# Source Process and Constitutive Relations of the 2011 Tohoku Earthquake Inferred from Near-Field Strong-Motion Data

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#### **SUMMARY:**

The spatial and temporal distribution of the stress on the fault plane of the  $M_w9.0~2011$  Tohoku, Japan, earthquake is calculated from kinematic inversion results. We have investigated the rupture process of the earthquake using the long-period strong-ground motion data. We calculate the stress changes using the finite-difference method to infer the constitutive relations and dynamic source parameters. The static stress drop at the large slip area along the trench, which is identified as an asperity, is about 25 MPa. Strength excess at the deeper margin of the asperity is about 3 MPa, although strength excesses on the most of the fault are generally small. The slip-weakening distances (Dc) obtained from the constitutive relations around the asperity are about 7 to 10 m. Dc of the asperity is unclear from the constitutive relations due to the complexity, but Dc obtained from peak slip velocities indicate about 15 to 22 m.

Keywords: 2011 Tohoku earthquake, Source process, constitutive relations

## **1. INTRODUCTION**

On 11 March 2011, the Tohoku earthquake of  $M_W9.0$  (Japan Meteorological Agency) struck the Tohoku region, Japan, and generated a huge tsunami. The 2011 Tohoku earthquake (the 2011 Off the Pacific Coast of Tohoku earthquake named by JMA) is the first M9 earthquake that is recorded by a lot of near-field strong-motion stations. During historical M9-class earthquakes (e.g., 2004 Sumatra, 1964 Alaska, 1960 Chile), few strong-motion seismometers near the source region were operated. Many kinematic source models are proposed using the strong-ground motion records for this earthquake.

A constitutive relation is a key element of dynamic descriptions of the seismic source which are based on models that satisfy the elastodynamics equation. Some dynamic source parameters are estimated from the constitutive relations. Many researchers investigated the constitutive relations of natural earthquakes (e.g, Ide and Takeo, 1997; Zhang *et al.*, 2003; Hok and Fukuyama, 2011). The constitutive relations are estimated with combining the slip evolution and stress evolution at each location on a fault plane. A spatio-temporal stress change can be estimated by solving the elastodynamic equation with a kinematic slip history estimated by an inversion analysis as boundary conditions.

The 2011 Tohoku earthquake is the first M9 event that is proposed many detailed kinematic source models. In this study, we will use a 3-D finite-difference method (FDM) proposed by Pitarka (1999) under appropriate boundary conditions to directly solve the elastodynamic equations and calculate the stress spatiotemporal distribution over a fault plane from the kinematic inversion result estimated by Yoshida *et al.* (2011). We apply this approach to study the dynamic source parameters of the 2011 Tohoku earthquake.



# **2. SOURCE MODEL**

We have investigated the rupture process of the 2011 Tohoku earthquake by the multi-time-window linear waveform inversion method using the long-period strong-ground motion data. Detail of the analysis is denoted in Yoshida *et al.* (2011).

We used the strong-motion data obtained from 37 stations of F-net and KiK-net (NIED) (Aoi *et al.*, 2004; Okada *et al.*, 2004), and 1 station of Hokkaido University (MYR, Sasatani *et al.*, 2002) for the inversion (Fig. 2.1). These stations are located on relatively hard rock. The data were windowed for 300 sec, starting at P-wave arrival time, and band-pass-filtered between periods of 200 s to 20 s (0.005–0.05 Hz) for waveform inversion.

We assumed a single planar fault model of 468 km long in strike and 228 km wide in dip. We assume N193°E and 10° as the strike and dip angles, respectively, referring to the JMA CMT solution. The rupture starting point is located at the hypocenter determined by JMA: 38.103°N, 142.861°E, 23.7 km. Theoretical Green's functions are calculated using the discrete wavenumber method (Bouchon, 1981) and the Reflection/Transmission coefficient matrix method (Kennett and Kerry, 1979) using a stratified medium. The velocity structure model proposed by Wu *et al.* (2008, 2009) for the waveform inversion of the 1979 and 2005 Miyagi-oki earthquake was used for calculating the theoretical Green's functions. We use multi-time-window linear waveform inversion procedure (e.g., Hartzell and Heaton, 1983) in which the moment-release distribution is discretized in both space and time. For discretization in space, we divide the fault plane into 39 in the strike direction and into 19 in the down-dip direction (making a total of 741 subfaults with an area of 12 km x 12 km). We use 8 smoothed ramp functions with duration of 16 seconds separated by 8 seconds' interval to represent the slip history of each subfault.

The seismic moment of this earthquake was estimated to be  $4.3 \times 10^{22}$  Nm (Mw 9.0). The inverted slip distribution shows a large slip area with a maximum slip of about 47 m which is located on the shallower part of the fault plane (Fig. 2.1). The shallow large slip area is identified as an asperity with the criteria of Somerville *et al.* (1999) (Fig. 2.2A). The moment distribution is dominated by the asperity (moment released area) in the shallower part of the fault plane. However, the peak moment-rate distribution indicates that high peak moment-rate areas extend to the deeper part of the fault plane (Fig. 2.2B). The feature of the ground motion suggested from the record section is well represented by the peak moment rate distributions, rather than the slip distributions. This source model is successfully validated by tsunami simulation (Petukhin *et al.*, 2012).



