A macro-element approach for modeling the nonlinear behaviour of monumental buildings under static and seismic loadings

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SUMMARY
The seismic assessment and subsequent rehabilitation of historical masonry structures constitutes a contemporary issue in most seismic regions and particularly in Europe in which the monumental buildings represent a huge cultural heritage that must be safeguarded. The different behaviour of masonry structures, compared to ordinary concrete and steel buildings, requires ad hoc algorithms capable of reproducing the nonlinear behaviour of masonry media and providing reliable numerical simulations. Refined finite element numerical models, such as the smeared cracked and discrete crack finite element models, able to predict the complex non-linear dynamic mechanical behaviour and the degradation of the masonry media, require sophisticated constitutive laws and a huge computational cost. As a consequence these methods are nowadays not suitable for practical application and extremely difficult to apply to large structures. In the past many authors developed simplified or alternative methodologies that, with a reduced computational effort, should be able to provide numerical results that can be considered sufficiently accurate for engineering practice purposes. However most of these methods are based on simplified hypotheses that make these approaches inappropriate for monumental buildings. In this paper a new modelling approach for the simulation of the seismic behaviour of monumental masonry buildings is presented. In particular a three dimensional discrete element model, able to predict the nonlinear behaviour of masonry shell elements, is presented as an extension of a previously introduced plane discrete-element conceived for the simulation of the in-plane behaviour of masonry plane elements. The proposed approach is based on the concept of macro-element discretization and has been conceived with the aim of capturing the nonlinear behaviour of a macro-portion of a masonry element and of the entire building, as an assemblage of several elements. The new macro-element enriches a larger computational framework, based on macro-element approach, devoted to the numerical simulation of the seismic behaviour of historical masonry structures.

Keywords: macro-element, discrete element, nonlinear response of masonry buidlings, monumental structures.

1. INTRODUCTION

The simulation of the nonlinear behaviour of a monumental masonry building under static and seismic loadings represents a challenging problem which rigorously requires the use of computationally expensive nonlinear finite element models and, above all, expert judgment. For this reason, the assessment and subsequent rehabilitation of historical masonry structures constitutes a contemporary issue in most regions and particularly in Europe in which the monumental buildings represent a huge cultural heritage that must be safeguarded. The different behaviour of masonry structures, compared to ordinary concrete and steel buildings, requires ad hoc algorithms capable of reproducing the nonlinear behaviour of masonry media and providing reliable numerical simulations. Refined finite element numerical models, such as the smeared cracked and discrete crack finite element models, able to predict the complex non-linear dynamic mechanical behaviour and the degradation of the masonry media, require sophisticated constitutive laws and a huge computational cost. As a consequence these methods are nowadays rarely used for practical application and extremely difficult to apply to large structures. In the past many authors developed simplified or alternative methodologies that, with a reduced computational effort, should be able to provide numerical results that can be considered sufficiently accurate for engineering practice purposes (Braga et al 1998; Chen et al. 2008; D'Asdia et al. 1995; Kappos et al. 2002; Magenes et al. 1998; Marques et al. 2011). However most of these

