A Novel Method Combined GM (1, *N*) Model with Site Correction for Seismic Intensity Rapid Estimation

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SUMMARY:
In developing countries, there are only small amounts of data for seismic intensity estimation after an earthquake. To solve this problem, this paper proposed a method combined GM (1, *N*) model with site correction. Firstly, the ground motion parameters peak ground acceleration (PGA), peak ground velocity (PGV) and spectrum intensity (SI) are proved to have closer correlations with seismic intensity through grey relational analysis. Secondly, PGA, PGV and SI are corrected by the method based on the site predominant periods (frequency). At last, GM (1, *N*) model with the modified PGA, PGV and SI is used to assess the seismic intensity. The experimental results showed that the estimation accuracy can be close to 75% and the errors were within 1 degree. Grey relational analysis and GM (1, *N*) model have no demand on the amounts of the data sample. The method has fast speed and can suppress the interference factors notably.

Keywords: seismic intensity, GM (1, N) model, site correction, grey relational analysis

1. INTRDUCTION

After an earthquake, if the full damage of an earthquake is not recognized by the central government in time, rescue and recovery efforts would be delayed. In order to make earthquake emergent response quickly, it is necessary to estimate the seismic intensity from the ground motion parameters rapidly (Wu, et al., 2002).For a big earthquake, it has a long rupture surface. The epicenter just represents the original rupture point in some way, and it usually isn't the maximum energy release region. For example, on the May 12th, 2008 Wenchuan earthquake, the area of Beichuan is the most damaged region, but there is more than 90km away from the epicenter Yingxiu (Dong, et al., 2011). Similarly, areas strongly affected by the Apr 14th, 2010 Yushu earthquake are also far away from the epicentral region (Wang, et al.,2012). The following three methods are usually used to derive the seismic intensity. First, the seismic intensity report is created by the actual damage survey after an earthquake

(Tobita, et al., 2007). Although this method can obtain the intensity information in accuracy, the work is time consuming and always takes several days or even tens of days. Obviously, this method cannot meet the urgent demand of emergency rescue. Second, the seismic intensity isoseismal map is estimated by the earthquake source parameters attenuation relationships (Azzaro, et al., 2006). This method is efficient but the accuracy is limited. Third, the relationships of the seismic intensity with ground motion parameters obtained by the regression analysis are used widely around the world. The regression analysis (Wald, et al., 1999; Wu, et al., 2003; Wang, et al., 2008) is based on large amounts of strong ground motion data. For China and other developing countries, where the strong earthquake station network is sparse, it is difficult to find enough records to set up regression relationships. Besides this, the relationships used in USA, Japan and Taiwan also have regional limit (Wang, et al., 2012). Therefore, the problem of how to get the seismic intensity information with limited strong ground motion records and uncertain factors is a hot point and urgent to be solved. In this paper, a method combined GM (1, *N*) model (Tong, et al., 2011) with site correction is proposed to obtain the relationships between the seismic intensity and the ground motion parameters for China and other station sparse areas. One significant difference from previous studies is that here we use both ground motion parameters PGA, PGV and SI to estimate the seismic intensity. Another difference is that the method can estimate the seismic intensity with smaller amounts of data instead of the statistical regression analysis based on large amounts of data.

2. METHOD

In the method combined GM (1, *N*) with site correction, the first step is grey relational analysis. Grey relational analysis is developed to assess the relationships between seismic intensity and ground motion parameters (acceleration, velocity, displacement, spectral amplitude and so on). The similarity of the data sequence curves which determines whether the data sequence has close link is compared in grey relational analysis. Ground motion parameters PGA, PGV and SI have the best correlations with seismic intensity through grey relational analysis. Secondly, PGA, PGV and SI are corrected by the method based on ground motion acceleration spectrum period and site condition. Thirdly, the corrected PGA, PGV and SI are used to assess the seismic intensity. At last, GM (1, *N*) is implemented to obtain the relationships between seismic intensity and the corrected PGA, PGV and SI. All the steps above reach the rapid estimation of the seismic intensity.

2.1. Grey relational Analysis

The high correlation parameters should be chosen to improve the estimation speed and accuracy. Grey relational analysis is used to obtain the relevancy between the seismic intensity and ground motion parameters. For the difference of ground motion parameters order of magnitude, it is necessary to be uniformed. The initial operator is used to work out this problem in this paper.

