Influence of the Plastic Hinges Non-Linear Behavior on Bridges Seismic Response

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SUMMARY:
This work intends to analyze the influence of the physical non-linearity of the plastic hinges in the seismic behaviour of bridges with piers of different lengths having the same cross- section and reinforcement. A plastic hinge model for design purposes was developed. The results indicate that the Eurocode hypothesis to neglect the post-yielding stiffness may fail to correctly limit the allowed behaviour factor due to the associated demanded ductility. The Eurocode doesn't efficiently take into account the effect of the piers being designed for stress values greater than the ones resulting from the design spectrum analysis. Although it's never against safety, this means that ductility demands on those piers will be considered higher than the real situation. In the end, this work presents a relation which allows the results of a non-linear analysis be obtained from the results of a design spectrum analysis.

Keywords: bridge, plastic hinge, hardening, behaviour factor, demanded ductility

1. INTRODUCTION

In high seismicity areas, like Portugal, bridge's piers are frequently designed mainly to withstand strong seismic action. Since strong earthquakes frequently lead to extended damages, forcing the structure to exploit its plastic reserves, it is important to properly study and model the influence of the physic non-linearity of the structural elements.

This work approaches particularly the study of plastic hinges, specifically their non-linear effects, in the longitudinal analysis of reinforced concrete bridges. Therefore, two methods of analysis will be compared to better understand what are the consequences of considering the results of a simpler and less accurate model when, compared to one that explicitly considers non-linear effects.

2. NON-LINEAR ANALYSIS

The physical non-linearity of the elements will be considered exclusively by concentrating its effects in plastic hinges.

The EN 1998-1 (CEN, 2004a) and EN 1998-2 (CEN, 2004b) define the method to calculate the parameters of the plastic hinges when a non-linear analysis is intended. These specifications are incompatible with the safety requirement methods stipulated by the EN 1992-1-1 (CEN, 2004). This happens because the EN 1998-2 (CEN, 2004b) recommends the use of mean values for the properties of the materials, while the EN 1992-1-1 (CEN, 2004) recommends design values.

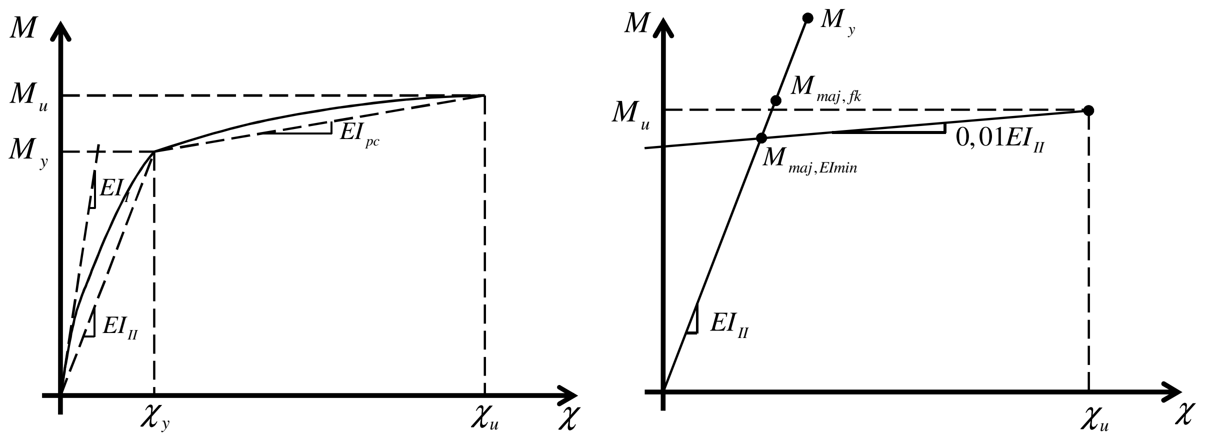
To solve the incongruity between the different codes a variant formulation of the plastic hinge is developed, which requires the definition of the constitutive laws for the materials. For the steel was assumed the bi-linear law presented in EN 1992- 1-1 3.2.7(2) (CEN, 2004).