methods are based on simplified hypotheses that make these approaches inappropriate for monumental buildings.
In this paper an original modelling approach for the simulation of the nonlinear behaviour of masonry buildings under static and seismic loadings is considered. The proposed approach is based on the concept of macro-element discretization (Marques et al. 2011) and has been conceived with the aim of capturing the nonlinear behaviour of a macro-portion of a masonry element and of an entire building, by means of an assemblage of several discrete-elements that can be characterised by different level of complexity according to the role of the element in the global model.
The basic element, developed for the simulation of the in-plane response, is constituted by an articulated quadrilateral with four rigid edges and four hinged vertices connected by two diagonal nonlinear springs; each of the rigid edges can be connected to other elements by means of discrete distributions of nonlinear springs with limited tension strength. This plane discrete-element has been applied for the simulation of the nonlinear behaviour of several masonry buildings, a detailed description of the plane macro-element is reported in reference (Caliò et al. 2012) where a validation with experimental results is also reported. The basic plane-element is intended for the simulation of the response of masonry walls in their own plane since the out-of-plane response is not taken into account. In order to overcome this significant restriction, that is common to many simplified approaches, the plane macro-element has been upgraded by introducing a third dimension and the needed additional degrees-of-freedom, with the aim to describe the out-of-plane kinematics of masonry walls (Pantò 2007, Caliò et al. 2008,). This enrichment of the element obviously introduces an increased computational effort, due to a larger number of degrees-of-freedom and to the introduction of additional non-linear links needed for describing the out-of-plane mechanical behaviour of the element. On the other hand, the increased complexity of the model is balanced by the fact that both the in-plane and out-of-plane mechanisms are incorporated in a unique macro-element. Nevertheless, in order to simulate the seismic response o monumental masonry buildings, in many cases it is also necessary to model the nonlinear behaviour of structural elements with a curved geometry, such as arches, vaults, domes whose role is fundamental both in the local and global behaviour of the buildings. With the aim to simulate the nonlinear behaviour of masonry structures with curved geometry the macro-element has been further upgraded (Cannizzaro 2011). This three dimensional discrete element model, able to predict the nonlinear behaviour of masonry shell elements, is presented as an extension of a previously introduced spatial discrete-element conceived for the simulation of both the in-plane and the out-of-plane behaviour of masonry plane elements. The computational cost of the proposed numerical approach is greatly reduced, compared to a traditional nonlinear finite element modeling. Since the equivalence between the masonry portion and the macro-element is based on a fiber discretization and the calibration of the model and the interpretation of the numerical results are simple and straightforward. The macro-elements introduced so far, enriches a larger computational framework, based on macro-element approach, devoted to the numerical simulation of the seismic behaviour of historical masonry structures.

2. THE PLANE ELEMENT

The basic element of the proposed approach has a simple mechanical scheme, Fig. 1. It is represented by an articulated quadrilateral constituted by four rigid edges connected by four hinges and two diagonal nonlinear springs. Each side of the panel can interact with other panels or elements or supports by means of a discrete distribution of nonlinear springs, denoted as interface. Each interface is constituted by n nonlinear springs, orthogonal to the panel side, and an additional longitudinal spring which controls the relative motion in the direction of the panel edge. In spite of its great simplicity, such a basic mechanical scheme is able to simulate the main in-plane failures of a portion of masonry wall subjected to horizontal and vertical loads, further details can be found in (Caliò et al. 2012).

2.1 The kinematics

According to the proposed discrete element approach, a masonry macro-portion is assimilated to an equivalent mechanical scheme in which the physical role of each component is simple and unambiguous.

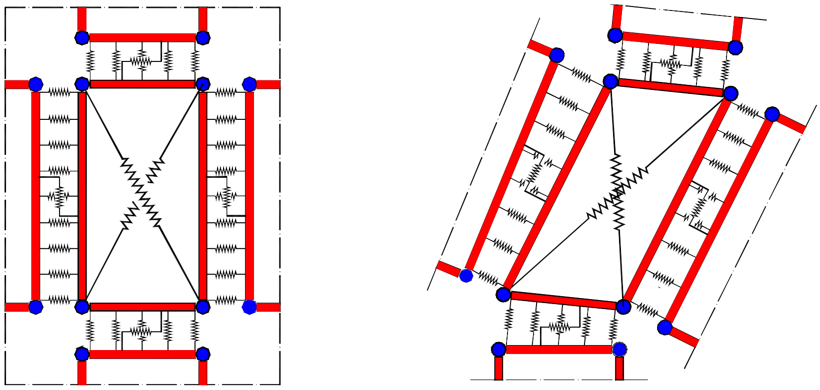


Figure 1. The basic macro-element: (a) undeformed configuration; (b) deformed configuration.

The degrees-of-freedom of the structural scheme are those associated to the in-plane motion of the basic elements. Each element exhibits three degrees-of-freedom, associated to the in-plane rigid-body motion, plus a further degree-of-freedom needed for the description of the shear deformability. The deformations of the interfaces are associated to the relative motion between corresponding panels, therefore no further Lagrangian parameters must be introduced in order to describe their kinematics.

