# Maximum Response Under Recorded and Principal Components of Ground Motion.

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#### SUMMARY:

A parametric study is conducted in order to investigate the influence of both the reference axes of ground motion and the seismic incident angle on structural response under three translational seismic components. Three discrete cases of seismic input are studied: i) the recorded components are applied along the structural axes; ii) the uncorrelated accelerograms are applied along the structural axes and iii) the uncorrelated accelerograms are applied along the principal axes of ground motion; that is the minor seismic component is not vertical while the major and intermediate components do not lie at horizontal plane. For each case the maximum response over all incident angles is computed as well as the response produced when the earthquake components are applied along the structural axes or the ground motion reference axes. The results show that the application of the recorded or uncorrelated earthquake components along the structural axes can significantly underestimate the response.

Keywords: Time history analysis; Maximum response; Principal earthquake components

## **1. INTRODUCTION**

In general, the seismic analysis is performed for the three translational seismic components. The recorded earthquake components can have any degree of correlation to each other. However, they can be rotated to the principal directions along which they are uncorrelated (Penzien and Watabe, 1975; Clough and Penzien, 1993). While it was initially considered that one principal direction of seismic motion is vertical (Penzien and Watabe, 1975) in more recent studies it has been found that a certain degree of correlation exists between the vertical component and the horizontal components of a ground motion (Hernandez and Lopez 2002, Lopez et al, 2006) and therefore the vertical direction is not always a principal direction.

Concerning the use of linear response history analysis modern seismic codes (UBC 1997; FEMA 356 2000; NEHRP (FEMA 450) 2003; EC8 2003; ASCE 41/06 2008) do not define clearly the orientation of the axes along which the accelerograms should be applied. It is common in engineering practice the accelerograms, correlated or uncorrelated, to be applied along the vertical and two horizontal, orthogonal directions which coincide with the structural axes i.e. the axes along which the vertical earthquake resisting structural elements are arranged in plan-view. It has been proved that the maximum response is a function of the seismic incident angle (Athanatopoulou, 2005; Athanatopoulou et al, 2005; Athanatopoulou and Avramidis, 2006) if the input accelerograms are applied along the principal axes of the ground motion. In this case the minor principal component is not vertical and the major and intermediate principal components lie in a plane which is not horizontal.

In the present paper a parametric study is conducted in order to investigate the influence of both the reference axes of seismic motion and the seismic incident angle on structural response within the context of Linear Response History Analysis. For this purpose a single storey R/C building is analysed under an ensemble of forty earthquakes. The time history analyses are performed for the following three discrete cases of seismic input: i) the recorded translational components are used as seismic input

along two horizontal and the vertical axes; ii) the uncorrelated accelerograms are applied along two horizontal and the vertical axes and iii) the uncorrelated accelerograms are applied along the principal axes of ground motion; that is the minor seismic component is not vertical while the major and intermediate components do not lie at horizontal plane. For each one of the above cases the maximum response corresponding to 37 angles of seismic incidence is computed ( $\theta$ =0°,10°,20°,...,360°). In addition the maximum response over all possible seismic incident angles is computed. The maximum response over all incident angles is compared with the corresponding one obtained by accelerograms applied along the structural axes. Furthermore the maximum response over all incident angles produced by case (i) and (ii) is compared with the corresponding one produced by case (iii).

#### 2. PRINCIPAL DIRECTIONS OF GROUND MOTION

The three recorded components of each earthquake  $\alpha_x(t)$ ,  $\alpha_y(t)$  and  $\alpha_z(t)$  can be rotated to the principal directions along which they are uncorrelated (Fig. 2.1). In order to determine the orientation of the principal axes the symmetric matrix  $\sigma$  needs to be generated as shown in Eqn. 2.1 (Penzien and Watabe, 1975; Clough and Penzien, 1993). In this equation  $\alpha_i(t)$  and  $\alpha_j(t)$  are the acceleration values at each time step and  $t_d$  the duration of the ground motion.

