

# Seismic Base Isolation Analysis for the Control of Structural Nonlinear Vibration

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## SUMMARY:

In this paper, structural nonlinear vibration control using base isolation system is studied. A nonlinear structure model with base isolation devices is developed to analyze the control effect of base isolated structure. A bi-linear hysteresis model is incorporated into the structure to mimic nonlinear behaviors. Simulation is made on this nonlinear structure with base isolation system. It can be concluded from the simulation that for nonlinear structure with base isolation if linear structural parameters are used to calculate the related control parameters, the control effect may be worse than the case without control. Therefore, in order to design the based isolation device for nonlinear structure, nonlinear response is recommended to calculate the related control parameters. Moreover, an active control method based on the negative stiffness idea is proposed to control the nonlinear base isolated structure, which is verified by the simulation.

*Keywords: structural nonlinear vibration; base isolation; negative stiffness*

## 1. INTRODUCTION

Base isolation is a widely used technique to suppress the vibration of structure under earthquake excitation (Lin and Zhou, 2002; Ni et al., 2006; Sun et al., 2006; Liu and Li, 2006; Shang and Qi, 2012). It can shift the fundamental frequency of the structure from the dominant period of the earthquake, and also increase the damping of the whole system. With the development of civil engineering, the structure becomes more and more flexible and higher. Even with the base isolation system, the superstructure may still exhibit the nonlinear behaviors. Therefore, it is of theoretical and practical value to study the structural nonlinear vibration with base isolation devices. Suy et al. (2007) studied a base isolation system composed of linear laminated rubber bearings and viscous dampers and nonlinear friction elements. The dynamic performance of the system was analyzed in both nonlinear transient and steady-state analyses. Kikuchi et al. (2008) investigated the seismic response of yielding isolated structures. The study considered both viscously damped and hysteretically damped isolation systems. Choi et al. (2008) investigated the applicability of the MR damper-based smart passive control system to the seismic protection of base isolated building structures with nonlinear isolation systems such as friction pendulum bearings and lead-rubber bearings. Ye et al. (2010) proposed a harmonic balancing method and a gradient method to solve two-degree-of-freedom system of LRB base-isolated building structure where the nonlinearity of both LRB isolation system and superstructure were taken into

consideration. Du et al. (2010) introduced an analytical method of nonlinear seismic response of high-rise isolated structure. Almost all the above studies focused on the nonlinearity of the base isolation part, while few of them considered the nonlinearity of the superstructure.

In this paper the structural nonlinear vibration is studied. A bi-linear hysteresis model is used to build the nonlinearity of the superstructure. Then simulation is conducted on this nonlinear model with base isolation devices. The analysis and conclusions are presented based on the numerical results.

## 2. THE NONLINEAR VIBRATION MODEL OF BASE ISOLATED STRUCTURE

The nth-story nonlinear shear-type structure with base isolation devices is shown in Fig. 2.1. Under the horizontal earthquake input  $\ddot{x}_g$ , the motion of equation of the super structure based on interstory drift is:

$$M\ddot{x}(t) + C\dot{x}(t) + F_n^e(x(t), \dot{x}(t)) = -Mh\ddot{x}_g(t) \quad (2.1)$$

where  $x(t) = [x_1, x_2, \dots, x_n]^T$  is the interstory drift of all levels.  $M_{(n \times n)}$  and  $C_{(n \times n)}$  are the mass and damping matrix of the structure, respectively.  $\eta_{(n \times 1)}$  is the earthquake input vector.

The hysteresis force of the nth story can be represented as:

$$F_n^e(x(t), \dot{x}(t)) = K_e x(t) + H_h f_h(x(t), \dot{x}(t)) \quad (2.2)$$

where  $K_{e(n \times n)}$  is the linear stiffness matrix of the nth story.  $H_{h(r \times 1)}$  the location matrix of hysteresis force.  $f_{h(r \times 1)}$  is the nonlinear hysteresis vector of the nth story.

The Eqn. 2.1 can be written as a state space model:

$$\dot{Z} = AZ - B_h f_h(Z) + G\ddot{x}_g \quad (2.3)$$

where  $Z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$  is the state vector.  $A = \begin{bmatrix} 0 & I \\ -M^{-1}K_e & -M^{-1}C \end{bmatrix}$  is the linear system matrix of the

structure.  $B_h = \begin{bmatrix} 0 \\ M^{-1}H_h \end{bmatrix}$  and  $G = \begin{bmatrix} 0 \\ -\eta \end{bmatrix}$ .