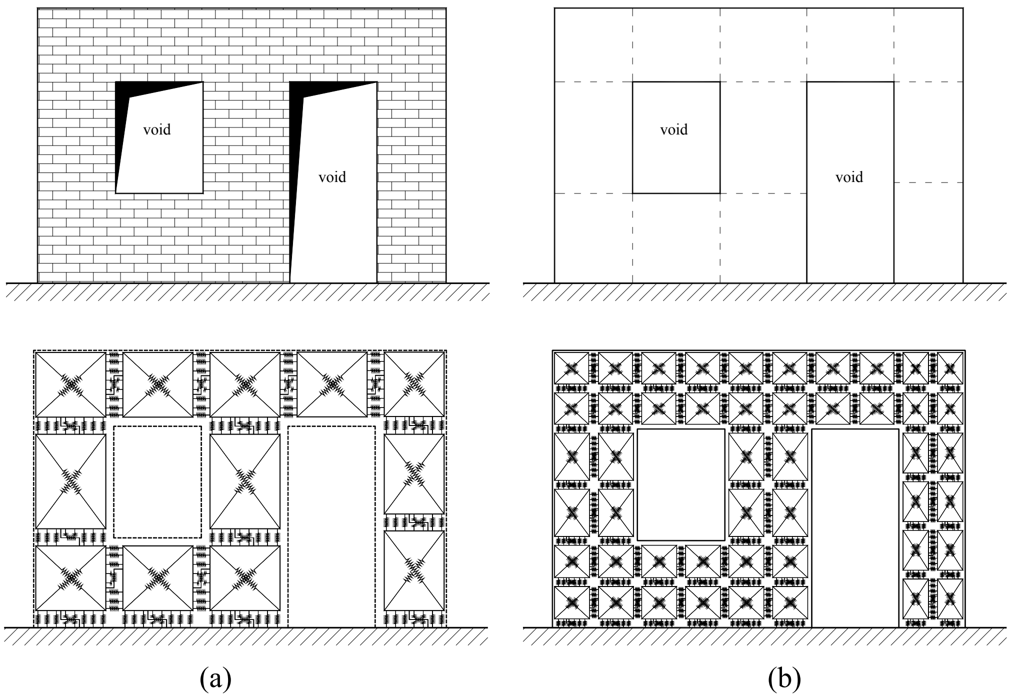


Figure 2: Masonry wall and corresponding macro-element discretization with different mesh resolutions.

Since the proposed modelling approach is suitable for describing the behaviour of a plane wall loaded in its own plane, a three-dimensional masonry building can be modelled as an assemblage of plane walls. Therefore, in this simplified approach the behaviour of the wall in the out-of-plane direction is not considered. In Fig. 2 it is shown how a simple masonry wall can be modelled by means of the proposed modelling approach. Namely, Figure 5a refers to a basic scheme composed by 12 panels and is characterised by 48 degrees of freedom, while in Figure 5b the same wall is modelled through a more refined mesh composed by 48 quadrilaterals which require 192 degrees of freedoms. The use of a more refined mesh is not mandatory however in some cases can provide more accurate results and a better description of the collapse mechanism.

2.2 Calibration of the nonlinear springs

The effectiveness of the simulation of the nonlinear behaviour relies on a suitable choice of the mechanical parameters of the model inferred by an equivalence between the masonry wall and a reference continuous model characterised by simple but reliable constitutive laws. This equivalence relies on a straightforward fiber calibration procedure, and is based only on the main mechanical parameters which characterise the masonry according to an orthotropic homogeneous medium (Caliò et al. 2012). It is worth to notice that in the proposed approach each macro-element inherits the plane geometrical properties of the modelled masonry portion, as a consequence, contrary to the simplified models based on equivalent frame element approach, there is no need to define an effective dimension of the structural element.

3. THE SPATIAL ELEMENT

The three-dimensional macro-element represents the natural upgrade of the plane macro-element described in the previous paragraph. In particular three additional degrees-of-freedom have been considered for the description of the out-of-plane kinematics and further nonlinear springs have been introduced in the interfaces in order to account for the three-dimensional mechanical behavior, Fig. 3.

