# Approximate Deaggregation Method for Determination of Design Earthquake Magnitudes for Australia

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#### SUMMARY:

Earthquake engineering in Australia relies on AS1170.4 to provide design ground motions. The Australian National Hazard Map that forms the basis of the ground motion maps in AS1170.4 indicates hazard level, and does not quantify the contribution of earthquake magnitude. Many earthquake engineering evaluations require design magnitude as an analysis parameter, such as for estimating the duration weighting factor for liquefaction evaluation or guiding the selection of seed acceleration-time histories for dynamic response analyses. An approximate magnitude deaggregation developed for this study was used to determine design earthquake magnitudes for Australia that are compatible with AS1170.4. The approximate-deaggregation-magnitude model considers a weighted average of magnitudes that are likely to produce a ground motion within specific Australian seismic hazard zones. This is accomplished using three weighting functions corresponding to: 1) likelihood of earthquake magnitude given design ground motion, 2) spatial variability of magnitude and 3) variability of ground motion intensity.

Keywords: earthquake, magnitude, deaggregation, Australia

#### **1. INTRODUCTION**

The Australian National Hazard Map that forms the basis of the ground motion maps in AS1170.4 indicates hazard level only, and does not quantify the contribution of earthquake magnitudes to the hazard. This leaves the earthquake engineering professional in Australia without guidance in selecting design earthquake magnitude. As a result, earthquake engineering practitioners in Australia have applied a number of different methodologies to assign earthquake design magnitude for site-specific studies. These methods range in sophistication, and include 1) estimating mean values from regional recurrence curves, 2) adopting the maximum historic earthquake magnitude for a given region, 3) performing sensitivity analyses considering a range of magnitudes, and 4) deaggregation of site-specific probabilistic seismic hazard analysis (PSHA). This study presents a procedure for determining design earthquake magnitude that is compatible with the current AS1170.4. The procedure approximates a deaggregation of the PSHA underlying the Australian National Hazard Map to compute a weighted average of the earthquake magnitudes that likely contribute to the hazard.

#### 2. PROBABILISTIC SEISMIC HAZARD ANALYSIS AND AS1170.4

The seismic hazard indicated in AS1170.4 is denoted Z, which is defined as peak ground acceleration with probability of exceedance of 10% in 50 years. Z is based on the Australian Hazard Map (McCue et al., 1993) and was determined using a PSHA described by Gaull et al., (1990). Ground motions determined using PSHA are quantified from a range of ground motion scenarios that consider magnitude, distance between site and source, fault type, fault and rupture geometry, site conditions, uncertainty, variability, as well as other aspects. The annual rate of ground motions predicted for specific earthquake scenarios are ranked in descending order of a measure of intensity, such as spectral acceleration, and the frequency of scenarios that meet or exceed the test intensity value are summed to

obtain return period. Scenarios that do not meet the test value do not contribute to the hazard. Analysis of the earthquake scenarios that contribute to the hazard, called deaggregation, is a useful tool to identify parameters of earthquake scenarios that control the ground motion hazard.

Deaggregation is a process for comparing the contribution of various aspects of the earthquake scenarios to the ground motion hazard using a histogram. Deaggregation is most commonly performed to identify the controlling pairs of magnitude and distance that contribute the most to the hazard, and similarly, which faults or other seismic sources control the hazard.

The relative contribution of earthquake magnitude to Z is estimated in this study by approximating a deaggregation of the seismic hazard. This is possible because the ground motion hazard given in AS1170.4 is generally controlled by a single source zone and thus presents a relatively simple model to back-analyse. The approximate-deaggregation-magnitude model developed for this study considers a weighted average of magnitudes that are likely to produce a specific Z.

# 3. APPROXIMATE-DEAGGREGATION-MAGNITUDE MODEL

The approximate-deaggregation-magnitude model considers a range of possible earthquake magnitudes that contribute to Z and comprises three weighting functions:

- 1. A weighting function to account for the likelihood of magnitude given occurrence of Z;
- 2. A weighting function to account for the contribution of larger magnitude earthquakes at greater site to source distance; and
- 3. A weighting function to represent the variability of ground motion.