The mass  $m_i$ , damping coefficient  $c_i$  and interstory stiffness  $k_i$  of each story are shown in Table 2.1.

The first three frequencies of the linear structure are 1.09、2.86 and 4.20Hz, respectively. The corresponding modal damping ratios are 2.00%、5.67% and 7.7%. The hysteresis model of restoring force is chosen to be a bi-linear model.

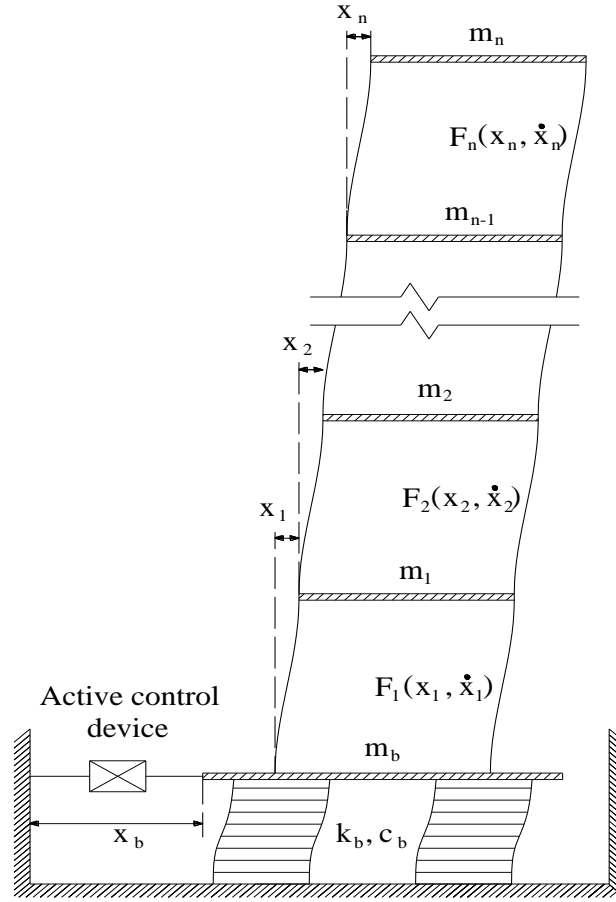


Figure 2.1. Structural nonlinear vibration model with base isolation devices

Table 2.1. Structural parameters of the 3-story nonlinear structural model

Story number	$m_i (\times 10^5 \text{ kg})$	$k_i (\times 10^5 \text{ kN/m})$	$c_i (\times 10^2 \text{ kN} \cdot \text{s/m})$
1	345.6	3.524	9.986
2	345.6	3.379	9.547
3	345.6	3.064	9.383

Base on the above derivation, the mass matrix  $\mathbf{M}$ , the linear stiffness matrix  $\mathbf{K}_e$  and the damping matrix  $\mathbf{C}$  can be written as:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ m_2 & m_2 & 0 \\ m_3 & m_3 & m_3 \end{bmatrix}, \quad \mathbf{K}_e = \begin{bmatrix} \alpha_1 k_1 & -\alpha_2 k_2 & 0 \\ 0 & \alpha_2 k_2 & -\alpha_3 k_3 \\ 0 & 0 & \alpha_3 k_3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_1 & -c_2 & 0 \\ 0 & c_2 & -c_3 \\ 0 & 0 & c_3 \end{bmatrix} \quad (2.4)$$

The bi-linear model is described by the Bouc-Wen hysteresis model, which is shown as:

$$F_{ni} = k_{ei}x_i(t) + f_{hi} = \alpha_i k_i x_i(t) + (1 - \alpha) k_i D_{yi} z_i \quad (2.5)$$