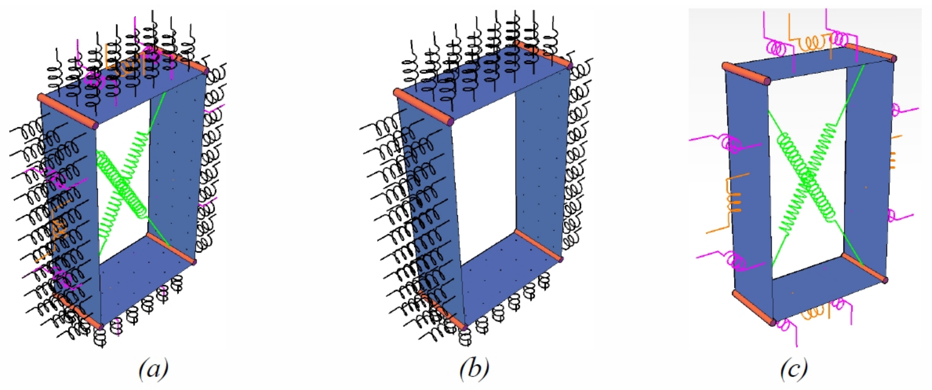


Figure 3. (a) The spatial macro-element: (b) the element with the representation of the orthogonal *NLinks* for the simulation flexural behaviour; (c) the element with the representation of the transversal and diagonal *NLinks* for the simulation of shear and torsional behaviour.

The kinematics of the spatial macro-element is therefore governed by 7 degrees-of-freedom able to describe both the rigid body motions and the in-plane shear deformability. The 3D-interface possesses m rows of n longitudinal (i.e. perpendicular to the planes of the interface) NLinks. Therefore each of the planes of the interface is subdivided, similarly to what is done in classical fibre models, in $m \times n$ sub-areas (Fig. 3b). Each longitudinal spring of the 3D interface represents a strip of masonry with base area equal to that of the sub-area which pertains to it. The number of rows m and the number of springs in each row n must be selected according to the desired level of accuracy of the nonlinear response. The 3D-interface possesses also further sliding springs. These are required to control the relative displacement of the panels perpendicular to their plane. To this purpose, two out-of-plane sliding NLinks, perpendicular to the plane of the panel and contained in the plane of the interface, have been provided (Fig. 3c). These NLinks control the out-of-plane sliding mechanisms of the panels and the torsion of the panels about the axis perpendicular to the plane of the interface.

4. A MACRO-ELEMENT FOR SHELL MASONRY STRUCTURES

The discrete element conceived to model shell masonry elements represents an extension of the spatial element, described in the previous paragraph, and its nucleus is constituted by an irregular articulated quadrilateral. Each macro-element, Fig. 4 is characterised by four rigid layer edges whose orientation and dimension is associated to the shape of the element and to the thickness of the portion of modelled masonry that must be represented, Fig 4.

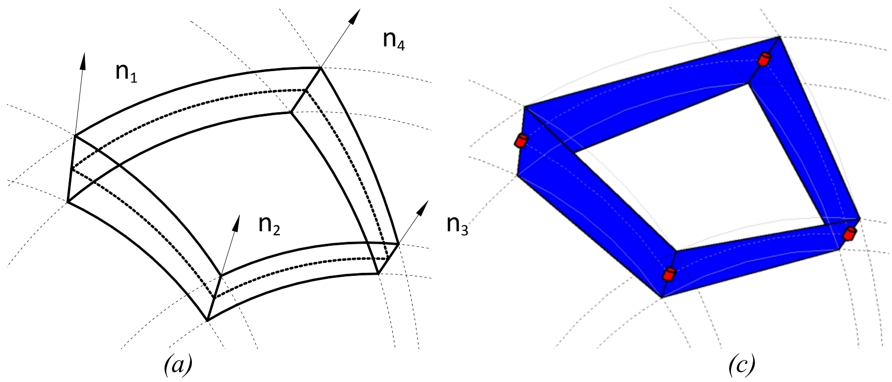


Figure 4 – Masonry portion and its equivalent mechanical model without the NLink representations.

The macro-element for shell structures constitutes an upgrade of the spatial element previously described, Fig 5. A diagonal spring simulates the shear deformation of the quadrilateral in its own medium plane (Fig. 5c), while spatial interfaces (Fig. 5b) govern the interaction with the adjacent elements or the external supports. These interfaces are in general skew relative to the medium plane of the element, and their movements are ruled by a discrete number of non linear springs. Each quadrilateral is defined by the geometric coordinates of his vertices, the four normal vectors to the surface and the thickness in these points (Fig. 4a).