# 3.1 Range of Earthquake Magnitudes that Contribute to Hazard

The range of magnitudes with potential to generate ground motions that meet or exceed Z is quantified using three values,  $M_{max}$ ,  $M_{int}$ , and  $M_{min}$ .  $M_{max}$  is taken as  $M_{Lmax}$  from Gaull et al. (1990). The minimum and intermediate magnitudes,  $M_{min}$  and  $M_{int}$ , correspond to earthquake scenarios with median plus one and median minus one standard deviation ground motions that are equal to Z, respectively. Determination of  $M_{max}$ ,  $M_{int}$ , and  $M_{min}$  is illustrated on Figure 1. Following Gaull et al. (1990), the attenuation model of Kanai (1961) is used for compatibility with the national hazard model.  $M_{min}$  and  $M_{int}$  were determined by assuming a site to source distance equal to the focal depth, h. That is, the site is located at the earthquake epicentre (earthquake occurs directly below the site).

A key feature of a PSHA is the explicit consideration of error. Although a full treatment of variability and uncertainty is beyond the scope of this paper, ground motion variability is acknowledged through consideration of ground motion plus and minus one standard deviation.

#### 3.2 Weighting Function for Likelihood of Magnitude

This weighting function accounts for the relative likelihood of magnitude given occurrence of Z. Gaull et al. (1990) modelled magnitude recurrence using the Gutenberg-Richter relationship, which includes an underlying exponential magnitude distribution. Therefore, an exponential distribution from  $M_{min}$  to  $M_{max}$  with activity and b-value given by Gaull et al. (1990) was assumed to represent the likelihood of magnitude given a ground motion, Z. This function is expressed in Eqn. 3.1 as:

$$fTE(M) = \frac{\alpha e^{-\alpha} (M - M_{min})}{1 - e^{-\alpha} (M_{max} - M_{min})} \text{ for } M_{min} < M < M_{max}$$
(3.1)

Where  $\alpha$  is ln(10) times the b-value.



Figure 1. Range of earthquake magnitudes that contribute to hazard

# 3.3 Weighting Function for Source to Site Distance

Earthquakes can occur anywhere within a source zone, but only larger magnitudes are capable of producing ground motions that meet Z at large source to site distance; therefore this weighting function accounts for the potential for larger magnitude earthquakes to contribute to the hazard at increased site to source distance. Figure 2 demonstrates this concept.

 $M_{min}$  is defined for a site located at the epicentre, but when the site is located offset of the epicentre, the minimum magnitude that can contribute to Z must be larger. In Figure 2, the minimum magnitude that can produce ground motions that meet Z at distance A is denoted  $M_{min,A}$ , which is larger than minimum magnitude that can produce ground motions that meet Z at the epicentre. Likewise is true for  $M_{min,B}$  compared with  $M_{min,A}$ , that is, at the greater distance, B, larger magnitude,  $M_{min,B}$ , is required. Similarly, an earthquake of  $M_{min}$  will only contribute to Z if it is located at the epicentre, but an earthquake of  $M_{min,B}$  will contribute to Z from anywhere within the areas defined by radius A and B, respectively. Thus the weighting function for site to source distance, fR(M), is specified as areas of concentric circles, which represent the locations of potential epicentres where M could produce Z.

In order to define fR(M), the value of R was determined for a range of magnitudes such that Z was achieved at +1 standard error. This process is similar to that described for determination of  $M_{min}$ , except the attenuation equation is rewritten with R as the unknown value. The areal weighting function, fR(M), is shown for circular areas in Eqn. 3.2:

$$fR(M) = \pi \left[ \left( \frac{\alpha e^{bM}}{e^{\ln Z - \sigma}} \right)^{\frac{2}{c}} - d^2 \right] \text{ for } M_{min} < M < M_{max}$$
(3.2)

Where a, b, c, d, and  $\sigma$  are given by Gaull et al. (1990).

Large magnitude earthquakes have potential to generate Z at distances that exceed the areal extent of a seismic source zone, as illustrated in the comparison of the identical tributary areas overlayed on Seismic Source Zones 5 and 5A, as shown in Figure 3. Therefore, consideration for the shape of the source zone must be given in fR(M). This is accomplished by limiting the area contribution (eg consider non-circular areas) where appropriate.



**Figure 2.** Tributary area for fR(M) for a given Z



Figure 3. Geometry of Seismic Source Zones 5 and 5A (Source: Gaull et al., 1990)

fR(M) represents possible epicentres and does not consider rupture geometry. Additionally, the extent of the seismic source zone is more accurately represented by a volume confined within the seismogenic zone, but for simplicity, area is used in this study. This assumption tends to produce smaller magnitudes, but the effect is small and is not likely to impact engineering applications.