The concrete will be considered having two possible constitutive laws. These are the "Parabola-Rectangle" presented in the EN 1992-1-1 3.1.7(1) (CEN, 2004) and the constitutive law for non-linear analysis, named in this work as "K-η", presented in the EN 1992-1-1 3.1.5(1) (CEN, 2004). The "K-η" law can be used with either design or mean values. If design values are used the result of a section analysis will be almost equal to the result obtained with the "parabola-rectangle" law. The EN 1992-1-1 (CEN, 2004) and EN 1998-2 (CEN, 2004b) specify that mean values should be used when performing a non-linear dynamic analysis.

2.1. Moment-Curvature Diagram

A simplified bi-linear moment-curvature (M-χ) diagram will be used to model the plastic hinge (Fig. 1a). The values of the points χ_y-M_y and χ_u-M_u are obtained by the Newton-Raphson method and the procedure is thoroughly explained by the author (Arriaga, 2010).

In this work the pair χ_y-M_y should be calculated prioritizing the correct modelling of the initial stiffness. Therefore, the "K-η" law with mean characteristics will be used to model the concrete behaviour. On the other hand, since the maximum bending moment allowed for the section is the design moment calculated according to the EN 1992-1-1 (CEN, 2004), the "parabola-rectangle" law (with design values) will be used to obtain the pair χ_u-M_u. Finally for numerical stability reasons there will only be considered values for the post-yielding stiffness (EI_{pc}) with positive derivative, with a minimum value of $0{,}01EI_{II}$. Also, to avoid inconsistency of data, a value greater than f_{ck} will not be allowed in the concrete when analysing the yielding point. To ensure these restrictions the stiffness EI_{II} will remain constant and the value of M_y will be reduced to M_{Maj}, as shown in Fig. 1b.



(a) (b)

M_y – Yielding moment
$M_{maj,fk}$ – Moment with the concrete stress equal to f_{ck}
$M_{maj,EImin}$ – Moment to ensure a minimum post-yielding stiffness of 1% of EI_{II}

Figure 1. Simplified M-χ Diagram

2.2. Moment-Rotation Diagram in a Plastic Hinge

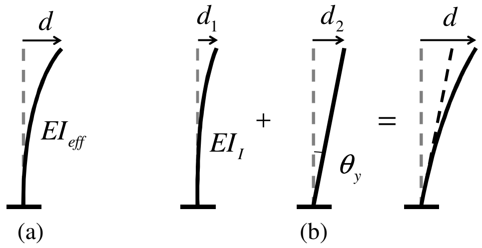
The moment-rotation (M-θ) diagram of a plastic hinge was developed to model the physically non-linear behaviour of a pier element. This diagram is obtained by considering the M-χ diagram for the section of the column and its corresponding characteristics. It is therefore necessary to establish a plastic hinge length for each pier. Two hinge lengths will then be created, the hinge length for the yielding moment L_{py} and the hinge length for the failure moment L_{pu}. The first establishes the transition from χ_y to θ_y and the second length defines the transition from χ_u to θ_u.

2.2.1. Hinge Length for the Yielding Moment

In common analysis programs, the user considers the uncracked stiffness (EI_I) in the elements. In reality, the elements will crack and the stiffness will be reduced accordingly. The EN 1998-2 (CEN, 2004b) proposes a corrected stiffness (EI_{eff}) to use in linear analysis that depends on the cracked (EI_{II}) and uncracked (EI_I) stiffness given by:

$$EI_{eff} = 0{,}08\ EI_I + EI_{II} \quad (1)$$

The following reasoning is then applied: what is the hinge length for the yielding moment (L_{py}) required to simulate the corrected stiffness EI_{eff} in a model constituted by a plastic hinge with initial stiffness K_θ and a pier with the uncracked stiffness EI_I . This is represented in Fig. 2. The displacement *d* represented in Fig. 2 is calculated for the corrected stiffness EI_{eff}. However, this corrected stiffness given by equation (1) establishes a fixed relationship between EI_I and EI_{II}, which is not accurate because it should vary with the reduced axial force (ν) affecting the pier in question. Since the value of ν should be around 0.2 for a well designed pier, the approximation of the EN 1998-2 may be applied without major errors.