Figure 2.1. Projection of the final slip distribution on the map. The star and squares indicate the rupture starting point and the strong-motion stations that are used for the inversion. Dotted line shows plate boundaries (Bird, 2003).



**Figure 2.2.** Upper: Distributions of coseismic slip obtained for the 2011 Tohoku earthquake. The star indicates the rupture starting point. The purple dashed box indicates the asperity identified with the criteria of Somerville *et al.* (1999). Lower: Distributions of peak moment rate together with the slip rate function at each subfault.

# **3. CALCULATION METHOD FOR STRESS CHANGES**

To determinate stress changes on the fault plane from the slip spatiotemporal distribution obtained by kinematic inversion, we used a 3-D finite difference method (FDM) presented by Pitarka (1999) to solve the following elastodynamic equations with appropriate boundary condition. In this method, the fourth-order staggered grid expressions for discrete spatial differential operators and the second-order approximate for time derivatives are used to model wave propagation in 3-D elastic media.

In the FDM coordinate system, we adopt x axis is the direction N103°E, y axis is the direction N193°E, and z axis is direction down (Fig. 3.1). During the calculation, we used four boundary conditions. The first set of boundary conditions is the free surface condition, which is the zero-stress formulation (Graves, 1996). The second set of boundary conditions is the continuity condition at each layer interface included in the medium, that is, the continuity of stresses and displacements. The third set of boundary conditions is an absorbing boundary condition (Clayton and Engquist, 1977) for the side and bottom boundaries that attenuates elastic waves reflected back from these artificial side and bottom boundaries of the model space. The fourth set of boundary conditions is for the points on the fault plane. For these points, we used the kinematic inversion results as the fault plane boundary conditions. The kinematic inversion results gave us the slip distributions on each subfault. Each subfault is like a point source with arbitrary focal mechanism; the moment tensor components can be represented as an equivalent distribution of body force couples centred at the grid point corresponding to each subfault. We used the moment tensor source formulation using stress components proposed by Pitarka (1999).

The target event has an oblique fault plane. Stress is calculated under the Cartesian coordinates of the finite-differential calculation. Stresses along dip and fault normal directions are taken by transformation of the stress tensor calculated by FDM on the Cartesian coordinates (Fig. 3.1).



Figure 3.1. Definition of the fault orientation parameters and slip direction.

# 4. CONSTITUTIVE RELATIONS AND DYNAMIC PARAMETERS OF THE 2011 TOHOKU EARTHQUAKE

The kinematic inversion indicates that the fault rupture process during this earthquake is very complex. In this study, we used the kinematic inversion result (Yoshida *et al.*, 2011) to analyse the stress distribution on the fault. The projection of the fault plane onto the surface is 468 km in length and 224.5 km in width. The fault is divided into 39 x 19 subfaults, and each subfault occupies a 12 km x 12 km area on the fault plane. The crustal structure is the same as the one used in the Green's function calculation for the kinematic inversion (Wu *et al.*, 2008, 2009). In order to represent the fault geometry whose dip angle is 10°, the grid sizes used in our FDM calculation were chosen as  $\Delta x = 11.8$  km,  $\Delta y = 12$  km, and  $\Delta z = 2.08$  km. The top of the fault plane for the FD calculation is aligned to the surface, although the top of the fault plane for the kinematic inversion is the depth of 8.1 km, because some direct observations of the sea bottom investigate that the faulting clearly reached to the sea bottom (Ito *et al.*, 2011). This modification affects the stress field near the top of the fault. The static stress drop on the fault plane used in the original kinematic inversion exceeds 40 MPa on the large slip area on the shallower part of the plane, but differences in most area of the fault between the two models are very little.



**Figure 4.1.** Snapshots of the slip rate  $v_d$  and shear stress  $\tau_{dn}$  in the dip direction. The history of  $v_d$  is determined by the inversion analysis, and that of  $\tau_{dn}$  is calculated by FDM. The star indicates the rupture starting point. In the FDM calculation, we used a oblique fault plane whose top depth is aligned to the surface.

Figure 4.1 shows the spatial and temporal distribution of the slip rate and shear stress on the fault plane. Although three components of the slip vector and six components of the stress tensor are determined everywhere on the fault plane, we show only shear stress along the dip direction  $\tau_{dn}$  and slip rate  $v_d$  since the rupture of this earthquake is dominated by dip slip. Stress increases of less than 5 MPa appears around a rupture front area.