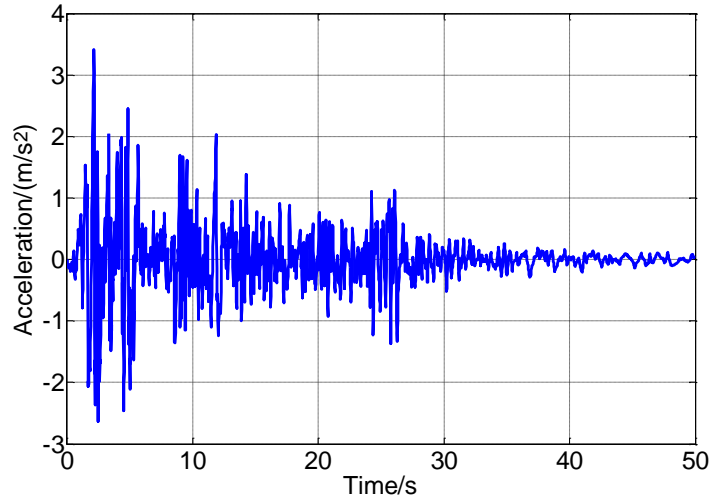
$$\dot{z}_i = D_{yi}^{-1} \left( A_i \dot{x}_i - b_i |\dot{x}_i| |z_i|^{n_i-1} z_i - g_i \dot{x}_i |z_i|^{n_i} \right) \quad (2.6)$$

where  $z_i$  is the dimensionless variable of the hysteresis displacement, the value of which is between  $\pm 1$ . The other parameters of this model are  $A_i = 1.0$ ,  $\beta_i = \gamma_i = 1.0$ ,  $n_i = 95$ . The coefficient  $\alpha_i$  represents the ratio between the post-yield stiffness and pre-yield stiffness, which is 0.1 for each story. The yielding displacement of each story is  $D_{yi} = 1.93, 1.76$  and  $1.47\text{cm}$ , respectively.

The location matrix  $\mathbf{H}_h$  and earthquake input vector  $\eta$  are

$$\mathbf{H}_h = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \eta = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (2.7)$$

In this paper, the earthquake input is the El Centro earthquake wave, which is shown in Fig. 2.2.



**Figure 2.2.** El Centro earthquake wave

### 3. SIMULATION ANALYSIS OF NONLINEAR BASE ISOLATED STRUCTURE

#### 3.1. Analysis of Linear Case

At first, we consider the simulation analysis of linear structure. The stiffness of the base layer is  $6.4841 \times 10^7 \text{ N/m}$  according to the first frequency of the structure. The damping coefficient of the base layer is changed from  $0 \text{ N.s/m}$  to  $4 \times 10^7 \text{ N.s/m}$ . Fig. 3.1. shows the simulation results between the structural response and the damping coefficient. Fig. 3.2 shows the simulation results between the response of base layers and the damping coefficient.