(a) – Displacement obtained with stiffness EI_{eff}
(b) – Displacement obtained with uncracked stiffness E_{II} plus rotation at its base

Figure 2. Definition of L_{py}

The yielding moment's hinge length (L_{py}) is defined as a function of L, which is the length between the base of the pier and the point of zero moment:

$$L_{py} = \alpha_y\ L \quad (2)$$

where α_y is defined by the following equation (Arriaga, 2010):

$$\alpha_y = \frac{EI_{II}}{3}\left(\frac{1}{EI_{eff}} - \frac{1}{EI_I}\right) \quad (3)$$

The value of α_y depends only of non-dimensional parameters like the reduced axial force ν or the reinforcement percentage ρ. This means that it can be used in different cross-sections if they present the same non-dimensional characteristics. Assuming some generic cross-sections common in piers of bridges, a parametric analysis was made.

On Fig. 3 the value of α_{ref} as a function of ν is represented, where α_{ref} is the value of α_y when $\rho = 1\%$. The value of α_y can also be related to the value of ρ. An analysis was made, and for usual values of reinforcement ($0{,}5\% \leq \rho \leq 1{,}5\%$) the expression (4) adjusts fairly well the actual values calculated when $\nu \geq 0{,}1$ (with an error always inferior to 6% in the rectangular cross-section and 3% for others).

$$\alpha_y\ (\rho,\nu) = \alpha(\nu)_{ref}\ [1 + (1 - \rho)\ (A + \nu B)] \quad (4)$$

where A and B are obtained from Table 2.1 and $\alpha(\nu)_{ref}$ is the value of α_{ref} acquired from the graphic of Fig. 3. The value of ρ must be expressed in percentage.

The parameter $\alpha_y\ (\rho,\nu)$ never exceeds the value of 0,1714, which is lower than 1/3 as established by the EN 1998-2. On the other hand, it wasn't possible to use the EN 1998-2 proposal for the bilinearization of the M-χ diagram, which means that even though the stiffness hypothesis are similar, the value for α_y here proposed cannot be directly compared with the one the EN 1998-2 suggests.

Table 2.1. Cross-section parameters A and B to calculate α_y (ρ,ν)

Cross-Section	A	B
Rectangular	0,36	0,04
Circular	0,12	0,10
Square	0,12	0,10
I Cross-Section	0,14	0,10
Hollow Rectangular	0,02	0,12

The yield rotation of the plastic hinge will be calculated according to the following expression:

$$\theta_y = \chi_y L_{py} = \chi_y L \alpha_y \quad (5)$$

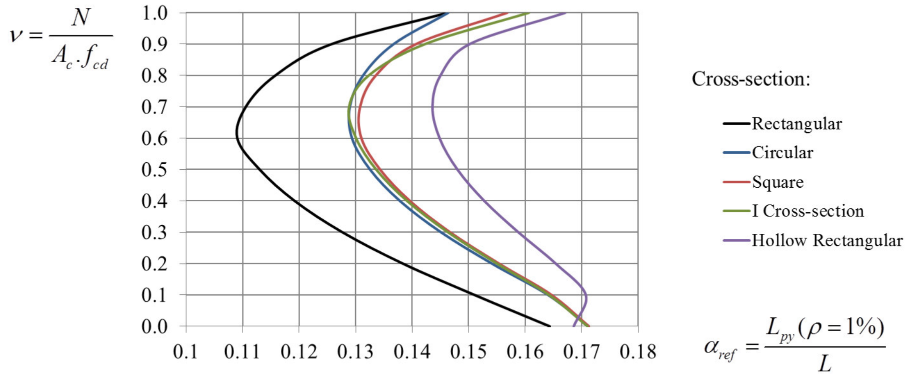


Figure 3. Value of α_{ref} as a function of ν for several cross-section types

2.2.2. Hinge Length for the Failure Moment

The length of the plastic hinge for the failure moment is used according to the following expressions:

