Viscously coupled shear walls: Concept, simplified analysis, and a design procedure

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SUMMARY:
This paper rigorously assesses the efficiency of using viscous dampers as the coupling elements in coupled shear walls. The parameter controlling the dynamic behavior of such systems is identified and its effect on various important responses is examined, thus, important insight to the effect of viscous dampers in those systems is gained. It is shown that the addition of fluid viscous dampers could effectively reduce important responses of walled structures. Those are: displacements, inter-story drifts, total accelerations, total base shear and overturning moment, and wall base shear and bending moment. In addition, the results of the analyses and the non-dimensional tables and graphs developed for important response parameters lead to a very simple "back of the envelope" method that could be easily implemented in practice for the purpose of initial design.

Keywords: Seismic Design, Viscous dampers, Coupled shear walls, Viscously coupled shear walls.

1. INTRODUCTION

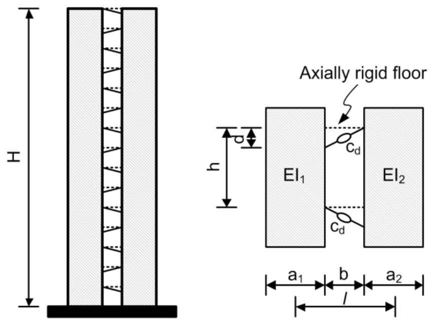
In the last few decades emphasize was drawn to improving the seismic performance of buildings. In parallel, nonstructural damage has gained attention as an additional performance measure besides structural damage. One of the approaches of attaining high seismic performance of structures relies on viscous fluid dampers (Constantinou and Symans, 1992). This approach was shown to be able to reduce both inter-story drifts and total accelerations, the main measures of structural and nonstructural damage (Lavan and Dargush, 2009).

Excessive research on retrofitting of structures using viscous dampers presented practical and optimal design methodologies (e.g. Lavan and Levy, 2010, and references therein). Most research, however, was focused on the seismic retrofitting of framed structures. Recently, it has been shown that mounting viscous dampers in creative locations could lead to an efficient response reduction in wall structures as well (Rahimian, 2002; Taylor, 2002; Madsen *et al*., 2003; Huang *et al*., 2006; Silvestri and Trombetti, 2007; Smith and Willford, 2007; Priestly *et al*., 2007; Toranzo *et al*., 2009; Sullivan and Lago, 2010). The efficiency of coupling adjacent walls by coupling beams to result coupled shear wall systems in reducing horizontal displacements and wall bending moments has been long recognized. This concept was also taken advantage of by Rahimian (2002) that made use of viscous dampers as coupling beams in coupled truss structures. Rahimian also implemented this concept in practice while designing the Torre Mayor building in Mexico City (Taylor, 2002). In contrast to conventional coupled shear walls, here displacement and force related responses are expected to reduce.

In this paper the parameter controlling the dynamic behavior of viscously coupled shear walls is first identified. Its effect on various responses of interest is then examined. The results reveal the efficiency of fluid viscous dampers in reducing important responses of walled structures. In addition, the non-dimensional tables and graphs developed for important response parameters lead to a very simple "back of the envelope" method that could be easily implemented in practice for the purpose of initial design.

2. STRUCTURAL MODEL

The structural system considers herein consists of two walls connected by viscous dampers that can only transfer shear forces due to shear velocity (Fig. 2.1). Note that the relative horizontal displacement between the walls is restrained by the floors.



(a) (b)

Figure 2.1. Structural systems considered in the study: a) viscously coupled shear walls, and b) parameters of the viscously coupled shear walls.

In order to attain approximate and simplified solutions the following assumptions are made:

- **A plane model is considered.** This, of course, limits the discussion to structures where the torsional response is limited.
- **All model parameters are assumed constant along the height.** While mass and stiffness are indeed close to uniform in most structures, the discussion is limited to constant damping along the height. Nonetheless, the conclusions drawn regarding feasibility of using viscous dampers in wall buildings, and the nature of the controlling parameter, are valid for other distributions of damping.
- **The behavior of the system can be approximated by linear analysis.** As use is made of viscous dampers the design objective would usually limit plasticity in the structure. Hence, the damped structure is expected to behave linearly or close to it, and linear analysis is expected to lead to reasonable approximation of the behavior. Moreover, as the considered structures are "regular", the “equal displacements approximation” is expected to lead to reasonable results in that range.
- **Axial deformations are neglected.** It is assumed that the axial deformation of the walls is small. This assumption is valid in cases where the damping is not too large and the deformations in the “damping system” (to be defined subsequently) concentrate in the dampers rather than mostly in axial deformations. For the range of damping typically used in buildings, this assumption seems reasonable.
- **The viscous dampers are linear and are assumed to be installed on relatively rigid diagonals.** Viscous dampers can be designed, in general, as nonlinear. Linear viscous dampers, however, have advantages over nonlinear dampers since the forces they produce are out of phase with the forces due to deformations. Hence, they are adopted in this study. Those are usually mounted on relatively axially stiff braces to fully utilize their efficiency.
- **The walls deform in bending only, i.e. shear deformations are neglected.** This assumption is often used for the analysis of elements of large span to depth ratio and has been extensively used for the analysis of conventional coupled shear walls.
- **The out-of-plane bending stiffness of the slabs is neglected.** In general, the out of plane bending stiffness of the slabs is indeed small. Note that their effect may become considerable in very tall buildings.
- **The foundations are assumed to be relatively rigid.**
- **When added dampers are considered, the inherent damping is neglected.** It is common to assume a relatively small inherent damping in structures retrofitted with viscous dampers. As the added damping would usually result in large damping ratios, the effect of the inherent damping on the responses is negligible.