The ground motion parameters sequence can be described as $X_i = (x_i(1), x_i(2), \cdots, x_i(n))$. The initial operation of X_i is the Eqn. 2.1.

$$X_i D_1 = (x_i(1)d_1, x_i(2)d_1, \cdots, x_i(n)d_1) \tag{2.1}$$

Where D_1 is the sequence initial operator; $x_i(k)d_1 = x_i(k) / x_i(1)$; $x_i(1) \neq 0$, $k = 1, 2, \cdots n$.

In the grey relational analysis, the seismic intensity sequence is X_0 and the ground motion parameters sequences are $X_1 \sim X_m$. (Eqn. 2.2)

$$\begin{aligned} X_0 &= (x_0(1), x_0(2), \cdots, x_0(n)) \\ X_1 &= (x_1(1), x_1(2), \cdots, x_1(n)) \\ &\cdots\cdots \\ X_i &= (x_i(1), x_i(2), \cdots, x_i(n)) \\ &\cdots\cdots \\ X_m &= (x_m(1), x_m(2), \cdots, x_m(n)) \end{aligned} \tag{2.2}$$

The grey correlation coefficient is the Eqn. 2.3 & 2.4:

$$\gamma(x_0(k), x_i(k)) = \frac{\min\limits_i \min\limits_k |x_0(k) - x_i(k)| + \xi \max\limits_i \max\limits_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \xi \max\limits_i \max\limits_k |x_0(k) - x_i(k)|} \tag{2.3}$$

$$\gamma(X_0, X_i) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x_0(k), x_i(k)) \tag{2.4}$$

Where ξ is the resolution coefficient and $\xi \in (0,1)$, which is usually taken as 0.5. $\gamma(x_0(k), x_i(k))$ is the grey correlation coefficient between each seismic intensity and ground motion parameter. $\gamma(X_0, X_i)$ is the grey correlation coefficient between the seismic intensity sequence X_0 and the ground parameter sequence X_i.

2.2. Site effect

The regional site conditions play an important role in the propagation of the seismic waves which affects the damage distribution and amplifies or decreases the strong ground motion (Lopez-Caballero , et al., 2010; Mundepi, et al., 2010; Santos, et al., 2011). Many researches study about the amplification effects of the soil and rock around the world (Yaghmaei-Sabegh, et al., 2011). In this article, we applied a method based on the site predominant periods (frequency) (the period of peak transfer function value) for site correction (Wang, 2010). The site predominant period (frequency) represents the focus band of the earthquake energy and has close relationship with the seismic intensity. The site calibration used in this paper is presented in the Eqn. 2.5 (Wang, 2010).

$$\lg(X_i^M) = \begin{cases} \lg(X_i) + 0.3 & T_P < 0.1 or \quad bedrock \\ \lg(X_i) + 0.2 & 0.1s \le T_P \le 0.2s \\ \lg(X_i) & 0.2s \le T_P \le 0.4s \\ \lg(X_i) - 0.3 & 0.4s \le T_P \le 1.0s \\ \lg(X_i) - 0.45 & T_P \ge 1.0s \end{cases} \tag{2.5}$$

Where T_P is the site predominant period, X_i is the original ground motion parameter value with the damping ratio 0.2, and X_i^M is the modified ground motion parameter value with the damping ratio 0.2.

2.3. GM (1, *N*) Model

GM (1, *N*) is a one order with *N* variables grey model. In the earthquake GM (1, *N*) model, the variables are determined by the number of the ground motion parameters. The original seismic intensity sequence can be described as the Eqn. 2.6:

$$X_1^{(0)} = (x_1^{(0)}(1), x_1^{(0)}(2), \cdots, x_1^{(0)}(n)) \tag{2.6}$$

Where $x_1^{(0)}(n)$ are the seismic intensities in different earthquake stations.