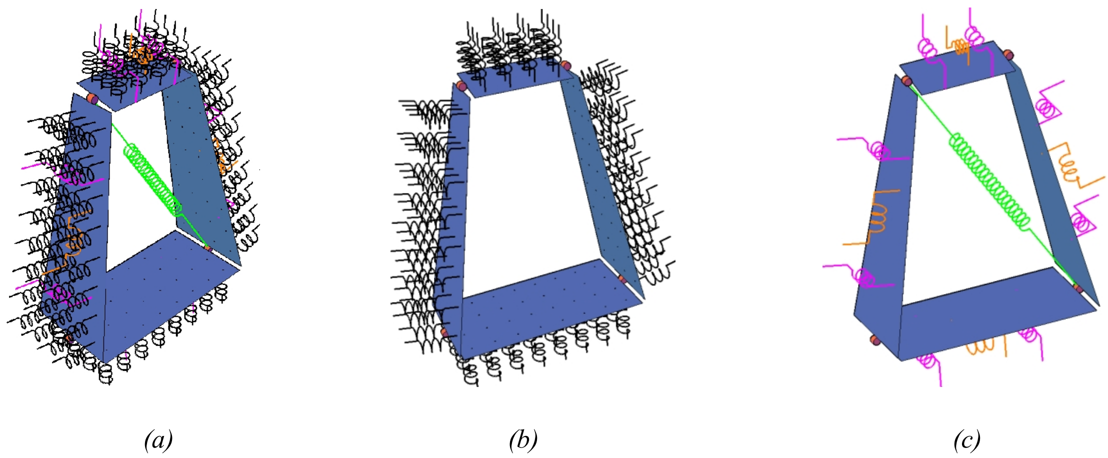
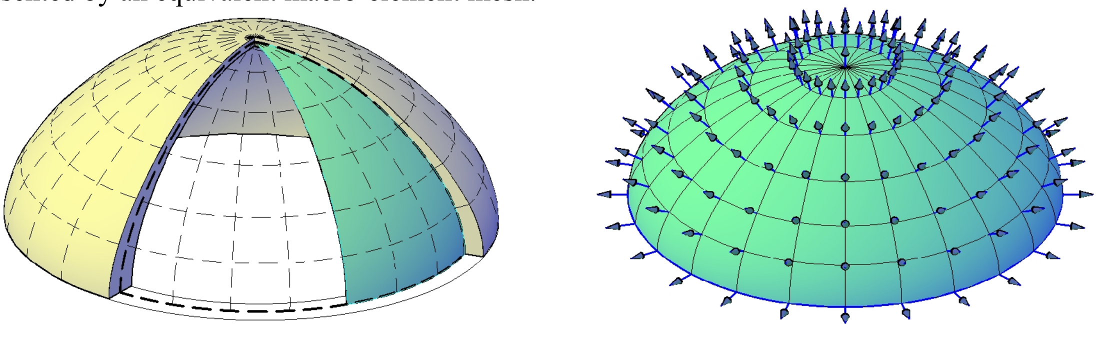


Figure 5 – (a) the quadrilater macro-element 'quad'. (b) the quad with the representation of the orthogonal NLinks for the simulation of membrane and flexural behaviour; (c) the quad with the representation of the transversal and diagonal NLinks for the simulation of shear and torsional behaviour

The kinematics of the spatial macro-element is governed by 7 degrees-of-freedom able to describe both the rigid body motions and the in-plane shear deformability. Each 3D-interface possesses *m* rows of *n* NLinks orthogonal to the interface (Fig. 4b). The number of rows *m* and the number of springs in each row *n* must be selected according to the desired level of accuracy of the nonlinear response. The 3D-interface possesses further sliding springs (Fig. 4c). These are required to control the relative displacement of the panels perpendicular to their plane and the sliding behaviour in its own plane.

4.1 From the physical model to its macro-portion representation

The first necessary operation is the transfer from the geometrical model to a mesh of masonry macro-portions. In Fig. 6, it is shown how a a dome can subdivided in macro-portion, in order to be represented by an equivalent macro-element mesh.