Under the aforementioned assumptions, the structural systems can be partitioned to two systems that work in parallel. Those are: the "stiffness system" resisting displacements and deforms in bending, and the "damping system" resisting velocities where the velocities are concentrated in the dampers. Note that the role of the walls as part of the damping system is to transfer the forces in the dampers due to the velocity. The partitioning of the total structural system to its two components is illustrated in Fig. 2.2. In the derivations that follow, reference is made to this partitioning.

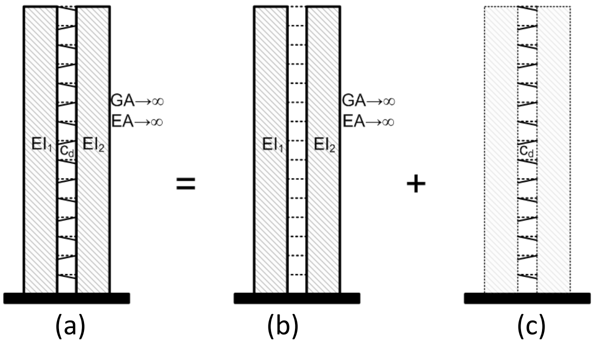


Figure 2.2. Partitioning of the total structural system (a) to its two parallel components: the "stiffness system" that resists displacements and deforms in bending (b), and the "damping system" that resists velocities where the deformations are concentrated in the dampers (c).

3. EQUATIONS OF MOTION

In cases where the number of stories is relatively large, the structural system described above could be modelled using a continuum approach (see e.g. Rosman, 1964). For the continuum model an analytical, or a semi-analytical, solution could be attained. The equation of motion of the continuum model of the system assuming constant values of the parameters throughout the height of the structure is given by:

$$m\frac{\partial^2 u(z,t)}{\partial t^2} - c\frac{\partial^3 u(z,t)}{\partial z^2 \partial t} + EI\frac{\partial^4 u(z,t)}{\partial z^4} = -m\frac{\partial^2 u_g(t)}{\partial t^2} \tag{3.1}$$

where u=horizontal relative displacement as a function of time, t, and height, z; m=mass per unit length; EI=flexural rigidity of the walls; c=added shear damping as given below; and u_g=ground acceleration. The sum of the first expression on the left hand side and the expression on the right hand side of Eqn. 3.1 represents the external forces required to maintain the absolute accelerations of the system at time t. The second expression on the left hand side represents the external forces required to maintain the velocities of the system at time t. The system resisting those forces is the "damping system" (Fig. 2.22c). The third expression on the left hand side of Eqn. 3.1 represents the external forces required to maintain the displacements of the system at time t. The system resisting those forces is the "stiffness system" (see Fig. 2.2b). The parameters take the values $EI=EI_1+EI_2$ and $c = c_d d^2 l^2 / h(h^2+b^2)$ (see parameters' description in Fig. 3.1b).

Equation **Error! Reference source not found.**3.1 is accompanied by the following boundary conditions (relative displacement and angle at the base, wall moment and total shear at the top all equal zero).

$$u(0,t) = \frac{\partial u(0,t)}{\partial z} = \frac{\partial^2 u(H,t)}{\partial z^2} = EI\frac{\partial^3 u(H,t)}{\partial z^3} - c\frac{\partial^2 u(H,t)}{\partial z \partial t} = 0 \tag{3.2}$$

where H=wall height. Eqn. 3.1 and the boundary conditions (Eqn. 3.2) can be brought to the following non-dimensional form:

$$\ddot{\delta}(\eta,\tau)-2\xi\dot{\delta}''(\eta,\tau)+\delta^{IV}(\eta,\tau)=-\ddot{\delta}_g(\tau) \tag{3.3}$$

$$\delta(0,\tau)=\delta'(0,\tau)=\delta''(1,\tau)=\delta'''(1,\tau)-2\xi\dot{\delta}'(1,\tau)=0 \tag{3.4}$$

where $\delta=u/H$; $\eta=z/H$; $\tau=t\sqrt{EI/mH^4}$; $2\xi=\sqrt{c^2/mEI}$; $\delta_g=u_g/H$, a dot represents a derivative with respect to τ and a tag represents a derivative with respect to η. As can be seen from Eqn. 3.3 and 3.4, the response is controlled by the excitation (hence time scaling parameters) and by ξ only. Note that the non-dimensional free vibration equation is dominated by a single compact parameter, namely ξ. Note also that, in contrast to coupled shear walls, the controlling parameter in the viscously coupled shear walls, $\xi=1/2\sqrt{c^2/mEI}$, does not depend on the height of the structure. This is a very important observation since it implies that this system can be efficient also for low-rise buildings. For a given solution of the non-dimensional Eqn. 3.3, $\delta(\eta,\tau)$, the responses of interest could be evaluated.