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}, \qquad \sigma_{ij} = \frac{1}{t_d} \int_0^{t_d} a_i(t) \cdot a_j(t) dt, \quad i, j = x, y, z$$
(2.1)

The recorded acceleration components  $\alpha_x(t)$ ,  $\alpha_y(t)$  and  $\alpha_z(t)$  along directions x, y, and z may be rotated to the principal components  $\alpha_I(t)$ ,  $\alpha_{II}(t)$  and  $\alpha_{III}(t)$  along the principal directions I, II, III (Fig. 2.1b) using the transformation of coordinates shown in Eqn. 2.2.  $\mathbf{\Phi}$  is the matrix whose columns are the three eigenvectors  $\mathbf{I}_I$ ,  $\mathbf{I}_{II}$  and  $\mathbf{I}_{III}$  of matrix  $\boldsymbol{\sigma}$  (Eqn 2.1) and superscript T denotes the transpose.



Figure 2.1. a) Recorded accelerograms along axes x, y, z and b) Uncorrelated accelerograms along principal axes of ground motion I, II, III.

#### **3. MAXIMUM RESPONSE UNDER THREE EARTHQUAKE COMPONENTS**

#### 3.1 Input axes of earthquake components

In order to investigate both the influence of the seismic input axes and the reference axes of ground motion on structural response the following three discrete cases are studied: i) Case REC $\theta$ SA.: The Recorded Earthquake Components  $\alpha_x(t)$  and  $\alpha_y(t)$  may form any angle  $\theta$  with the Structural Axes X and Y respectively, while accelerogram  $\alpha_z(t)$  is applied along the vertical axis Z (Fig. 3.1a), ii) Case PEC $\theta$ SA: The Principal Earthquake Components  $\alpha_I(t)$  and  $\alpha_{II}(t)$  are horizontal and may form any angle  $\theta$  with the Structural Axes X and Y respectively, while accelerogram  $\alpha_{III}(t)$  is applied along the vertical axis Z (Fig. 3.1b) and iii) Case PEC $\theta$ PA: The Principal Earthquake Components  $\alpha_I(t)$  and  $\alpha_{II}(t)$  lie in the plane formed by ground motion principal axes I and II and may form any angle  $\theta$  with the Principal Axes I and II of ground motion, while accelerogram  $\alpha_{III}(t)$  is applied along the principal axis III (Fig. 3.1c). In this case components  $\alpha_I(t)$  and  $\alpha_{II}(t)$  lie on a plane which is perpendicular to axis III.



Figure 3.1. Orientation of ground motion components for cases: a) REC@SA, b) PEC@SA and c) PEC@PA

#### 3.2 Maximum response in cases REC0SA and PEC0SA

For these cases REC0SA and PEC0SA one earthquake component remains vertical. The maximum value of a response parameter max*R* as well as the corresponding critical angle of seismic incidence  $\theta_{cr}$  can be calculated using Eqns. 3.1 and 3.2 (Athanatopoulou, 2005). For case REC0SA:  $R_{,0}(t)$  is the time history of a response parameter produced when the earthquake components  $a_x(t)$  and  $a_y(t)$  are applied along the structural axes X and Y respectively (Fig. 3.2a),  $R_{,90}(t)$  is the time history of a response parameter produced when the horizontal earthquake components  $a_x(t)$  and  $a_y(t)$  are rotated by an angle  $\theta=90^{\circ}$  in relation to structural axes X and Y (Fig. 3.2b) and  $R_{,z}(t)$  is produced when the vertical earthquake component  $a_z(t)$  is applied along axis Z (Fig. 3.2c). Obviously, for case PEC0SA the procedure for the determination of max*R* and  $\theta_{cr}$  is similar.

$$\max R = \max(\sqrt{R_{,0}^{2}(t) + R_{,0}^{2}(t)} + R_{,2}(t)) = \sqrt{R_{,0}^{2}(t_{cr}) + R_{,0}^{2}(t_{cr})} + R_{,2}(t_{cr})$$
(3.1)

$$\theta_{cr} = \tan^{-1} \left( \frac{R_{,90}(t_{cr})}{R_{,0}(t_{cr})} \right)$$
(3.2)