$$\theta_u = \theta_y + \theta_{p,d} \quad (6)$$

with $\theta_{p,d}$ defined by:

$$\theta_{p,d} = \frac{\theta_{p,u}}{\gamma_{R,p}} \quad (7)$$

where, according to the EN 1998-2, $\gamma_{R,p}$ = 1,4 and $\theta_{p,u}$ is given by:

$$\theta_{p,u} = (\chi_u - \chi_y) L_p (1 - \frac{L_p}{2L}) \quad (8)$$

The definition of L_p is shown by the following equation:

$$L_p = 0{,}10\, L_p + 0{,}015 f_{yk} d_{bL} \quad (9)$$

where f_{yk} is the characteristic yield stress for the longitudinal bars (in MPa) and d_{bL} is their diameter (in m). This equation is only valid for piers with a shear span ratio $L/d \geq 3{,}0$.

3. NUMERICAL MODEL

A computer program was developed to analyse two dimensional structures with a plastic hinge model.

A non-linear spring element was developed to simulate the non-linear behaviour of the plastic hinge. It was considered as a bi-linear behaviour defined with tree parameters. These parameters are the elastic stiffness K_l, the post-yielding stiffness K_{pc} and the yielding moment M_y. The parameter θ_u, which refers to the failure rotation, was defined as a stopping criterion.

The procedure for the definition of the plastic hinge's stiffness in each instant is presented in Arriaga (2010). Basically, the procedure begins by defining the positive and negative yield boundaries ($F_y(\theta)$ and $-F_y (\theta - \theta_y)$) where:

$$F_y(\theta) = M_y + (\theta - \theta_y)\, K_{pc} \tag{10}$$

It is common in bridges to use sliding bearings to isolate longitudinally the piers from the deck. The model of the sliding bearing will be a horizontal non-linear spring with a yielding force $F_y = F_{Friction}$. The value for $F_{Friction}$ is obtained by the following equation:

$$F_{Friction} = N\, \mu_a \tag{11}$$

where N corresponds to the axial force on the pier and μ_a is the friction coefficient associated to the sliding bearing. To simulate the frictional system the spring properties should be $K_l = \infty$ and $K_{pc} = 0$. To avoid numerical problems, K_l will be taken as the pier stiffness in elastic regimen (thus replacing the pier itself) and K_{pc} will take a very low value, generally 0,5% of K_l.

4. SEISMIC ANALYSIS OF A TWO PIER FRAME

Consider the one bay frame represented in Fig. 4. The two piers are named pier one (on the left) and pier two (on the right). The beam is rigid, with mass M and its connection to the piers is pinned. The length of the pier 2 (L_2) is always grater than the length of the pier 1 (L_1) and their quotient is represented by β ($\beta = L_2/L_1$).

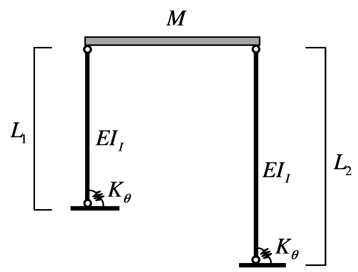


Figure 4. Two pier frame

Two analysis were made, one considering a set of ten earthquakes and the non-linear response of the structure, and other based on an elastic response spectrum obtained as the average of the spectra of the ten earthquakes considered in the first analysis. Therefore the following variables will be considered for the base of the piers:

- $M_{i,el}$ - Elastic bending moment in pier i.
- $M_{i,NL}$ - Maximum bending moment that the pier i is subject to, during the non-linear step-by-step analysis.
- $M_{i,Ed}$ - Maximum bending moment in pier i, obtained by the elastic spectrum analysis, affected by a behaviour factor q.