4. EIGEN ANALYSIS

Caughey and O'kelly (1965) have shown that a continuously damped linear dynamic system would possess classical normal modes if two conditions are met. The first condition requires that the differential operators acting on the velocity and the displacement commute. This condition is satisfied by the equation of motion at hand (Eqn. 3.3). The second condition requires that the boundary conditions on the higher order operator would be derivable from a compatible set of boundary conditions on the lower order operator. This condition is not satisfied by the boundary conditions of the problem at hand (Eqn. 3.4). Hence, complex mode shapes are expected.

Let us first solve the non-dimensional free vibration equation (the homogeneous counterpart of Eqn. 3.3) by assuming a solution of the form $\delta(\eta,\tau)=\varphi(\eta)e^{\lambda\tau}$ where $\varphi(\eta)$= a complex function (mode shape) and λ = a complex number. Substituting the assumed solution to the homogeneous counterpart of Eqn. 3.3 with its B.C. given by Eqn. 3.4, leads to the following equation and B.C. for $\varphi(\eta)$:

$$\varphi^{IV}(\eta)-2\xi\lambda\varphi''(\eta)+\lambda^2\varphi(\eta)=0 \tag{4.1}$$

$$\varphi(0)=\varphi'(0)=\varphi''(1)=\varphi'''(1)-2\xi\lambda\varphi'(1)=0 \tag{4.2}$$

Equation 4.1 is an ordinary linear differential equation with constant coefficients. Its solution may be written as $\varphi(\eta)=e^{\beta\eta}$ where β = a complex number. Substitution of this solution in Eqn. 4.1 implies that $\beta^4-2\xi\lambda\beta^2+\lambda^2=0$, thus $\beta=\pm\sqrt{\lambda\left(-\xi\pm\sqrt{\xi^2-1}\right)}$. Hence, the solution of Eqn. 4.1 is of the form

$$\varphi(\eta)=\left[A_1e^{\eta\sqrt{\lambda\left(-\xi+\sqrt{\xi^2-1}\right)}}+A_2e^{\eta\sqrt{\lambda\left(-\xi-\sqrt{\xi^2-1}\right)}}+A_3e^{-\eta\sqrt{\lambda\left(-\xi+\sqrt{\xi^2-1}\right)}}+A_4e^{-\eta\sqrt{\lambda\left(-\xi-\sqrt{\xi^2-1}\right)}}\right] \tag{4.3}$$

where A_1-A_4 are complex constants of integration whose values are to be determined by imposing the boundary conditions. Imposing the B.C. from Eqn. 4.2, and assuming $\lambda\neq 0$ (to avoid the trivial solution) one obtains a set of linear homogeneous algebraic equations for the constants A_1 through A_4. For $\varphi(\eta)$ to have a nontrivial solution the determinant of the coefficient matrix in this equation should vanish. This leads to an equation in λ for which an infinite number of solutions exist. For each value of λ, corresponding values of the constants A_2 through A_4 can be obtained by substituting A_1=1+I (where $I=\sqrt{-1}$), for example, and solving the remaining three equations in the set of homogeneous algebraic equations. Using the attained values for λ and the corresponding constants A_1 through A_4 in Eqn. 4.3 leads to the mode shapes. Observe that each set of complex eigenvalue and eigenvector is accompanied by its complex conjugate as a solution. It can be shown that for those the circular frequency of vibrations is the absolute of λi, or $\overline{\omega}_i = |\lambda_i|$, and the damping coefficient is $\overline{\xi}_i = -\mathrm{Re}(\lambda_i)/\overline{\omega}_i$ where Re( ) is the real part of a complex number. For reasonable values of damping, all eigenvalues are expected to be complex, i.e. underdamped. The above procedure was executed for various values of ξ. Modal parameters for the first three modes are given in Table 4.1.