**Figure 3.2.** Orientation of ground motion components  $\alpha_x(t)$ ,  $\alpha_y(t)$  and  $\alpha_z(t)$  and corresponding response parameters a)  $R_{,0}(t)$ , b)  $R_{,90}(t)$ , c)  $R_{,z}(t)$  and d) maxR

#### 3.3 Maximum response in case PEC0PA

In case PEC0PA the minor principal earthquake component  $\alpha_{III}(t)$  is applied along the axis III which forms an angle  $\psi$  with the vertical axis Z. In this case the maximum value of a response parameter max*R* and the corresponding critical angle  $\theta_{cr}$  can be calculated using Eqns. 3.3 and 3.4.  $R_{P,0}(t)$  is the time history of a response parameter produced when the uncorrelated accelerograms  $\alpha_I(t)$  and  $\alpha_{II}(t)$  are applied along the principal axes I and II respectively (Fig. 3.3a),  $R_{P,90}(t)$  is the time history of a response parameter produced when the uncorrelated accelerograms  $\alpha_I(t)$  and  $\alpha_{II}(t)$  are rotated by 90° around principal axis III (Fig. 3.3b) and  $R_{III}(t)$  is produced when the uncorrelated accelerogram  $\alpha_{III}(t)$ is applied along axis III (Fig. 3.3c).

$$\max R = \max(\sqrt{R_{P,0}^{2}(t) + R_{P,90}^{2}(t)} + R_{III}(t)) = \sqrt{R_{P,0}^{2}(t_{cr}) + R_{P,90}^{2}(t_{cr})} + R_{III}(t_{cr})$$
(3.3)

$$\theta_{cr} = \tan^{-1} \left( \frac{R_{P,90}(t_{cr})}{R_{P,0}(t_{cr})} \right)$$
(3.4)



**Figure 3.3.** Orientation of ground motion components  $\alpha_{I}(t)$ ,  $\alpha_{II}(t)$  and  $\alpha_{III}(t)$  and corresponding response parameters a)  $R_{P,0}(t)$ , b)  $R_{P,90}(t)$ , c)  $R_{III}(t)$  and d) maxR

#### 4. INVESTIGATED BUILDING - GROUND MOTIONS

The investigated structural model considered in the present study represents a reinforced concrete single storey symmetric building (Fig. 4.1). The mass is concentrated in the joints shown in Fig. 4.1. The recorded accelerograms of the forty ground motions which were used in the analyses are presented in Table 4.1 and were obtained from the PEER strong motion database (No 1-33 and No 38-40) and from the CESMD database (No 34-37). The analyses are performed by using the SAP2000 computer program. The three recorded correlated components of each ground motion are rotated to the principal directions I, II, III and the corresponding principal uncorrelated components are computed. Angle  $\psi$  (Fig. 3.3) for the specific earthquakes varies between 1.0° and 51.6°. The relative and the exceedance frequency of the inclination angle  $\psi$  are shown in Fig. 4.2. The mean value of these angles is found to be 13.8° with a standard deviation of 8.8°. The most frequent case of inclination between the vertical and the principal axis III ranges between 5° and 10°.



