., et al. (2011). Simplified Model of High-Rise Isolated Structure for Earthquake Response Analysis. *Advanced Materials Research*. **250**, 2007-2010.

Jangid, R. (2007). Optimum lead-rubber isolation bearings for near-fault motions. *Engineering Structures*. **29**:**10**, 2503-2513.

Jia, G. and Shi, Z. (2010). A new seismic isolation system and its feasibility study. *Earthquake Engineering and Engineering Vibration*. **9**:**1**, 75-82.

Johnson, E.A., Ramallo, J.C., et al. (1998). Intelligent base isolation systems. *Second World Conference on Structural Control*. Kyoto, Japan.

Kunde, M. and Jangid, R. (2003). Seismic behavior of isolated bridges: A-state-of-the-art review. *Electronic Journal of Structural Engineering*. **3**:**2**, 140-169.

Kushwaha, M.S., Halevi, P., et al. (1993). Acoustic band structure of periodic elastic composites. *Physical Review Letters*. **71**:**13**, 2022-2025.

Lu, L.Y. and Lin, G.L. (2009). Improvement of near-fault seismic isolation using a resettable variable stiffness damper. *Engineering Structures*. **31**:**9**, 2097-2114.

Moon, B., Kang, G., et al. (2003). Mechanical properties of seismic isolation system with fiber-reinforced bearing of strip type. *International applied mechanics*. **39**:**10**, 1231-1239.

Naeim, F. and Kelly, J.M. (1999). Design of seismic isolated structures: from theory to practice, Wiley.

Ramallo, J. and Spencer Jr, B. (2002). “Smart” Base Isolation Systems. *Journal of Engineering Mechanics*. **128**, 1088.

Redondo, J., Sánchez-Morcillo, V., et al. (2011). The Potential for Phononic Sound Diffusers (PSD). *Building Acoustics*. **18**:**1**, 37-46.

Shi, Z., Cheng, Z., et al. (2010). A New Seismic Isolation Method by Using a Periodic Foundation. *Earth and Space 2010: Engineering, Science, Construction, and Operations in Challenging Environments*. Honolulu, Hawaii, USA.

Skinner, R.I., Robinson, W.H., et al. (1993). An introduction to seismic isolation, Wiley, Chichester ; New York.

Spyrakos, C., Koutromanos, I., et al. (2009). Seismic response of base-isolated buildings including soil-structure interaction. *Soil Dynamics and Earthquake Engineering*. **29**:**4**, 658-668.

Tsang, H.H. (2008). Seismic isolation by rubber–soil mixtures for developing countries. *Earthquake engineering & structural dynamics*. **37**:**2**, 283-303.

Xiang, H., Shi, Z., et al. (2010). Seismic Isolation of Buildings with a New Type of Periodic Foundations. *Earth and Space 2010: Engineering, Science, Construction, and Operations in Challenging Environments*. Honolulu, Hawaii, USA.

Yang, Z., Dai, H., et al. (2010). Acoustic metamaterial panels for sound attenuation in the 50–1000 Hz regime. *Applied Physics Letters*. **96**, 041906.