Table 4.1. Modal properties for various damping magnitudes

Mode	ξ=0.0		ξ=0.1		ξ=0.2		ξ=0.3		ξ=0.4	
	$\overline{\omega}_i$	$\overline{\xi}_i$	$\overline{\omega}_i$	$\overline{\xi}_i$	$\overline{\omega}_i$	$\overline{\xi}_i$	$\overline{\omega}_i$	$\overline{\xi}_i$	$\overline{\omega}_i$	$\overline{\xi}_i$
1	3.516	0.000	3.525	0.132	3.553	0.266	3.606	0.403	3.697	0.546
2	22.035	0.000	22.067	0.148	22.181	0.300	22.454	0.462	23.197	0.649
3	61.697	0.000	61.704	0.126	61.730	0.255	61.808	0.391	62.098	0.551

Note that the values attained for ξ=0, as expected, coincide with the values known from the free vibrations analysis of a bending cantilever. Note also that the natural frequencies vary very little with the addition of ξ. The damping ratio of each mode, on the other hand, varies significantly with ξ, again, as expected.

$$\omega_i = \sqrt{\frac{EI}{mH^4}}\,\overline{\omega}_i \qquad ; \qquad \xi_i = \overline{\xi}_i \tag{4.4}$$

5. COMPLEX MODAL SPECTRAL ANALYSIS

Procedures for modal spectral analysis have been proposed to discrete systems with nonproportional damping (see e.g. Song *et al.*, 2008). The derivations that follow, for a modal spectral analysis of the continuous system, are inspired by that method. For that purpose the state space formulation is used in the following discussion.

5.1. Eigen analysis in the state space

Equation 3.3 can be formulated in state space notation as:

$$\mathbf{A}\dot{\mathbf{y}}(\eta,\tau)+\mathbf{B}\mathbf{y}(\eta,\tau)=-\mathbf{L}\ddot{\delta}_g(\tau) \tag{5.1}$$

where

$$\mathbf{A}=\begin{bmatrix}0 & 1\\ 1 & -2\xi(\ )''\end{bmatrix} \ ; \ \mathbf{B}=\begin{bmatrix}-1 & 0\\ 0 & (\ )^{IV}\end{bmatrix} \ ; \ \mathbf{L}=\begin{Bmatrix}0\\1\end{Bmatrix} \ ; \ \mathbf{y}(\eta,\tau)=\begin{Bmatrix}\dot{\delta}(\eta,\tau)\\ \delta(\eta,\tau)\end{Bmatrix} \tag{5.2}$$

Assume the following solution to the homogeneous counterpart of Eqn. 5.1:

$$\mathbf{y}(\eta,\tau)=\boldsymbol{\psi}(\eta)e^{\lambda\tau} \tag{5.3}$$

where $\boldsymbol{\psi}(\eta)=\{\varphi_{\dot{\delta}}(\eta) \quad \varphi_{\delta}(\eta)\}^T$. Substitution of this solution to the homogeneous counterpart of Eqn. 5.1 leads to the following eigenproblem:

$$\lambda\mathbf{A}\boldsymbol{\psi}(\eta)+\mathbf{B}\boldsymbol{\psi}(\eta)=\mathbf{0} \tag{5.4}$$

This set of equations reveals that $\varphi_{\dot\delta}(\eta)=\lambda\varphi_{\delta}(\eta)$ and leads to the following eigenproblem for $\varphi_{\delta}(\eta)$: $\lambda^2\varphi_{\delta}(\eta)-2\xi\lambda\varphi''_{\delta}(\eta)+\varphi^{IV}_{\delta}(\eta)=0$. This problem is identical to the eigenproblem that was solved in the previous section. This is attained by setting $\varphi_{\delta}(\eta)\equiv\varphi(\eta)$. It should be noted that if λ and $\boldsymbol{\psi}(\eta)$ are solutions of this eigenproblem then λ^* and $\boldsymbol{\psi}^*(\eta)$ also satisfy this eigenproblem. In addition, the eigenfunctions possess orthogonality properties (not shown or derived here).

5.2. Modal analysis

For the purpose of uncoupling the state equations (Eqn. 5.1) their solution is first written as a sum of modal contributions as:

$$\mathbf{y}(\eta,\tau)=\sum_{j=1}^{\infty}\boldsymbol{\psi}_j(\eta)z_j(\tau) \tag{5.5}$$

Substitution of the assumed solution from Eqn. 5.5 to Eqn. 5.1, premultiplying by $\boldsymbol{\psi}_i^T$, integrating from $\eta=0$ to $\eta=1$ and interchanging the order of summation and integration lead to

$$\sum_{j=1}^{\infty}\int_0^1\boldsymbol{\psi}_i^T(\eta)\mathbf{A}\boldsymbol{\psi}_j(\eta)\dot z_j(\tau)d\eta+\sum_{j=1}^{\infty}\int_0^1\boldsymbol{\psi}_i^T(\eta)\mathbf{B}\boldsymbol{\psi}_j(\eta)z_j(\tau)d\eta=-\int_0^1\boldsymbol{\psi}_i^T(\eta)\mathbf{L}d\eta\ddot\delta_g(\tau) \tag{5.6}$$