**Figure 4.2**. a) Relative and b) Exceedance frequency for the inclination angle  $\psi$ .

No	Date	Earthquake	Ms	Station Name	*Site	t <sub>d</sub>	**D <sub>rup</sub>	PGA (g)		
		Name			Class	(sec)	(Km)	$a_x(t)$	$a_{\rm v}(t)$	$a_z(t)$
1	15/10/1979	El Centro	6.5	Superstition Mtn	D	28	24.6	0.11	0.2	0.08
2	28/06/1992	Landers	7.4	SilentValley.	В	55	51.7	0.05	0.04	0.03
3	28/06/1992	Landers	7.4	Twentynine Palms	С	50	41.4	0.08	0.06	0.04
4	28/06/1992	Landers	7.4	Amboy	D	50	69.2	0.11	0.15	0.08
5	18/10/1989	Loma Prieta	7.1	Point Bonita	В	40	88.6	0.07	0.07	0.03
6	18/10/1989	Loma Prieta	7.1	Piedmont Jr High	В	40	73.0	0.07	0.08	0.03
7	18/10/1989	Loma Prieta	7.1	SF-Pacific Heights	В	28	76.1	0.06	0.05	0.03
8	18/10/1989	Loma Prieta	7.1	SF - Rincon Hill	В	40	74.1	0.08	0.09	0.03
9	18/10/1989	Loma Prieta	7.1	GoldenGate Bridge	С	28	79.8	0.23	0.12	0.05
10	18/10/1989	Loma Prieta	7.1	Hollister	В	30	30.6	0.04	0.06	0.04
11	18/10/1989	Loma Prieta	7.1	Sierra Point	В	40	68.2	0.06	0.11	0.03
12	18/10/1989	Loma Prieta	7.1	Berkeley LBL	С	40	79.3	0.06	0.12	0.03
13	18/10/1989	Loma Prieta	7.1	Coyote Lake Dam	D	40	20.8	0.16	0.18	0.09
14	17/01/1994	Northridge	6.7	Mt Wilson	В	40	35.9	0.23	0.13	0.09
15	17/01/1994	Northridge	6.7	Antelope Buttes	В	28	46.9	0.05	0.07	0.03
16	17/01/1994	Northridge	6.7	Wonderland Ave	В	30	20.3	0.11	0.17	0.07
17	17/01/1994	Northridge	6.7	Wrightwood	В	60	64.7	0.06	0.04	0.02
18	17/01/1994	Northridge	6.7	Littlerock	В	40	46.6	0.03	0.06	0.03
19	17/01/1994	Northridge	6.7	San Gabriel	С	35	39.3	0.14	0.26	0.07
20	28/06/1992	Landers	7.4	Desert Hot Springs	D	50	21.8	0.17	0.15	0.12
21	18/10/1989	Loma Prieta	7.1	Diamond Heights	С	40	77.0	0.10	0.11	0.04
22	09/02/1971	San Fernando	6.6	Pasadena	С	29	31.7	0.09	0.11	0.10
23	09/02/1971	San Fernando	6.6	Pearblossom Pump	С	17	39.0	0.14	0.10	0.05
24	23/10/2011	Eastern Turkey	7.2	Bitlis	С	76	108.7	0.10	0.09	0.04
25	03/09/2010	New Zealand	7.0	Darfield	С	80	5.7	0.49	0.46	0.31
26	03/09/2010	New Zealand	7.0	Greendale	С	80	7.0	0.75	0.68	0.95
27	03/09/2010	New Zealand	7.0	Heathcote Valley	С	80	46.7	0.56	0.62	0.28
28	13/05/1995	Kozani, Greece	6.4	Florina	С	32	74.1	0.03	0.02	0.02
29	17/01/1994	Northridge	6.7	LA Fire Station	D	40	7.1	0.59	0.58	0.55
30	25/04/1992	Cape Mendocino	7.0	Cape Mendocino	С	30	7.0	1.50	1.04	0.52
31	17/05/1976	Gazli	7.3	Karakyr	С	16	5.5	0.61	1.72	0.26
32	17/08/1999	Kocaeli	7.8	Izmit	В	30	7.2	0.22	0.15	0.15
33	25/04/1992	Cape Mendocino	7.1	Petrolia	С	36	8.2	0.59	0.66	0.16
34	17/08/1999	Kocaeli	7.8	Duzce	D	27	15.4	0.31	0.36	0.19
35	17/08/1999	Kocaeli	7.8	Yarimca	D	35	4.8	0.35	0.27	0.23
36	23/12/1985	Nahanni, Canada	6.9	Site 1	С	21	9.6	1.10	0.98	2.09
37	28/06/1966	Parkfield	6.1	Temblor pre-1969	С	30	16.0	0.36	0.27	0.14
38	28/06/1966	Parkfield	6.1	Cholame	D	26	12.9	0.27	0.25	0.08
39	09/02/1971	San Fernando	6.6	Pacoima Dam	A	42	1.8	1.23	1.16	0.65
40	16/09/1978	Tabas, Iran	7.4	Tabas	В	33	2.1	0.84	0.85	0.69

Table 4.1. Ground motions.