(a) (b)

Figure 6. An example of subdivision of a dome by means of macro-portion.

The average size of each of these macro-portions is, by hypothesis, contained in a plane. In the more general case, this representation is always possible by means of triangular flat elements. However, generally, the geometric regularity of the surfaces of the structural elements involved allows a geometric representation of the physical model by means a mesh of quadrilateral elements and a limited number of triangular elements. In figure 6 a macro-element representation of a dome is reported.

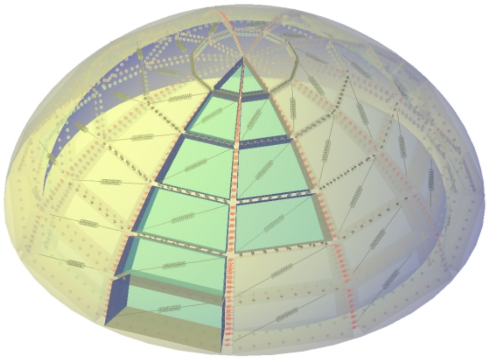


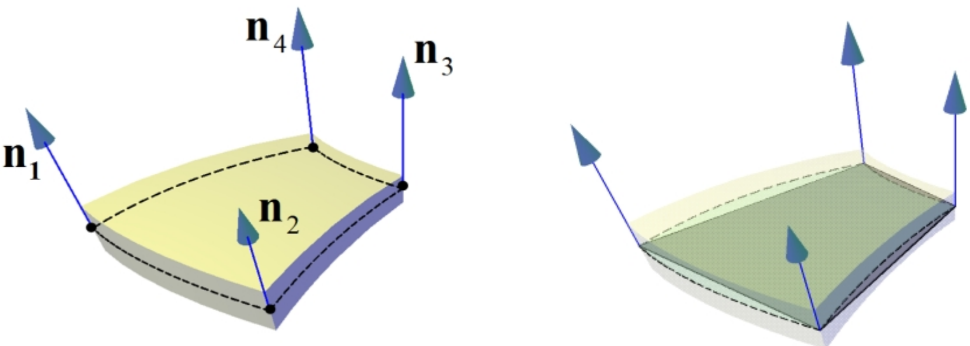
Figure 7. discrete-element meshing of a dome.

4.2 From the macro-portion representation to a macro-element discretization

The macro-element must be able of simulating the mechanical behaviour typical of masonry structures with curved geometry subjected to static and dynamic loadings. Although complex, the nonlinear response of this structures is highly governed by the specific geometric configuration and the load conditions. However, if one refers to a portion of an element of vaults, it is possible to identify typical criteria of local crisis, which together determine the crisis of a masonry vaulted structure. The basic idea is to conceive an equivalent mechanical model able to simulate the nonlinear behaviour of the masonry portion by means of an approach in which the membrane and the flexural response of the masonry element are governed by a fibre discretization, along the main directions of the element, while the shear and torsional responses are simulated by a discrete number of nonlinear links.

4.3 Calibration of the Nonlinear Links

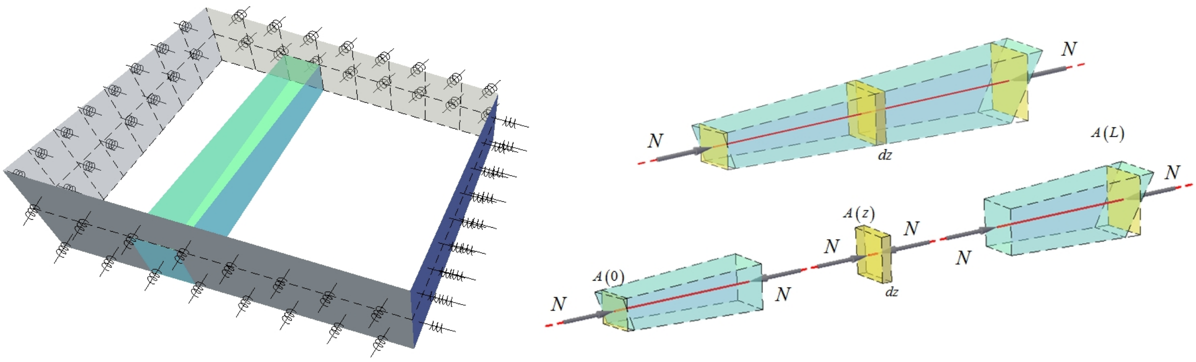
The calibration procedure is based on the assumption that the behaviour of a continuously curved surface can be adequately represented by flat elements, Fig. 8. In the sub-division of an arbitrary shell into flat elements generally both triangular and quadrilateral elements should be used. For the sake of conciseness, here only the quadrilateral shape element is described in more details. Each element must be representative of the corresponding finite portion of the shell cut out by plane sections which are located to the edges of the irregular quadrilateral and whose orientations and thicknesses are associated to the actual represented shell.