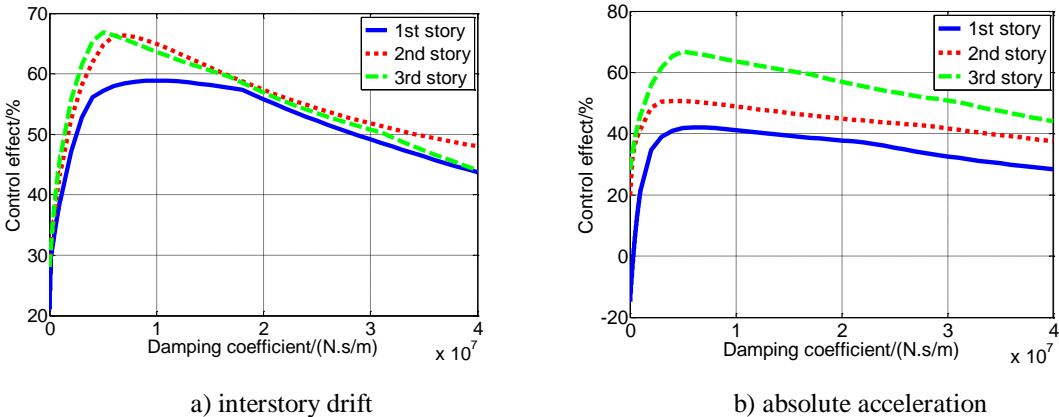


Figure 3.1. Structural response with base isolation device

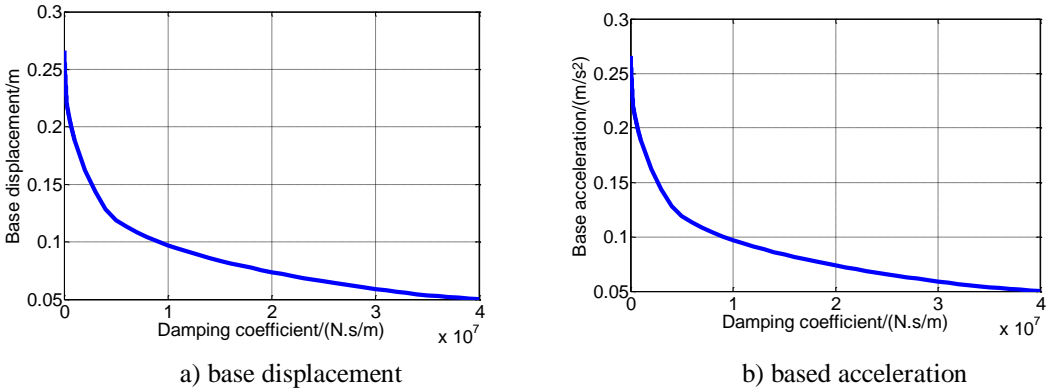


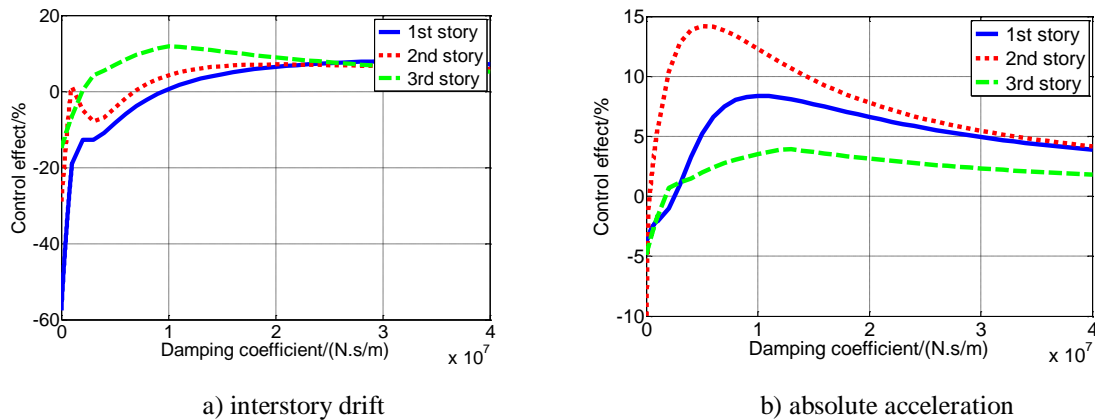
Figure 3.2. Response of the base layer

From the simulation results, it can be concluded that there exists the optimal value of the damping coefficient. In this case, when the damping coefficient is about  $5 \times 10^6 \text{ N.s/m}$ , the control effect of displacement will be 50%-70% and the control effect of acceleration will be 40%-70%. However, under the optimal case, the base displacement or acceleration may be excessive large, which will lead to the damage of the base.

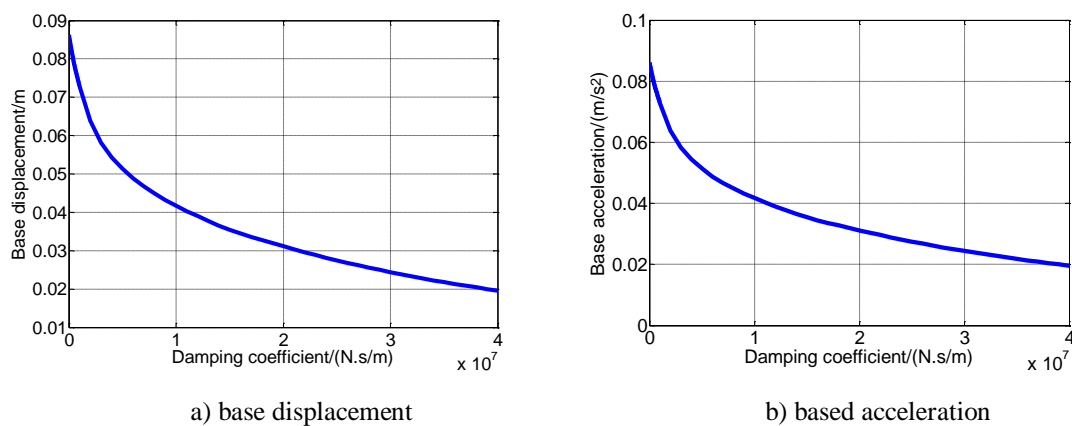
### 3.2. Analysis of Nonlinear Case

#### 3.2.1. Simulation based on linear response

For the nonlinear base isolated structure, we firstly use the linear structural parameters to design the base isolation system. Therefore, the stiffness of the isolation layer is still  $6.4841e+007\text{N.m}$  based on the linear analysis. The damping coefficient of the base is changed from  $0\text{ N.s/m}$  to  $4e7\text{N.s/m}$ . Fig. 3.3. shows the simulation results between the structural response and the damping coefficient. Fig. 3.4. shows the simulation results between the response of base layers and the damping coefficient.