\*Site Class according to FEMA356, \*\*D<sub>rup</sub>: Closest distance to Fault Rupture

# 5. INFLUENCE OF SEISMIC INCIDENT ANGLE

To investigate the influence of seismic incident angle on structural response the single storey symmetric R/C building shown in Fig. 4.1 is analysed for the three cases of orientation of the earthquake components: REC $\theta$ SA, PEC $\theta$ SA and PEC $\theta$ PA. The analyses are performed for 36 angles of seismic incidence  $\theta$  (i.e.  $\theta=0^{\circ}$ ,  $10^{\circ}$ ,  $20^{\circ}$ , ...,  $350^{\circ}$  (degrees)). In order to quantify the effect of seismic incident angle on structural response the ratio max $|R_{,\theta}(t)|/max|R_{,0}(t)|$  in respect to the incident angle  $\theta$  is plotted. Figs 5.1 and 5.2 show this ratio for response quantity  $\sigma_B(t)$  (normal stress) in column C1 bottom section and Figs. 5.3 and 5.4 for bending moment  $M_3(t)$  in beam BX11 start section. max $|R_{,\theta}(t)|$  is the maximum absolute value of the response parameter for incident angle  $\theta$  produced by each case under consideration and max $|R_{,0}(t)|$  is the maximum absolute value of the response parameter for location of the response parameter for the response parameter for

produced when the recorded earthquake components  $\alpha_x(t)$ ,  $\alpha_y(t)$  and  $\alpha_z(t)$  are applied along the structural axes. It must be noted that the response quantity  $\sigma_B(t)$  is calculated on the basis of response quantities N(t),  $M_2(t)$  and  $M_3(t)$  in column C1 bottom section.



**Figure 5.1.** Ratio max $|\sigma_B(t)|/\text{max}| \sigma_{B,0}(t)|$  vs angle  $\theta$  under earthquakes No 1 to 20 for a) REC $\theta$ SA, b) PEC $\theta$ SA and c) PEC $\theta$ PA.



**Figure 5.2.** Ratio max $|\sigma_B(t)|/\text{max}| \sigma_{B,0}(t)|$  vs angle  $\theta$  under earthquakes No 21 to 40 for a) REC $\theta$ SA, b) PEC $\theta$ SA and c) PEC $\theta$ PA.

It can be seen in Figs. 5.1 and 5.2 that the maximum absolute value of the response quantity  $\sigma_B(t)$  is affected by incident angle  $\theta$  for all three cases of application of the earthquake components. In Figs. 5.1a and 5.2a it can be seen that for REC $\theta$ SA the response parameter max $|\sigma_B(t)|$  attains its maximum value under earthquake No 30 for incident angle  $\theta=120^\circ$ . This maximum value is by 124.00% larger than the value produced for incident angle  $\theta=0^\circ$ . For PEC $\theta$ SA max $|\sigma_B(t)|$  attains its maximum value under the same earthquake for incident angle  $\theta=320^\circ$  and it is by 123.00% larger than max $|\sigma_{B,0}(t)|$  (Figs. 5.1b and 5.2b). For PEC $\theta$ SA max $|\sigma_B(t)|$  attains its maximum value under earthquake No 20 for incident angle  $\theta=260^\circ$  and it is by 129.00% larger than max $|\sigma_{B,0}(t)|$  (Figs. 5.1c and 5.2c).