(a) (b)

Figure 1. (a) Quadrilateral portion of masonry structures; (b) Flat element representation.

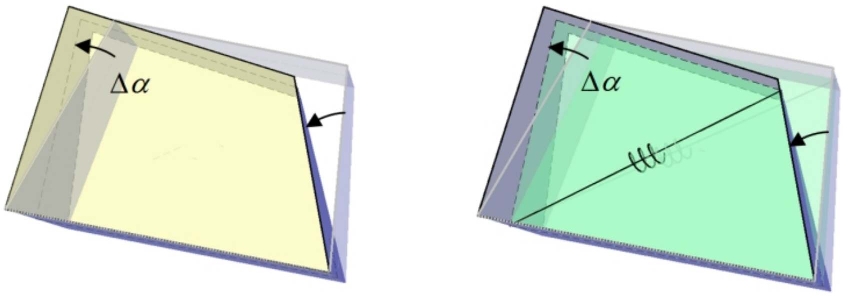
The membrane and the bending behaviour of the element is governed by the nonlinear links orthogonal to the rigid layer edges, while three additional transversal links control both the in-plane and out-of plane sliding shear and the twisting of two adjacent layers. A single diagonal nonlinear link governs the in-plane nonlinear shear behaviour of the quadrilateral element. The orthogonal NLinks that govern the membrane and flexural response are calibrated by means of a fibre modelling approach. The calibration procedures of each nonlinear link are based on the mechanical characterization of masonry and the geometric properties of the elements. Masonry, considered as a continuous homogeneous solid, will be modelled considering different constitutive laws for each fundamental behaviour: membrane, bending, shear, sliding, and torque.



(a) (b)

Figure 9. Representation of the generic fiber corresponding to two Nlinks.

The discrete model has been thought as a fibre model, and each element of the mechanical scheme simulates the behavior of a corresponding fibre. Each orthogonal link is calibrated in order to be equivalent to the corresponding portion of masonry that represents, according to pertinent volume Fig. 9a. Similar to the procedure adopted for the plane element (Caliò et al. 2012), the calibration procedure consists of two phases. In the first one the mechanical properties of two links, which correspond to the fibers of each of the two elements connected by the interface, are calculated. In the second phase the two links in series will be condensed in a single equivalent nonlinear link. Each of the two links can be calibrated considering a nonlinear elastic-plastic beam with a variable section (Fig. 9b). The transversal links govern the in-plane and the out-of-plane sliding of the shell element according to a Mohr-Coulomb law. In the elastic range the diagonal shear link is calibrated by imposing an energy equivalence with a continuous reference elastic model, Fig.10.



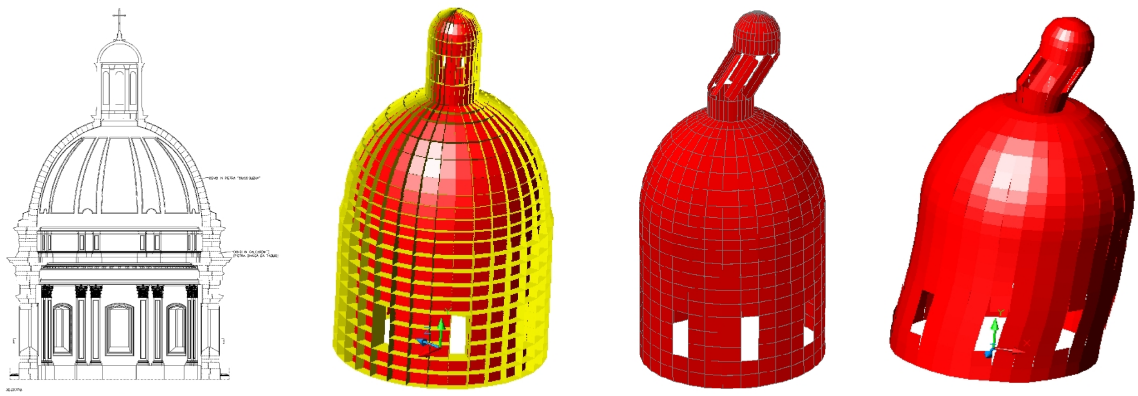
(a) (b)