Integrating by parts the terms in Eqn. 5.6 having expressions involving various orders of derivatives of $\varphi_i(\eta)$ and $\varphi_j(\eta)$, and using the orthogonality conditions with the boundary conditions of Eqn. 4.2 one obtains the following equation for mode *i*:

$$a_i\dot z_i(\tau)+b_iz_i(\tau)=-\int_0^1\varphi_i(\eta)l(\eta)\ddot\delta_g(\tau)d\eta \tag{5.7}$$

where

$$a_i=\int_0^1\left[2\lambda_i\varphi_i^2(\eta)+2\xi\varphi_i'^2(\eta)\right]d\eta \qquad b_i=\int_0^1\left[-\lambda_i^2\varphi_i^2(\eta)+\varphi_i''^2(\eta)\right]d\eta \tag{5.8}$$

A similar equation could be derived for the conjugate solution. It should be noted that in the range of damping reasonable for buildings no overdamped modes are to be expected (see Table 4.1), hence such modes are not dealt with herein. The contribution of the complex valued mode *i* and its conjugate to the Laplace transform of the state vector is obtained by transforming their contribution in Eqn. 5.5 using the Laplace transform as:

$$\mathbf{y}_i(\eta,s)=\left[\boldsymbol{\psi}_i(\eta)Z_i(s)+\boldsymbol{\psi}_i^*(\eta)Z_i^*(s)\right]\cdot\ddot\Delta_g(s) \tag{5.9}$$

where $\ddot\Delta_g(s)$ is the Laplace transform of the ground acceleration, or, using Eqn. 5.2:

$$\begin{Bmatrix}\dot\Delta_i(\eta,s)\\ \Delta_i(\eta,s)\end{Bmatrix}=\left[\begin{Bmatrix}\lambda\varphi_i(\eta)\\ \varphi_i(\eta)\end{Bmatrix}Z_i(s)+\begin{Bmatrix}\lambda^*\varphi_i^*(\eta)\\ \varphi_i^*(\eta)\end{Bmatrix}Z_i^*(s)\right]\cdot\ddot\Delta_g(s) \tag{5.10}$$

where $\Delta_i(\eta,s)$ is the Laplace transform of $\delta_i(\eta,\tau)$. Substituting here $Z_i(s)$, the Laplace transform of $z_i(\tau)$, obtained by taking the Laplace transform of Eqn. 5.7, and using Eqn. 5.8, it can be shown that:

$$\begin{Bmatrix} \dot{\Delta}_i(\eta,s) \\ \Delta_i(\eta,s) \end{Bmatrix} = \begin{Bmatrix} A_{V,i}(\eta) \\ A_{D,i}(\eta) \end{Bmatrix} Q_{V,i}(s) + \begin{Bmatrix} B_{V,i}(\eta) \\ B_{D,i}(\eta) \end{Bmatrix} Q_i(s) \tag{5.11}$$

where $Q_i(s) = H_i(s)\ddot{\Delta}_g(s) = -1/\left(s^2 + 2\xi_i\omega_i s + \omega_i^2\right)\cdot\ddot{\Delta}_g(s)$ is the Laplace transform of the response $q_i(\tau)$ of a SDOF system to the equation of motion $\ddot{q}_i(\tau) + 2\xi_i\omega_i\dot{q}_i(\tau) + \omega_i^2 q_i(\tau) = \ddot{\delta}_g(\tau)$ and $Q_{V,i}(s) = H_{V,i}(s)\cdot\ddot{\Delta}_g(s) = -s/\left(s^2 + 2\xi_i\omega_i s + \omega_i^2\right)\cdot\ddot{\Delta}_g(s)$ is the Laplace transform of the velocity $\dot{q}_i(\tau)$ of this SDOF system. Also, $A_{D,i}(\eta) = 2R_i^R(\eta)$, $B_{D,i}(\eta) = 2\omega_i\left(\xi_i R_i^R(\eta) - \sqrt{1-\xi_i^2}R_i^I(\eta)\right)$, $A_{V,i}(\eta) = -2\omega_i\left(\xi_i R_i^R(\eta) + \sqrt{1-\xi_i^2}R_i^I(\eta)\right)$, $B_{V,i}(\eta) = -2\omega_i^2 R_i^R(\eta)$, $R_i(\eta) = r_i\varphi_i(\eta)$, $r_i = \int_0^1 \varphi_i(\eta)l(\eta)d\eta \Big/ a_i$, $R_i^R(\eta)$ is the real part of $R_i(\eta)$, and $R_i^I(\eta)$ is the imaginary part of $R_i(\eta)$. Applying inverse Laplace transform to Eqn. 5.11 the contribution of the complex valued mode i and its conjugate to the displacements and velocities in the time domain is obtained:

$$\begin{Bmatrix} \dot{\delta}_i(\eta,\tau) \\ \delta_i(\eta,\tau) \end{Bmatrix} = \begin{Bmatrix} A_{V,i}(\eta) \\ A_{D,i}(\eta) \end{Bmatrix} \dot{q}_i(\tau) + \begin{Bmatrix} B_{V,i}(\eta) \\ B_{D,i}(\eta) \end{Bmatrix} q_i(\tau) \tag{5.12}$$

Responses of interest can also be brought to the following general form:

$$x_{0,i}(\eta,\tau) = A_{0,i}(\eta)\dot{q}_i(\tau) + B_{0,i}(\eta)q_i(\tau) \tag{5.13}$$

where $x_{0,i}(\eta,\tau)$ is the contribution of the mode i and its conjugate to the response of interest and $A_{0,i}(\eta)$ and $B_{0,i}(\eta)$ are its corresponding coefficients.