From Fig. 5.1a it can be seen that for REC $\theta$ SA the critical angle for  $\sigma_B(t)$  under earthquake No 20 is  $\theta$ =80° while under earthquake No 3 is  $\theta$ =90°. Hence, different ground motions have different critical angles for the same response parameter. From Fig. 5.1b it can be seen that for PEC $\theta$ SA the critical angle under earthquake No 20 is  $\theta$ =320° and under earthquake No 3 is  $\theta$ =120° and from Fig. 5.1c it can be seen that for PEC $\theta$ SA the critical angle under earthquake No 3 is  $\theta$ =260° and under earthquake No 3 is  $\theta$ =100°. Therefore, the above conclusion is valid for all three cases of application of the earthquake components.

In Figs. 5.3a,b and 5.4a,b it can be seen for REC $\theta$ SA and PEC $\theta$ SA the ratio max $|R_{,\theta}(t)|/max|R_{,0}(t)|$  is stable for any incident angle  $\theta$ . This means that the specific response parameter is not affected by the orientation of the two horizontal translational components of ground motion because the response parameter M<sub>3</sub> under the horizontal components is zero due to symmetry. In Figs. 5.3c and 5.4c it can be seen that for PEC $\theta$ PA the orientation of the principal components  $\alpha_I(t)$  and  $\alpha_{II}(t)$  in respect to the principal axes I, II affects significantly the maximum absolute value of this response parameter. Bending moment  $M_3(t)$  attains its maximum absolute value under earthquake No 32 for incident angle

 $\theta$ =110° and it is by 187.00% larger than max| $M_{3,0}(t)$ | (Fig. 5.4c).

From Fig. 5.2c it can be seen that for PEC $\theta$ PA under earthquake No 30 the critical angle for max $|\sigma_B(t)|$  is  $\theta$ =120°, while under the same earthquake the critical angle for max $|M_3(t)|$  is  $\theta$ =240° (Fig. 5.4c). This means that the same earthquake components may lead to different critical angles for different response parameters in the same building. Moreover all these Figures clearly indicate that the seismic input along the structural axes does not produce conservative estimation of structural response.



**Figure 5.3.** Ratio max $|M_3(t)|/\max|M_{3,0}(t)|$  vs angle  $\theta$  under earthquakes No 1 to 20 for a) REC $\theta$ SA, b) PEC $\theta$ SA and c) PEC $\theta$ PA.



**Figure 5.4.** Ratio  $\max|M_3(t)|/\max|M_{3,0}(t)|$  vs angle  $\theta$  under earthquakes No 21 to 40 for a) REC $\theta$ SA, b) PEC $\theta$ SA and c) PEC $\theta$ PA.

#### 6. COMPARISON

#### **6.1 Individual ground motions**

In order to quantify the differences among the results produced by the aforementioned cases the relative variation rations (%)  $RV_{0,i}$ ,  $RV_{P.A,i}$ ,  $RV_{max,i}$  are presented in Figs. 6.1. 6.2 and 6.3 respectively. These ratios are defined by Eqns. 6.1-6.3. In these equations  $\max|R_{\theta cr,i}|$ , for i=1,2,3 is the peak absolute value of a response parameter which corresponds to angle  $\theta = \theta_{cr}$  for each of the three cases REC0SA, PEC0SA and PEC0PA respectively. The calculation of  $\max|R_{\theta cr,1}|$  and  $\max|R_{\theta cr,2}|$  is carried out with Eqn. 3.1 while the calculation of  $\max|R_{\theta cr,3}|$  with Eqn. 3.3.  $\max|R_{0,i}|$ , for i=1,2,3 is the peak absolute value of a response parameter which corresponds to incident angle  $\theta = \theta^{o}$  for each of the three cases.