#### 3.4 Weighting Function for Ground Motion Variability

In Figure 1,  $M_{min}$  is the minimum magnitude that has potential for generating ground motions that meet or exceed Z. However, essentially no earthquake scenarios using  $M_{min}$  will meet or exceed Z because the criteria is narrowly limited to earthquakes located directly below the site that produce ground motions at one or more standard deviations above the median.

In contrast, likely any scenario using  $M_{max}$  will exceed the hazard because the full range of modelled ground motion variability (ie ±1 standard deviation from the median) is predicted to be above Z. In fact, examination of Figure 1 indicates that between  $M_{int}$  and  $M_{max}$ , the full range of variability in ground motion considered will meet or exceed the hazard. Therefore, this observation is adapted as a weighting function for ground motion variability such that magnitudes above  $M_{int}$  are weighted at 100%; magnitude of  $M_{min}$  is weighted at 0; and a linear relationship is used between  $M_{min}$  and  $M_{int}$ . The weighting function for ground motion variability, fV(M), is shown in Figure 4 and given as Eqns. 3.3 and 3.4:

$$fV(M) = \frac{M - M_{min}}{M_{int} - M_{min}} \text{ for } M_{min} < M < M_{int}$$
(3.3)

$$fV(M) = 1 \text{ for } M_{int} < M < M_{max}$$

$$(3.4)$$

#### 3.5 Weighted Design Magnitude

The weighted magnitude,  $M_{weighted}$ , (units of  $M_L$ ) is determined by integrating and normalizing the product of the three weighting functions, given in Eqn. 3.5 and shown normalized in Figure 5:

$$M_{weighted} = \frac{1}{N} \int_{M_{min}}^{M_{max}} M \cdot fV(M) \cdot fTE(M) \cdot fR(M) dM$$
(3.5)

Where N is a normalization term computed from Eqn. 3.6 as:

$$N = \int_{M_{min}}^{M_{max}} fV(M) \cdot fTE(M) \cdot fR(M) dM$$
(3.6)



Figure 4. Weighting function for ground motion variability

**Figure 5.** Normalized Weighting Functions for a Typical Seismic Source

The weighted earthquake design magnitude,  $M_{Ww}$ , is obtained by converting  $M_{weighted}$  in units of ML to units of  $M_W$  using the ratio of  $M_W$  to  $M_L$  given by Allen et al. (2011):

$$M_{WW} = M_{weighted} \cdot \frac{M_W}{M_L} \tag{3.7}$$

The  $M_{Ww}$  calculation was implemented in a GIS to produce Australia-wide maps of  $M_{Ww}$  as shown in Figure 6.

# 4. COMPARISON OF APPROXIMATE-DEAGGREGATION-MAGNITUDE MODEL WITH SITE-SPECIFIC PSHA

The  $M_{Ww}$  model was created specifically to work with the Australian National Hazard Map and the work of Gaull et al. (1990), but a better method for evaluation of seismic hazards and design magnitude is to conduct site-specific PSHA. Ground motion hazard and design magnitude determined through PSHA are not expected to agree with the Australian National Hazard Map or  $M_{Ww}$  because the methods used by Gaull et al. (1990) have now been superseded. Regardless, comparison of  $M_{Ww}$  with PSHA is insightful. The Kanai (1967) ground motion prediction equation (GMPE), which is several generations old, is significantly different from a more modern GMPE, such as Sadigh (1997) or any number of next generation GMPEs. Comparison of the Sadigh (1997) and Kanai (1967) relationships

provides insight into the differences between  $M_{Ww}$  and a PSHA. Median values of the two GMPE are compared on Figure 7. The Sadigh (1997) GMPE predicts greater ground motion at short distance than the Kanai (1967) relationship, which implies that the low end of the magnitude range should contribute more to the design magnitude.

A comparison of  $M_{Ww}$  to the contribution of magnitude in a simple PSHA model was conducted using Arup's in-house PSHA software, Sismic (Oasys, 2012). Sismic was set up to run the analyses using the Sadigh (1997) GMPE with up to 1 standard deviation of ground motion (to be compatible with the assumptions made in this study). The magnitude contribution from the PSHA is compared with the normalized weighting functions on Figure 8 where the heavy dots are the computed values of  $M_{Ww}$ . As expected, the magnitude contribution for the PSHA indicates lower magnitudes control the ground motion in comparison with  $M_{Ww}$ .