The relational ground motion parameters sequences are the Eqn. 2.7:

$$\begin{aligned} X_2^{(0)} &= (x_2^{(0)}(1), x_2^{(0)}(2), \cdots, x_2^{(0)}(n)) \\ X_3^{(0)} &= (x_3^{(0)}(1), x_3^{(0)}(2), \cdots, x_3^{(0)}(n)) \\ &\cdots\cdots \\ X_N^{(0)} &= (x_N^{(0)}(1), x_N^{(0)}(2), \cdots, x_N^{(0)}(n)) \end{aligned} \tag{2.7}$$

Where $x_N^{(0)}(n)$ are the ground motion parameter values in different earthquake stations, $N = 2 \cdots N$.

GM (1, *N*) model can be described as the Eqn. 2.8:

$$x_1^{(0)}(k) + az_1^{(1)} = \sum_{i=2}^{N} b_i x_i^{(1)}(k) \tag{2.8}$$

Where $X_i^{(1)} = (x^{(1)}(1), x^{(1)}(2) \cdots x^{(1)}(k))$ is the accommodation generation sequence of the original ground motion parameter sequence $X_i^{(0)}$ and $i = 1, 2, \cdots N$; $x^{(1)}(k)$ is the Eqn. 2.9; $Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \cdots z^{(1)}(n))$ is the close mean generation sequence of the sequence $X_1^{(1)}$. $z^{(1)}(k)$ is the Eqn. 2.10.

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)(k = 1, 2, \cdots n) \tag{2.9}$$

$$z^{(1)}(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1)), \quad k = 2, 3, \cdots n \tag{2.10}$$

In the GM (1, *N*) model, -*a* is named system development coefficient; $b_i x_i^{(1)}(k)$ is the drive term; b_i is the drive coefficient; $a = [a, b_1, b_2, \cdots b_N]^T$ is the parameter sequence.

Define:

$$B = \begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) & \cdots & x_N^{(1)}(2) \\ -z_1^{(1)}(3) & x_2^{(1)}(3) & \cdots & x_N^{(1)}(3) \\ \vdots & \vdots & & \vdots \\ -z_1^{(1)}(n) & x_2^{(1)}(n) & \cdots & x_N^{(1)}(n) \end{bmatrix}, Y = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \vdots \\ x_1^{(0)}(n) \end{bmatrix} \tag{2.11}$$

The least square estimation $\hat{a}$ of the parameter sequence *a* is:

$$\hat{a} = (B^T B)^{-1} B^T Y \tag{2.12}$$

The whitlization equation of GM (1, *N*) model in the Eqn. 2.8 is the Eqn. 2.13:

$$\frac{dx^{(1)}}{\mathrm{dt}}+ax_1^{(1)}=b_2x_2^{(1)}+b_3x_3^{(1)}+\ldots\ldots+b_Nx_N^{(1)} \tag{2.13}$$

The solution of GM (1, *N*) model whitlization equation is the Eqn. 2.14:

$$x^{(1)}(t)=e^{-at}\left[x_1^{(1)}(0)-t\sum_{i=2}^{N}b_ix_i^{(1)}(0)+\sum_{i=2}^{N}\int b_ix_i^{(1)}(t)e^{at}dt\right] \tag{2.14}$$

When the variation scale of $X_i^{(1)}(i=1,2,\cdots N)$ is small, $\sum_{i=2}^{N}b_ix_i^{(1)}(k)$ can be taken as a grey constant, and the approximate time response equation of the Eqn. 2.8 is the Eqn. 2.15:

$$\hat{x}_1^{(1)}(k+1)=(x_1^{(1)}(0)-\frac{1}{a}\sum_{i=2}^{N}b_ix_i^{(1)}(k+1))e^{-ak}+\frac{1}{a}\sum_{i=2}^{N}b_ix_i^{(1)}(k+1) \tag{2.15}$$

Where $x_1^{(1)}(0)$ is taken as $x_1^{(0)}(1)$.

The inverse reduction of GM (1, *N*) model is the Eqn. 2.16:

$$\hat{x}_0^{(1)}(k+1)=a^{(1)}\hat{x}_1^{(1)}(k+1)=\hat{x}_1^{(1)}(k+1)-\hat{x}_1^{(1)}(k) \tag{2.16}$$

$\hat{x}_0^{(1)}(k+1)$ in the Eqn. 2.16 is the predicted seismic intensity by GM (1, *N*) model.