$$RV_{0,i} = \frac{\max \left| R_{\theta_{cr,i}} \right| - \max \left| R_{0,i} \right|}{\max \left| R_{0,i} \right|} \cdot 100, \quad i = 1, 2, 3$$
(6.1)

$$RV_{P.A.,i} = \frac{\max \left| R_{\theta cr,3} \right| - \max \left| R_{,0,i} \right|}{\max \left| R_{,0,i} \right|} \cdot 100, \quad i = 1,2$$
(6.2)

$$RV_{max,i} = \frac{\max \left| R_{\theta cr,i} \right| - \max \left| R_{\theta cr,i} \right|}{\max \left| R_{\theta cr,i} \right|} \cdot 100, \quad i = 1,2$$

$$(6.3)$$

From Figure 6.1a it can be seen that for  $\sigma_B(t)$  the value of  $RV_{0,i}$  for a specific case of application of the earthquake components depends on the earthquake record. For example earthquake No 2 gives value of  $RV_{0,1}$  equal to 54%, while earthquake No 20 gives value of  $RV_{0,1}$  equal to 113%. From Figure 6.1b it can be seen that the same is valid for response quantity  $M_3(t)$  and relative variation  $RV_{0,3}$ . Since the value of  $M_3(t)$  for cases REC0SA and PEC0SA is not affected by incident angle  $\theta$ , for the specific response quantity  $RV_{0,1}$  and  $RV_{0,2}$  are equal to zero under all of the earthquakes used in the analyses.









From Figure 6.2a it can be seen that for  $\sigma_B(t)$  the ratio  $RV_{P,A,,1}$  varies between 0% and 129% and  $RV_{P,A,,2}$  between 1% and 92%. Also for  $M_3(t)$  in beam start BX11 ratio  $RV_{P,A,,1}$  varies between 0% and 187% and  $RV_{P,A,,2}$  between 1% and 264% (Figure 6.2b). This means that the maximum response for case PEC0PA, can be significantly greater than the response produced when the recorded or principal components of the same earthquake are applied along the structural axes X, Y and Z, which is the most common engineering practice.

From Figure 6.3a it can be seen that for  $\sigma_B(t)$  ratio  $RV_{max,1}$  varies from -7% to 8% and ratio  $RV_{max,2}$ 

varies from -10% to 13%. These ratios under a specific earthquake may take both positive and negative values. This means that under a specific earthquake the maximum value of  $\sigma_B(t)$  can be greater for any of the three cases of application of the earthquake components. For  $M_3(t)$  ratio  $RV_{max,1}$  varies from 1% to 187% and ratio  $RV_{max,2}$  varies from 1% to 264% (Figure 6.3b). It can be seen that for the specific response quantity these ratios take only positive values. This means that the critical value of  $M_3(t)$  under a specific earthquake is always greater for case PEC0PA.

#### 6.2 Comparison due to 3 or 7 earthquake records

Seismic code provisions (EC8, NEHRP, FEMA356 and ASCE 41/06) suggest that when three time history data sets are used as seismic input, the maximum value of each response parameter must be used for design, while in the case of seven or more time history data sets, the average value of each response parameter may be permitted to determine the design acceptability. In order to compare the three cases of application of the earthquake components REC0SA, PEC0SA and PEC0PA the values of the ratios  $RV_{0,i}$ ,  $RV_{P.A,i}$ ,  $RV_{max,i}$  corresponding to response parameters  $\sigma_B(t)$  and  $M_3(t)$  are calculated due to three or seven earthquakes. From the forty earthquake ground motions of Table 1, all possible 9,880 combinations of 3 records are considered. For each one of these 3 record combinations, the maximum values of the response quantities are determined for angles  $\theta=0^{\circ}$  and  $\theta=\theta_{cr}$ . Similarly from the 40 earthquake ground motions all possible 18,643,560 combinations of 7 records are considered. For each one of these 7 record combinations, the average values of the response quantities are determined for angles  $\theta=0^{\circ}$  and  $\theta=\theta_{cr}$ . For these response values the ratios  $RV_{0,i}$ ,  $RV_{P.A,i}$ ,  $RV_{max,i}$  are calculated. In Fig. 6.4 the minimum value, the average value minus the standard deviation, the average value, the average value plus the standard deviation and the maximum value of each ratio corresponding to all the above mentioned combinations are plotted.