Figure 6.  $M_{Ww}$  for (a) 500 and (b) 1000 year return period ground motions



**Figure 7.** Comparison of ground motion prediction equations



Figure 8. Comparison of magnitude contribution

#### **5. CONCLUSION AND DISCUSSION**

Earthquake engineering evaluations require design magnitude as an analysis parameter, such as for estimating the duration weighting factor for liquefaction evaluation or guiding the selection of seed acceleration-time histories for dynamic response analyses. A typical method for selecting magnitude is to consider the earthquake scenarios that contribute the greatest amount to the ground motion hazard through an examination of the magnitude deaggregation of a PSHA. In Australia, this information is not readily available in AS1170.4 (2007) however; the Australian Hazard Model provides information on the broad seismic sources quantified into source areas, making it possible to estimate the relative contribution of magnitude to the hazard by making a few simplifying assumptions. The earthquake design magnitude selection model developed in this study approximates the magnitude deaggregation and, the authors feel, provides a basis to guide earthquake design magnitude selection in Australia.

The method may be applied to an individual site using AS1170.4 and Gaull et al. (1990) to determine Z and seismic source zone parameters for input into the proposed approximate deaggregation. For convenience, Table 1 summarizes  $M_{Ww}$  computed for sites summarized on Table 3.2 of AS1170.4. In the absence of a site specific determination, the values listed in Table 1 may be used for design.

Location	$Z_{500}^{a}$	M <sub>Ww500</sub> <sup>b</sup>	$Z_{1000}^{c}$	M <sub>Ww1000</sub> <sup>d</sup>	Z <sub>2500</sub> <sup>e</sup>	M <sub>Ww2500</sub> <sup>f</sup>
Adelaide	0.10	5.1	0.13	5.2	0.18	5.3
Albany	0.08	5.9	0.10	6.1	0.14	6.3
Albury/Wodonga	0.09	5.0	0.12	5.1	0.16	5.3
Alice Springs	0.08	4.9	0.10	5.0	0.14	5.1
Ballarat	0.08	5.0	0.10	5.1	0.14	5.3
Ballidu	0.15	6.3	0.20	6.5	0.27	6.7
Bathurst	0.08	4.9	0.10	5.1	0.14	5.2
Bendigo	0.09	5.0	0.12	5.1	0.16	5.3
Brisbane	0.05	4.7	0.07	4.8	0.09	4.9
Broome	0.12	5.9	0.16	6.1	0.22	6.2
Bundaberg	0.11	5.8	0.14	5.9	0.20	6.1
Burnie	0.07	4.9	0.09	5.0	0.13	5.2
Cairns	0.06	5.0	0.08	5.1	0.11	5.2
Camden	0.09	5.0	0.12	5.1	0.16	5.3
Canberra	0.08	4.9	0.10	5.1	0.14	5.2
Carnarvon	0.09	6.2	0.12	6.3	0.16	6.5
Coffs Harbour	0.05	4.7	0.07	4.8	0.09	4.9
Cooma	0.08	4.9	0.10	5.1	0.14	5.2
Corrigan	0.14	6.3	0.18	6.4	0.25	6.6
Cunderdin	0.22	6.6	0.29	6.7	0.40	6.9
Dampier	0.12	5.3	0.16	5.5	0.22	5.6
Darwin	0.09	4.4	0.08	4.5	0.11	4.6
Derby	0.09	5.8	0.12	5.9	0.16	6.1
Dowerin	0.20	6.5	0.26	6.7	0.36	6.8
Dubbo	0.08	4.9	0.10	5.1	0.14	5.2
Esperance	0.09	4.4	0.08	4.5	0.11	4.6
Geelong	0.10	5.1	0.13	5.2	0.18	5.4
Geraldton	0.09	4.4	0.08	4.5	0.11	4.6
Gippsland	0.10	5.1	0.13	5.2	0.18	5.4
Gladstone	0.09	5.8	0.12	5.9	0.16	6.0
Gold Coast	0.05	4.7	0.07	4.8	0.09	4.9
Goomalling	0.16	6.3	0.21	6.5	0.29	6.7
Gosford	0.09	4.9	0.12	5.0	0.16	5.1
Goulburn	0.09	5.1	0.12	5.2	0.16	5.4
Grafton	0.05	4.7	0.07	4.8	0.09	4.9
Hobart	0.03	4.4	0.04	4.6	0.05	4.7
Karratha	0.12	5.3	0.16	5.5	0.22	5.6
Katoomba	0.09	5.0	0.12	5.1	0.16	5.3
Kellerberrin	0.14	6.3	0.18	6.4	0.25	6.6
Latrobe Valley	0.10	5.1	0.13	5.2	0.18	5.4
Launceston	0.04	4.6	0.05	4.7	0.07	4.8
Lismore	0.05	4.7	0.07	4.8	0.09	4.9
Lorne	0.10	5.1	0.13	5.2	0.18	5.4
Mackay	0.07	4.8	0.09	4.9	0.13	5.0
Maitland	0.10	5.2	0.13	5.3	0.18	5.4