Pier 1 has always grater internal forces than pier 2, hence the variables x (ductility coefficient in force) and q (behaviour factor) will be defined as following:

$$x = \frac{M_{1,el}}{M_{1,y}} \tag{12}$$

$$q = \frac{M_{1,el}}{M_{1,Ed}} \tag{13}$$

To include the influence of the section hardening, the parameter k will be defined as the ratio between the post-yield stiffness ($K_{\theta,pc}$) and the linear stiffness ($K_{\theta,l}$), as presented in the following equation:

$$k = \frac{K_{\theta,pc}}{K_{\theta,l}} \tag{14}$$

The cross-sections and reinforcements of both piers were considered identical, defined by the characteristics of pier 1, resulting in the same values for k, M_y and M_u. Several models were analysed considering all the combinations presented in Table 4.1, where T is the period of the structure controlled through the mass of the beam (M).

Table 4.1. Parameter values used in the analysis

k	0,01	0,30			
x	2,0	2,5	3,0		
T (s)	0,25	0,5	1,0	1,5	2,0
β	1,00	1,25	1,50	1,75	

4.1. Design Bending Moment in Pier 2

The design bending moments in the piers of a bridge are obtained through the response spectrum affected by a behaviour factor assuming an elastic distribution along the piers. This only corresponds to the real distribution of moments when the piers have been designed to have they're resistant moments equal to the ones obtained by the analysis.

In this work is made the hypothesis that all the piers have the same yielding and resistant moments, either for constructive of for regulatory reasons, which means that the distribution of moments obtained by the response spectrum won't correspond to the real distribution.

On Fig. 5 the quotient of $M_{2,NL}/M_{2,Ed}$ for pier 2 as a function of the behaviour factor calculated for that structure through equation (13) is represented. The graphics are drawn for two different values of x.

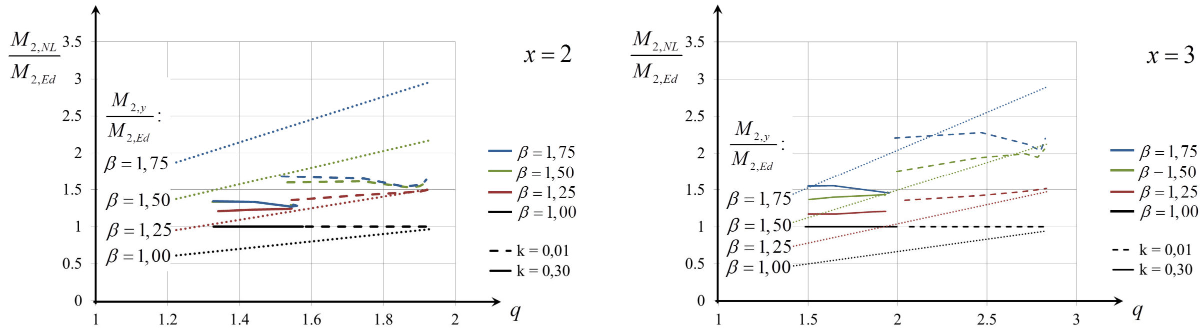
Also represented on Fig. 5 is the normalized yielding moment in pier 2 ($M_{2,y}/M_{2,Ed}$), given by the equation:

$$\frac{M_{2,y}}{M_{2,Ed}} = \frac{\beta^2}{x} q \tag{15}$$

where the variables β, x and q were already defined.

Consequently, from Fig. 5 one may calculate the value of the real bending moment considering the non-linear behaviour. This is done by using the result of the response spectrum analysis multiplied by a value extracted from the graphics displayed in this figure.