5.3. Modal spectral analysis

The peak contribution of the mode i and its conjugate to the response of interest can be estimated by means of a response spectrum analysis (see Song *et al.*, 2008) as $x_{0,\max,i}(\eta) = \sqrt{\omega_i^2 A_{0,i}^2(\eta) + B_{0,i}^2(\eta)}\cdot q_{\max,i}$. Here $x_{0,\max,i}(\eta)$ is the peak contribution of the mode i and its conjugate to the response of interest and $q_{\max,i}$ is the peak value of $q_i(\tau)$. In contrast to Song *et al.* (2008) where the coefficients $A_{0,i}$ and $B_{0,i}$ were multiplied by the spectral displacement to attain a desired response, it is proposed here to use somewhat different coefficients that would be multiplied by the spectral pseudo acceleration. This leads to

$$x_{0,\max,i}(\eta) = \left(\sqrt{\omega_i^2 A_{0,i}^2(\eta) + B_{0,i}^2(\eta)} \Big/ \omega_i^2\right)\cdot\left(\omega_i^2 q_{\max,i}\right) = R_{0,i}\cdot S_{a,i} \tag{5.14}$$

where $R_{0,i} = \sqrt{\omega_i^2 A_{0,i}^2(\eta) + B_{0,i}^2(\eta)} \Big/ \omega_i^2$ and $S_{a,i}$ is the spectral pseudo acceleration. As the periods of the various modes are widely spread (see Table 4.1), thy can be combined using the SRSS rule leading to a good approximation of the peak response. Similar expressions for $R_{0,i}$ were derived for additional responses in the non-dimensional domain. Figure 5.1 presents graphs of these coefficients as function of height. Each graph presents the coefficient using 5 values of ξ: 0, 0.1, 0.2, 0.3, 0.4. The first row in Fig. 5.1 presents the contribution of the first pair of conjugate modes while the second and third rows present the contributions of the second and third pairs of modes, respectively. Those can then be brought to the dimensional domain as follows: $R_{D,i} = mH^4/EI\cdot R_{d,i}$; $R_{ID,i} = mH^3/EI\cdot R_{id,i}$; $R_{A,i} = R_{a,i}$; $R_{TS,i} = mH\cdot R_{ts,i}$; $R_{WS,i} = mH\cdot R_{ws,i}$; $R_{DS,i} = mH\cdot R_{ds,i}$; $R_{TM,i} = mH^2\cdot R_{tm,i}$; $R_{WM,i} = mH^2\cdot R_{wm,i}$; $R_{DM,i} = mH^2\cdot R_{dm,i}$.