Table 1. Summary of M<sub>Ww</sub> for Site Class B at locations in Table 3.2 of AS 1170.4, excluding islands.

Location	$Z_{500}^{a}$	M <sub>Ww500</sub> <sup>b</sup>	Z <sub>1000</sub> <sup>c</sup>	M <sub>Ww1000</sub> <sup>d</sup>	Z <sub>2500</sub> <sup>e</sup>	M <sub>Ww2500</sub> <sup>t</sup>
Meckering	0.20	6.5	0.26	6.7	0.36	6.8
Melbourne	0.08	5.0	0.10	5.1	0.14	5.3
Mittagong	0.09	5.0	0.12	5.1	0.16	5.3
Morisset	0.10	5.2	0.13	5.3	0.18	5.4
Newcastle	0.11	5.2	0.14	5.3	0.20	5.4
Noosa	0.08	5.7	0.10	5.8	0.14	5.9
Northam	0.14	6.3	0.18	6.4	0.25	6.6
Orange	0.08	4.9	0.10	5.1	0.14	5.2
Perth	0.09	4.4	0.08	4.5	0.11	4.6
Port Augusta	0.11	5.1	0.14	5.2	0.20	5.4
Port Hedland	0.12	5.3	0.16	5.5	0.22	5.6
Port Lincoln	0.10	5.1	0.13	5.2	0.18	5.3
Port Macquarie	0.06	4.8	0.08	4.9	0.11	5.0
Port Pirie	0.10	5.1	0.13	5.2	0.18	5.3
Robe	0.10	5.4	0.13	5.5	0.18	5.8
Rockhampton	0.08	4.9	0.10	5.0	0.14	5.1
Shepparton	0.09	4.9	0.12	5.0	0.16	5.1
Sydney	0.08	4.9	0.10	5.1	0.14	5.2
Tamworth	0.07	4.8	0.09	4.9	0.13	5.0
Taree	0.08	5.0	0.10	5.2	0.14	5.3
Tennant Creek	0.13	5.9	0.17	6.1	0.23	6.2
Toowoomba	0.06	4.8	0.08	4.9	0.11	5.0
Townsville	0.07	4.8	0.09	4.9	0.13	5.0
Tweed Heads	0.05	4.7	0.07	4.8	0.09	4.9
Uluru	0.08	4.9	0.10	5.0	0.14	5.1
Wagga Wagga	0.09	5.0	0.12	5.1	0.16	5.3
Wangaratta	0.09	5.0	0.12	5.1	0.16	5.3
Whyalla	0.09	5.0	0.12	5.1	0.16	5.3
Wickepin	0.15	6.3	0.20	6.5	0.27	6.7
Wollongong	0.09	5.0	0.12	5.1	0.16	5.3
Wongan Hills	0.15	6.3	0.20	6.5	0.27	6.7
Woomera	0.08	4.9	0.10	5.0	0.14	5.1
Wyndham	0.09	5.8	0.12	5.9	0.16	6.1
Wyong	0.10	5.2	0.13	5.3	0.18	5.4
York	0.14	6.3	0.18	6.4	0.25	6.6

Table 1 continued. Summary of M<sub>Ww</sub> for Site Class B at locations in Table 3.2 of AS 1170.4, excluding islands.

a  $Z_{\rm 500}$  is Z for 500 yr return period given on Table 3.2 of AS1170.4.

b Weighted design magnitude,  $M_{Ww}$  for  $Z_{500}$ 

c  $Z_{1000}$  is Z for 1000 yr return period (AS1170.4) =  $1.3 \cdot Z_{500}$ 

d Weighted design magnitude,  $M_{Ww}$  for  $Z_{1000}$ 

e  $Z_{2500}$  is Z for 2500 yr return period (AS1170.4) =  $1.8 \cdot Z_{500}$ 

f Weighted design magnitude,  $M_{Ww}$  for  $Z_{2500}$ 

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