3. EXAMPLE ANALYSIS

Ten Northridge earthquake intensity records in 1994 were chosen to verify the method proposed in this paper. In the process of building GM (1, *N*) model, the main sequence X_0 is the seismic intensity sequence and the subsequences $X_1, X_2, \cdots X_N$ are the ground motion parameters sequences. The subsequences $X_1, X_2, \cdots X_N$ are peak ground acceleration (PGA), peak ground velocity (PGV), peak spectrum acceleration (PSA), peak spectrum velocity (PSV), peak spectrum displacement (PSD), and spectrum intensity (SI), respectively. First, grey relational analysis is preceded. Grey relational coefficients of the seismic intensity and ground motion parameters are shown in table 3.1. It can be seen that PGA, PGV and SI have closer relationships with the seismic intensity. So the ground motion parameters PGA, PGV and SI are taken as the factor sets of GM (1, *N*) model. The data of the factor sets are shown in table 3.2.

Table 3.1. Grey Related Coefficients

Intensity	PGA	PGV	PSA	PSV	PSD	SI
The Grey Related Coefficients	0.5404	0.5263	0.4864	0.5260	0.5120	0.5402

Table 3.2. The Factor Sets Of GM (1, *N*) Model

NO.	STATION NAME	LAT.	LONG.	PGA(cm/s^2)	PGV(cm/s)	SI(cm/s)	MMI
1	Norwalk	33.920	-118.070	234.9	21.6	39.1185	7
2	Alhambra Fremont School	34.070	-118.150	374	21.7	37.1641	8
3	Altadena Eaton Canyon Park	34.177	-118.096	299	10.2	19.2436	6
4	Castaic Old Ridge Route	34.564	-118.642	67.2	4.3	8.0801	4
5	LA Baldwin Hills	34.009	-118.361	150	7.7	16.6568	5
6	Century City North	34.063	-118.418	97.5	6.6	14.2878	5
7	Rosamond Godde Ranch	34.827	-118.265	73.8	3.6	7.4053	4
8	Newhall La County	34.390	-118.530	57.2	3.7	7.724	4
9	Riverside Airport	33.951	-117.446	56.8	1.3	2.2329	5
10	Pacoima Kagel Canyon	34.288	-118.375	155	7.8	16.3145	5

In order to decrease the effect of the ground motion parameters data order of magnitude, a log algorithm is used for the original parameter sequences $X_1, X_2, \cdots X_N$. The log sequences are $X_{1\lg}, X_{2\lg}, \cdots X_{N\lg}$. Then GM (1, *N*) Model was built through the steps in the section 2. The predicted seismic intensity data derived by GM (1, *N*) method the modified GM (1, *N*) method (combined with the site correction) are shown in table 3.3. The seismic intensity estimation errors of the ten stations and the errors percentage are shown in the Fig. 3.1 and Fig. 3.2. The seismic intensity estimation errors state the accuracy of the proposed method. The error=0 means that the estimation seismic intensity is the same with the actual seismic intensity, the error=1 or -1 means that the estimation seismic intensity is one intensity degree higher or lower than the actual seismic intensity and so forth. It can be seen from the Fig. 3.1 and Fig. 3.2 that the accuracy of the modified GM (1, *N*) method (combined with the site correction) is higher than GM (1, *N*) Model method.

Table 3.3. The Seismic Intensity Estimation Results With Different Methods

NO.	Station	Actual Intensity	GM(1,*N*)	Error	Modified GM(1,*N*)	Error
1	Norwalk	7	7	0	7	0
2	Alhambra Fremont School	8	7	-1	7	1
3	Altadena Eaton Canyon Park	6	6	0	6	0
4	Castaic Old Ridge Route	4	5	1	4	0
5	LA Baldwin Hills	5	5	0	5	0

6	Century City North	5	5	0	6	-1
7	Rosamond Godde Ranch	4	5	1	4	0
8	Newhall La County	4	5	1	4	0
9	Riverside Airport	5	5	0	5	0
10	Pacoima Kagel Canyon	5	6	1	5	0