Figure 2. Equivalence between the continuous and the discrete model.

The continuous model is represented by shear deformable plate with variable thickness subjected to the same displacement field of the discrete element. The current yielding forces are associated to the reaching of the limits of tensile or compressive stresses in the reference continuous model. Further details on the calibration approach can be found in (Cannizzaro 2011).

5 NUMERICAL APPLICATION

The application reported in the following is relative to the dome of the Cathedral of Noto in the eastern part of Sicily. The Cathedral was built after the strong earthquake which hit the eastern Sicily in 1693, its plan is a three nave basilica structure with a dome resting on a tall drum supported by four large arches at the centre of the transept. On March 13 1996 the central part of the right lateral nave collapsed together with a large part of the dome. The application here reported is relative to a push-over analysis of the original dome subjected to mass-proportional horizontal loading. The mechanical characteristics of the dome, reported in table 1, and its geometry have been derived by the referenced papers (Binda et al. 2003, Tringali et al. 2003).



(a) (b) (c) (d)

Figure 11. (a) Section of the dome (figure from reference (Tringali et al. 2003)); (b) The macro-element discretization; (c) the deformed configuration in the condition of incipient collapse considering an unreinforced masonry lantern; (d) the deformed configuration in the condition of incipient collapse considering a linear elastic behaviour of the lantern.

The dome has been modeled by means of 824 macro-elements 792 of which are quadrilateral and 32 are triangular.

Table 1. Mechanical characteristics of masonry

E Young modulus [kN/cm²]	G Shear modulus [kN/cm²]	σ_t Tensile strength [kN/cm²]	σ_c Compressive strength [kN/cm²]	w Unit weight [kN/m³]
90	37.5	0.005	0.0716	17

Two different models have been analyzed, in the first the whole dome is characterized by an unreinforced masonry structure, in the second model it has been assumed that the lantern has been reinforced by steel elements in the columns in order to avoid its premature collapse. In the first model the collapse is concentrate at the lantern level while the other parts of the dome still exhibit a quasi-elastic behaviour, in the second model the collapse in characterized by the cracking of the dome in the meridian directions and the shear damage of the piers in the drum. Fig. 12 reports the push-over curve of the dome expressed as the base shear normalized with respect to the total weight (10886 kN) of the dome as a function of a target point corresponding to the top of the dome for both the considered models.

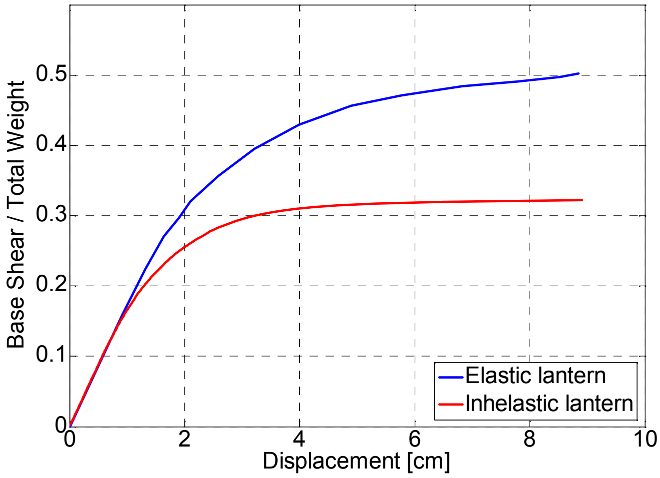


Figure 12. Push-over analysis curve**.**

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