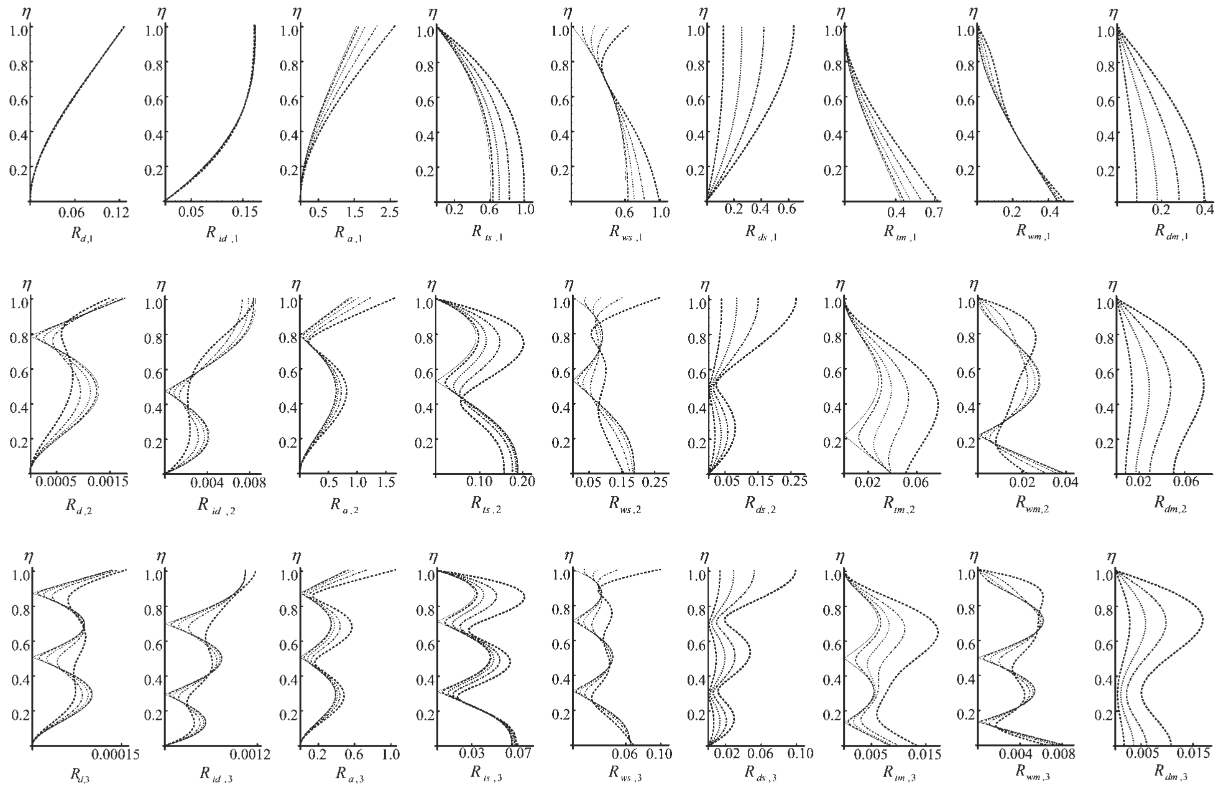


Figure 5.1. R coefficients for various responses (columns) for the first three modes (rows) using various damping values (ξ=0 continuous, ξ=0.1 dashed, ξ=0.2 dotted, ξ=0.3 dashed-dotted, ξ=0.4 dashed thick).

6. DISCUSSION ON THE EFFECT OF ξ

The effect of ξ is explored by plotting (Fig. 6.1) the R coefficients as they are multiplied by the value of $R_{\xi,i} = S_a(\xi_i)/S_a(0.05) = \sqrt{0.07/(0.02+\xi_i)}$. Here $R_{\xi,i}$ is the conversion factor from pseudo-acceleration computed for 5% damping, $S_a(0.05)$, to pseudo-acceleration for a damping ratio ξ_i, $S_a(\xi_i)$. This expression is adopted from Eurocode 8 (CEN) and was found by Priestly *et al*. (2007) to better suite time history responses than other expressions. To obtain the contribution of each pair of conjugate modes to the desired response, the plots in Fig. 6.1 are to be multiplied by the pseudo acceleration at the relevant period, with 5% damping, irrespective of ξ_i. The plots in Fig. 6.1 for the various values of ξ can thus be directly compared to gain some insight on the effect of ξ. The values presented for ξ=0 represent the structure with no added damping, but with 5% Caughey damping, i.e. 5% damping in every mode. Note that here the analysis of the viscously damped structures ignores the inherent damping in each mode. This approximation is often made when damping is added to the structure and its deformations considerably reduce. As can be seen, a ξ value of 0.2 leads to a reduction of circa 40%-50% in all responses except those in the damping system. Values of ξ>0.2 may lead to a further slight reduction in those responses. This, however, will be accompanied by a large increase in the forces in the damping system. In turn, this would lead to a large increase in the peak forces in the dampers and axial forces in the walls.

Hence it is suggested using values of ξ≤0.2. It should be noted that other approximations for the effect of damping ratio on S_a are available. Those are expected to lead to the same conclusions with a slightly different threshold value for the maximum suggested ξ. They may also lead to a different reduction in the pseudo acceleration, hence, the responses. As can also be seen from Fig. 5.1, the higher modes effect on various responses is expected to have the same characteristics as the effect of higher modes on the undamped system. That is, in relatively tall buildings, shear forces and bending moments are expected to be affected by higher modes. Nevertheless, the addition of damping seems to somewhat reduce its magnitude. Another point worth noting is the increase of wall shear forces in the higher stories compared with the undamped structure. This result is expected, and has long been recognized in the case of coupled shear walls and wall-frame systems.