Figure 4.2 shows the spatial distribution of static stress changes (static stress drop). The distribution of the stress drop is very complex, and is similar to the distribution of the peak moment rate. The maximum stress drop in  $\tau_{dn}$  is about 25 MPa at the region of large slip area. This maximum stress drop is relatively high comparing to those for other plate-boundary earthquake, e.g. about 10 MPa (Yagi, 2004), but it seems to be reasonable due to large slip of about 47 m at the asperity. Moderate stress drop (~ 10 MPa) of the deeper part of the initial rupture point corresponds to the high peak moment rate area on the deeper part of the fault plane. Small stress increases of less than 10 MPa appear in wide area of the deeper part of the fault plane. Fig. 4.3 shows the spatial distribution of dynamic stress drop. The spatial distribution of the dynamic stress drop is very similar with that of the static stress drop.

Figure 4.4 shows the distribution of strength excess on the fault. The deeper margin of the asperity has rather high strength (about 3 MPa). This relatively high strength excess suggests to cause that the rupture progression suspended before the rupture of the asperity. The stress increase just before the rupture of the asperity (at the frame of 40 s in Fig. 4.1) is recognized. Distribution of the high strength area corresponds to the low peak moment rate, small total slip area (Figs. 2.2 and 4.4). Variety of the strength suggests causing the complex slip distribution and the rupture progression.



Figure 4.2. Spatial distribution of stress change (static stress drop) for the 2011 Tohoku earthquake. The star indicates the rupture starting point. Depth of the top of the fault plane is aligned to the surface.



**Figure 4.3.** Spatial distribution of dynamic stress drop for the 2011 Tohoku earthquake. The star indicates the rupture starting point. Depth of the top of the fault plane is aligned to the surface.



Figure 4.4. Spatial distribution of strength excess for the 2011 Tohoku earthquake. The star indicates the rupture starting point. Depth of the top of the fault plane is aligned to the surface.

The distribution of slip and stress can be used to give the constitutive relation at each grid point in space. Figure 4.5 shows the relations plotted at each time step up to 200 s after the rupture initiation. At almost point, stress decreases with increasing slip (slip weakening) as seen in frictional experiments of rocks. Obviously, the slip weakening rate  $(du_d/d\tau_{dn})$  in the large slip point (A) is significantly smaller than those of the other parts.

The relations between slip velocity and stress (right panel of Fig. 4.5) indicate that velocity weakening behaviour is not unclear. This figure also shows that slips repeat twice (B, C, D, F) or three-time (E), except for the point A. Slip at these points stopped (slip velocity dropped to zero) once, and then restarted. These repetitions of the slip correspond to the three-stage rupture obtained from the kinematic inversion (Yoshida *et al.*, 2011).

Another important parameter in the slip-weakening model is critical slip-weakening distance Dc. Dc is defined as the slip displacement required for the local strength in the breakdown zone behind the rupture front to degrade to a residual friction stress level. The constitutive relations indicate the slip-weakening distances (Dc) are about 7 m for point C, and about 5 m for point D. However, these slip-weakening distances from the constitutive relations are upper bound because of the limitation due to the resolution of the kinematic inversion. For point A, B, E and F, Dc are not identified clearly due to the complex behaviour of the constitutive relations.

Mikumo *et al.* (2003) presented an approach to estimate the critical slip-weakening distance Dc based on the relation between the breakdown time of shear stress,  $T_b$ , and the time of peak slip velocity,  $T_{pv}$ . Figure 4.6 show the spatial distribution of Dc estimated based on peak slip velocity. The Dc for point C and D are 9.6 m and 6.9 m, respectively. At these points both Dc obtained from the constitutive relations and  $T_{pv}$  are similar results. The Dc obtained from  $T_{pv}$  on the asperity (points A and B) are 20.4 m and 14.8 m, respectively. These values are very large, and are limited by the inversion resolution, so that the validity of these values should be checked.



**Figure 4.5.** (Left)The constitutive relation between slip and stress on the fault plane. (Right) The relation between slip rate and stress. Each trace is the function calculated at the corresponding location in the bottom figure of slip distribution. Depth of the top of the fault plane is aligned to the surface.



Figure 4.6. Spatial distribution of the estimated Dc values on the fault of the 2011 Tohoku earthquake obtained with  $T_{pv}$  (Mikumo *et al.*, 2003). Depth of the top of the fault plane is aligned to the surface.

### **5. SUMMARY**

Stress evolution is estimated from the kinematic inversion results on the entire plane of the 2011 Tohoku earthquake fault. The relations between stress and slip, and between stress and slip rate on the fault, show that most locations on the fault plane followed the slip-weakening model during the

rupture. The static stress drop at the large slip area along the trench is about 25 MPa. Dynamic strength excess are similar to the static stress drop. Strength excess at the bottom of the large slip areas is about 3 MPa, although strength excesses on the most of the fault are generally small. The constitutive relations indicated that the slip weakening rate in the large slip point is significantly smaller than those of the other parts. The slip-weakening distances (Dc) around the asperity obtained from the constitutive relations are about 7 to 10 m. Dc of the asperity is not clear from the constitutive relations, but Dc obtained from the peak slip velocity indicates about 15 to 22 m.

#### AKCNOWLEDGEMENT

We thank the National Research Institute for Earth Science Disaster Prevention and Hokkaido University to provide the strong-motion data.

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