The first consideration to be made is that the normalised moment (on the ordinate) presents a different behaviour whether it is above or below the yielding moment line (represented by a dotted line). If the moment is above the yielding moment, then it is mildly increasing. If the moment is below to the dotted line, then it will decrease rapidly. It is also visible that when the value of x is fixed, the maximum value of q will be strongly limited by the value of k. This implies that a plastic hinge with a higher value of k (greater hardening) will require a higher value of x to guarantee the same value of q.



(a) (b)

Figure 5. Normalised moment in pier 2 as a function of q, for x=2 (a) and for x=3 (b).

Since the value of $M_{2,NL}$ is strongly related to the value of $M_{2,y}$, a greater value of k implies a smaller value of $M_{2,NL}$. This happens because of the restriction on the behavior factor (q) given by the parameter k.

4.2. Demanded Ductility

The discrepancy between the values of q and x are particularly relevant when evaluating the “demanded ductility" (Costa, 1990) of a pier. This parameter is defined by the ratio between the displacement of the non-linear analysis (d_t) and the yielding displacement (d_y). On Fig. 7 this parameter is represented as a function of the period of the structure, for the pier 1, since this is the pier with the greatest ductility demand.

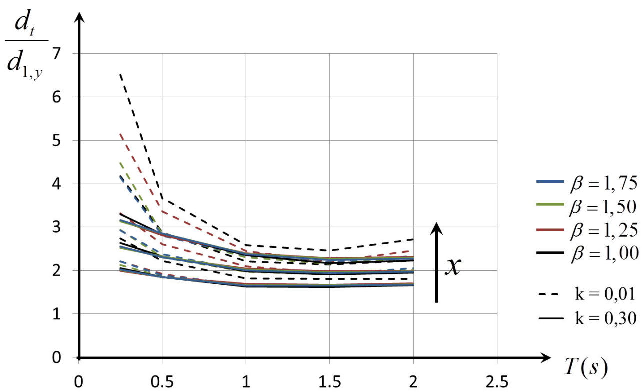
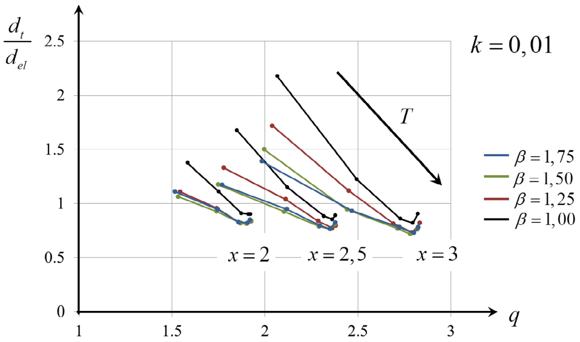


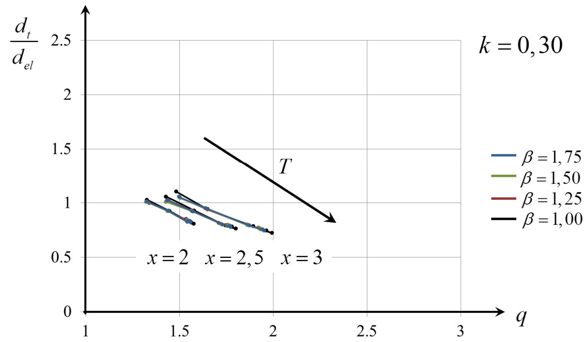
Figure 6. Demanded ductility in pier 1 as a function of T.

By observing the Fig. 6 it's easy to conclude that, for values of period greater than 1 second, the ductility demand is lower than x. It is also possible to observe that the ductility demand is significantly higher for greater values of x. The value of k only affects this parameter directly by reducing its dependence from the value of β. Nevertheless, since a higher value of k implies a higher value of x for the same q, k is indirectly a very strong influence on the ductility demand.