**Figure 6.4.** Ratios  $RV_{0,i}$ ,  $RV_{P.A,i}$ ,  $RV_{max,i}$  a) for  $\sigma_B(t)$  when 3 records are considered, b) for  $\sigma_B(t)$  when 7 records are considered, c) for  $M_3(t)$  when 3 records are considered, d) for  $M_3(t)$  when 7 records are considered.

From Fig. 6.4a,b it can be seen that for  $\sigma_B(t)$  when 3 or 7 earthquake records are considered the mean values of  $RV_{0,1}$ ,  $RV_{0,2}$  and  $RV_{0,3}$  vary between 24 and 30%. These variations may take maximum values up to 129%. For response quantity  $M_3(t)$  the mean values of  $RV_{0,1}$  and  $RV_{0,2}$  are equal to zero (Fig. 6.4c,d). When 3 earthquake records are considered the mean value of  $RV_{0,3}$  is equal to 42% and when 7 records are considered it is equal to 37%. This variation takes maximum value of 190% (Fig. 6.4c,c). In Fig. 6.4a,b it can be seen that for  $\sigma_B(t)$  when 3 or 7 earthquake records are considered the mean values of  $RV_{P,A,1}$  and  $RV_{P,A,2}$  vary between 24 and 29% (Fig. 6.4b). For  $M_3(t)$  the mean values of these ratios vary between 35% and 42% (Fig. 6.4c,d). For the specific response quantities the mean value of  $RV_{P,A,1}$  is greater than  $RV_{P,A,2}$ . For response quantity  $\sigma_B(t)$  the mean values of ratios  $RV_{max,1}$  and  $RV_{max,2}$  are significantly smaller compared to the rest of the ratios  $RV_{0,i}$  and  $RV_{P,A,i}$ . When 3 or 7 earthquake records are considered the mean values of  $RV_{a,a,i}$ . When 3 or 7 earthquake records are considered the mean values of  $RV_{a,a,i}$  and  $RV_{max,2}$  take significant values. When 3 or 7 earthquake records are considered the mean values of ratios  $RV_{max,1}$  and  $RV_{max,2}$  vary between 35% and 42% (Fig. 6.4c,d).

### 7. CONCLUSIONS

Three cases of application of the earthquake components were presented: a) REC $\theta$ SA, b) PEC $\theta$ SA and c) PEC $\theta$ PA (Fig. 3.1). From the results of the present study the following conclusions were drawn:

- The selection of the recorded or corresponding principal earthquake components as well as the selection of the seismic input axes differentiates the critical angle and the corresponding maximum value of a response quantity.
- Considering each ground motion individually the maximum response for each case of application of the earthquake components which corresponds to the critical angle can be up to 190% greater than the response produced for angle  $\theta=0^{\circ}$ . Considering 3 ground motions this percentage may take values up to 190% while when 7 records are considered it may take values up to 150%.
- Considering each ground motion individually the critical response for case PEC0PA which corresponds to the critical angle can be up to 264% greater than the response produced when the recorded or principal components of the same earthquake are applied along the structural axes. When 3 ground motions are considered this percentage takes maximum value equal to 264% while for 7 records it takes maximum value equal to 154%. For all the possible earthquake combinations the mean value of this percentage is also significant and ranges between 24 and 42%. The calculation of the critical response for case PEC0PA is the most rational, since for this case the inclination of the principal direction with regard to the vertical axis is taken into account.
- In general, the critical response can be greater for any of the three cases REC0SA, PEC0SA or PEC0PA. Considering each ground motion individually the difference in the critical response between the three cases can be up to 13%, considering 3 ground motions also up to 13% and considering 7 ground motions up to 11%. For response parameters whose value depends only on the vertical earthquake component the critical response is greater for case PEC0PA. In this case considering each ground motion individually the difference can be up to 260%, considering 3 ground motions up to 154%.

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