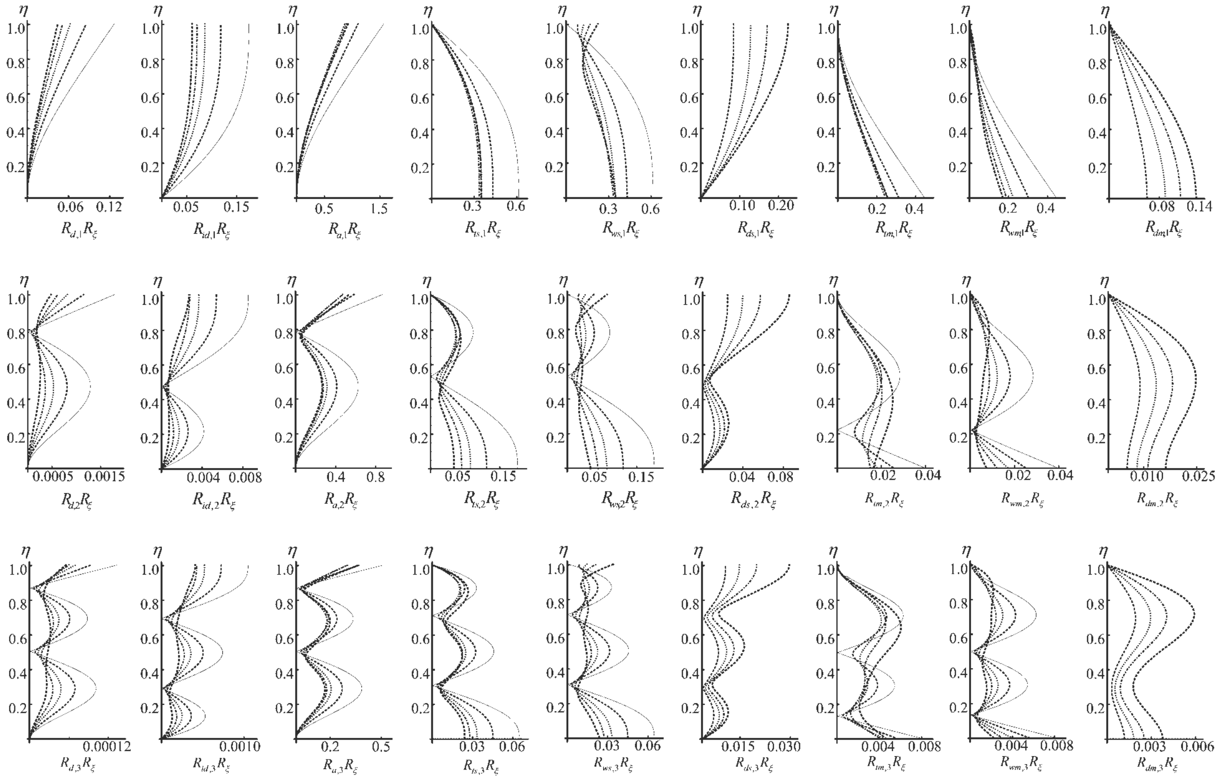


Figure 6.1. *R* coefficients multiplied by R_ξ for various responses (columns) for the first three modes (rows) using various damping values (ξ=0 continuous, ξ=0.1 dashed, ξ=0.2 dotted, ξ=0.3 dashed-dotted, ξ=0.4 dashed thick).

7. CONCLUSIONS

In this work the natural frequencies and damping ratios, as well as height-wise distributions of important modal responses, were derived for viscously coupled shear walls or wall-viscous frames. The table and graphs presented can easily be used in the design process for initial design to form a simple performance-based design algorithm where the required uniform damping for a desired response is computed. The derivations and results lead to the following observations:

1. Under the assumptions made, a single parameter controls the response reduction of viscously coupled shear walls or wall-viscous frames w.r.t the corresponding uncoupled wall system.
2. In contrast to conventional coupled shear walls and wall-frame structures, the controlling parameter in the viscously coupled shear walls, $\xi = 1/2\sqrt{c^2/mEI}$, does not depend on the height of the structure. This is a very important observation since it implies that this system can be efficient also for low rise buildings. Of course, depending on the aspect ratio, shear deformation of the wall system (not considered here) may play a role in those systems and should be accounted for.
3. The addition of damping can lead to a considerable reduction in most responses of interest (up to 60% for cases where the relation given by Eurocode 8 for the effect of damping on SDOF systems' response holds) in most responses of interest. These are: displacements, inter-story drifts, total accelerations, total and wall shear, overturning moment and wall bending moments.
4. Increasing ξ further than 0.2 may lead to a slight further reduction in the responses listed in Item 2 above. This, however, will be accompanied by a large increase in the forces in the damping system. In turn, this would lead to a large increase in dampers’ peak forces and in

columns/walls axial forces. Hence, it is suggested using values of $\xi \leq 0.2$. Based on R_ξ from Eurocode 8 and the derivations in this paper for $\xi=0.2$ circa 50% reduction in most response values can be attained. The force in the most loaded damper in this case would be circa $F_D = 19\, h/l \cdot \sqrt{d^2+b^2}/d$ % of the base shear of the undamped structure, and the damper stroke would approximately be $stroke = 0.5 \cdot l \cdot d / \sqrt{d^2+b^2} \cdot ID_{undamped}$ where $ID_{undamped}$ is the inter-story drift of the undamped system (%).

5. From Item 4 it can be concluded that for the same response reductions in two buildings having the same geometric properties *h*, *d*, *b*, and *l*, but with a different height, *H*, mass, *m*, and stiffness *EI*, the ratio of the force at the most loaded damper to the undamped base shear, would be similar. That is, the design damper forces are expected to be smaller in lower buildings, showing, again, the applicability of the concept to low raise buildings.
6. In cases where the higher modes effect in the undamped structure is considerable, it is expected to remain considerable in the damped structure as well. Its effect, however, is expected to be somewhat less pronounced with increasing ξ.

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