4.3. Pier Top Design Displacement

As in section 4.1, the values obtained from the step-by-step analysis and the response spectrum analysis will be compared, but in this case regarding the displacement of the top of the piers (d_t and d_{el} respectively). On Fig. 7 these two variables are compared for k = 0,01 and 0,30.





(a) (b)

Figure 7. Normalised displacement for k=0,01 (a) and for k=0,30 (b).

It is observable from figure 7 that the conclusions taken from the section 4.2 regarding the value of k still apply. It was said that a greater value of k reduces the importance of β for the response and limits the value of q. From Fig. 7 it's easy to observe that small values of period imply displacements greater than the elastic, and values above the limit of $1.25T_C$, established by the EN 1998-2, are inferior to the elastic displacement. This is taken into account by the EN 1998-2 in 2.3.6.1(6)P where, for periods inferior to $1.25T_C$ the displacement is amplified by a ductility factor and for the remaining periods the displacement is equal to the elastic displacement (Arriaga, 2010).

Finally, for high values of k and T, the hypothesis of the EN 1998-2 can be excessively conservative resulting in design displacements much higher than the actual displacements. A reduction of the displacement should be considered since it is in the high-period structures that the displacement is higher and more important for design purposes.

4.4. High-Period Analysis

The response of a high-period structure calculated with the acceleration response spectrum won't be accurate due to the relationship established between the acceleration and the displacement spectra. This is why the elastic response of high-period structure will be calculated by step- by-step analysis.

On Fig. 8 the beam displacement obtained from a non-linear step-by-step analysis normalised with the elastic displacement is represented. The structure considered has a hardening factor k = 0,01, a ductility coefficient in force x = 2 and a relation of piers β = 1. Additionally, the period of the structure will go from 0,25 to 5 seconds. As observed in the Fig. 8 the displacement tends to the elastic displacement when the period (T) of the structure increases. It is also observable that the displacements are lower than the elastic displacement when $T \geq T_0$, confirming the hypothesis of the EN 1998-2, which considers them equal to the elastic displacement as a conservative position. These observations were verified in all structures with k = 0,01.

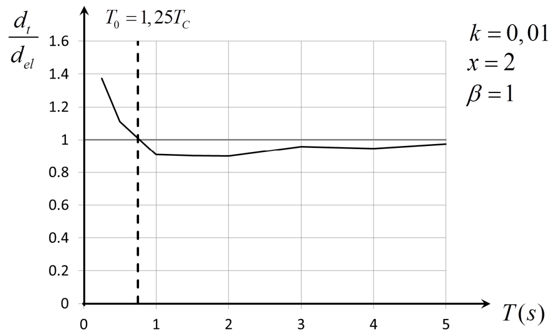


Figure 8. Normalised displacement as a function of T (k=0,01; x=2; β=1)

Since that, for high-period structures, the displacement is approximately equal to elastic displacement, a relationship can be established between the behaviour factor (q) and the ductility in force coefficient (x), give by the following equation:

$$q = \frac{x}{1 + (x - 1)\,k} \tag{16}$$

This expression was compared to the results obtained for the behaviour factor from the analysis of the high-period structures and presented an error of about 2%.

5. CASE STUDY OF A REAL BRIDGE

Using the bridge model presented in Fig. 9, two analyses were made, an elastic analysis by response spectrum and a time non-linear analysis. The parameters of each pier were calculated according to the methods described earlier, resulting in the parameters presented on Table 3 (Arriaga, 2010).