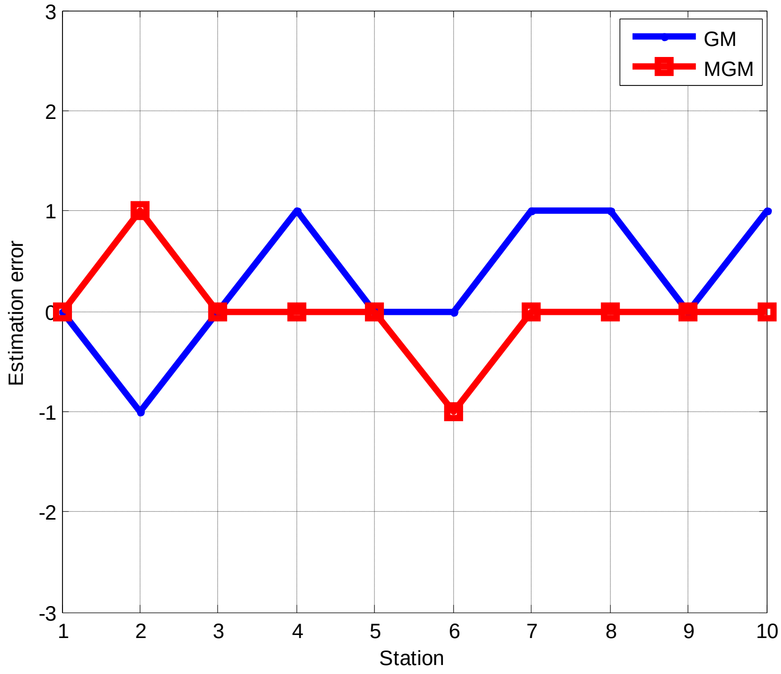
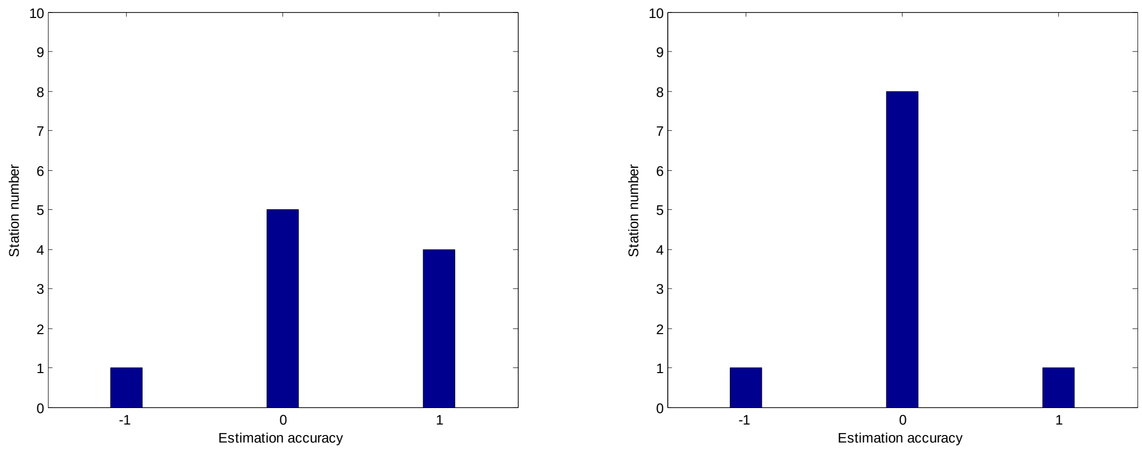


Figure 3.1. The seismic intensity estimation errors comparison



(a) The estimation accuracy of GM(1, N) model

(b) The estimation accuracy of GM(1, N) model combined with site correction

Figure 3.2. The estimation accuracy of the two methods

4. CONCLUSIONS

The method combined GM (1, *N*) Model with site correction can be used to estimate the seismic intensity quickly and effectively. The grey relational analysis proves that PGA, PGV and SI have high correlations with the seismic intensity. The experimental results show that the seismic intensity estimation accuracy can be close to 75% and the estimation errors are within one degree. The site predominant periods (frequency) used for site correction improve the estimation accuracy greatly, because it represents the focus energy of the ground motion. The estimation errors show that the proposed GM (1, *N*) method (combined with the site correction) is better than GM (1, *N*) Model method. Compared to the traditional regression analysis, this model only needs four data at least. It means that it can be adapted to the conditions where the seismic records are small and the data distribution is unknown. This method creates a good seismic intensity estimation direction for the countries where the earthquake network is sparse.

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