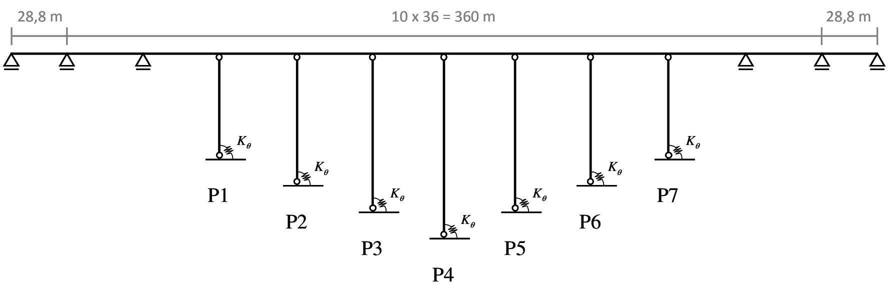


Figure 9. Bridge Model

Table 5.1. Parameters of the piers and respective plastic hinges

	P1/P7	P2/P6	P3/P5	P4
q (q')	1,81 (1,88)			
x	1,93			
L_p (m)	20	25	30	35
β_p	1	1,25	1,5	1,75

On Fig. 10 the bending moments calculated in each pier are represented, one set for the elastic analysis with the design spectrum and other for the non-linear step-by-step analysis. As observed, the behaviour factor q considered on the analysis was such that the values of the bending moment in both piers 1 and 7 were the same. The parameters in Table 5.1 were used to calculate, for each pier, a correction factor based on the results presented on Fig. 5. This correction factor, $M_{p,NL}/M_{p,Ed}$, attempts to modify the values of the response spectrum analysis so that the values of the bending moments become closer to the real response, as if calculated by a non-linear analysis.

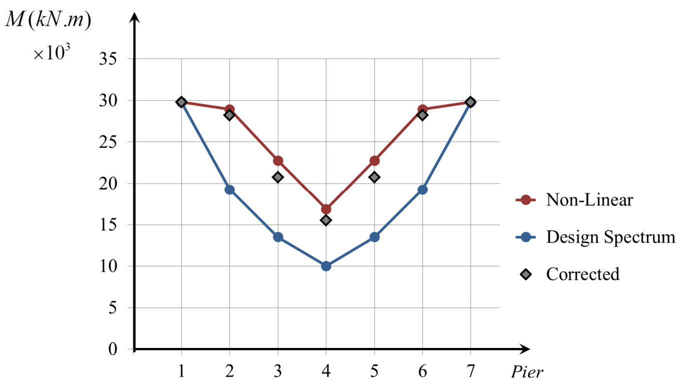


Figure 10. Comparison of the bending moments obtained with different methodologies

Although the results on Fig. 5 were obtained for a two pier structure, it is very clear from the results presented in the Fig. 10 that the corrected values also give good results on a multi-pier structure.

6. CONCLUSIONS

One of the plastic hinge's characteristics was its α_y value. For the usual types of bridge cross-sections, if $\nu \leq 0,4$, the α_y parameter varies in a short range of values, between 0,10 and 0,17, and is in most cases greater than 0,14. The analysis of this variable enables a better knowledge of the quotient EI_{eff}/EI_I, which will be between 20% and 40% when $\nu \leq 0,1$. It was also observed that the value of ρ doesn't affect strongly the value of α_y and consequently the value of EI_{eff}/EI_I.

If the hardening factor (k) of a plastic hinge increases then the ductility in force factor (x) also increases, keeping the behaviour factor (q) constant. The demanded ductility, given by d_t/d_y, increases for x larger values. Consequently, the value of the behaviour factor should be more restricted in cross-sections that imply a considerable hardening factor like the circular cross-section.

Comparing two elements, the first with a high hardening value (k = 0,3) and the second with a low hardening value (k = 0,01), it was observed that an increase of 25% in the value of q could imply an increase of 35% in the demanded ductility for the first case and only 11% for the second.
Also, a higher value of k implies a reduced displacement, where this effect is most relevant for low values of period.

It was made clear that the increase in bending moment on a cross-section due to non-linear effects will never be enough to put in danger the safety of that element. Also, the safety margin will increase if β, x or k increase. The value of q will not interfere in the safety margin because it will only translate the effect of the period of the structure.

If period is greater than 2s, then the displacement will be approximately equal to the elastic displacement. This implies a correlation between x, q and k that allows an estimate of the maximum behavior factor (q) that is possible for a set of k and x.

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