**Figure 3.3.** Structural response with base isolation device

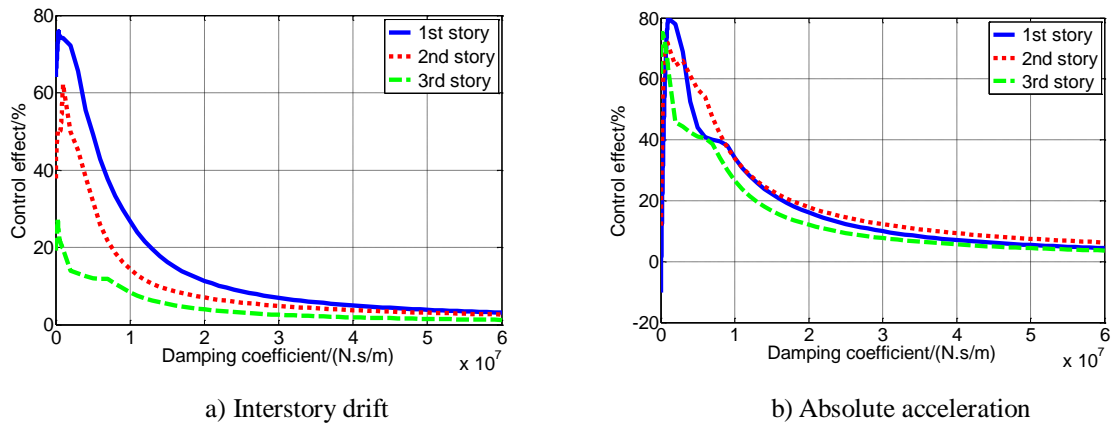


**Figure 3.4.** Response of the base layer

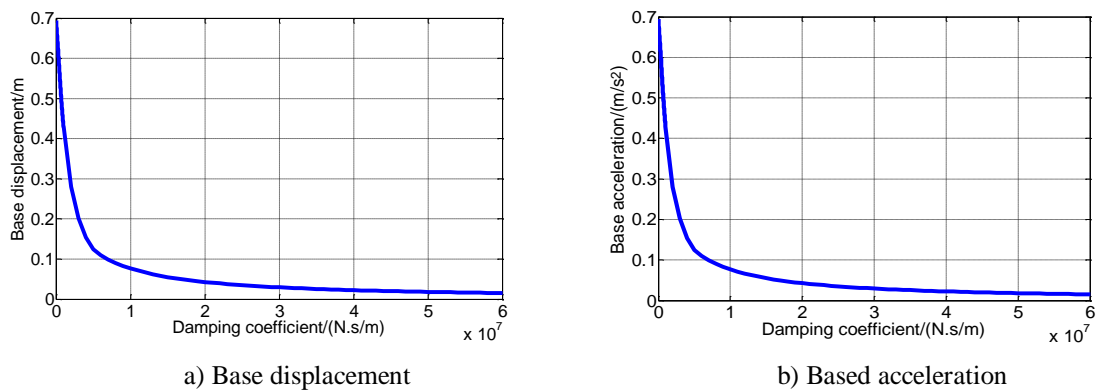
From the above results, we can see that if linear structural parameters are used to calculate the related control parameters of nonlinear base isolated structure, the control effect is not very good. Close to the optimal damping coefficient, the control effect is only 10%-15%. Moreover, for some damping coefficient, even negative control effect can be achieved, which means that the control effect with base isolation is even worse than that without base isolation. Therefore, for nonlinear base structure design, it is not suggested to use the linear structural parameters.

### 3.2.2. Simulation based on nonlinear response

In order to improve the above simulation results, the nonlinear response will be used to design the base isolation system. Base on the analysis of nonlinear structural response under earthquake input, the dominant frequency of the structure is 0.33 Hz. Using this dominant frequency, the stiffness of the isolation layer is  $4.9117e+006\text{N.m}$ . The damping coefficient of the base is changed from 0 N.s/m to  $4e7\text{N.s/m}$ . Fig. 3.5. shows the simulation results between the structural response and the damping coefficient. Fig. 3.6. shows the simulation results between the response of base layers and the damping coefficient.



**Figure 3.5.** Structural response with base isolation device



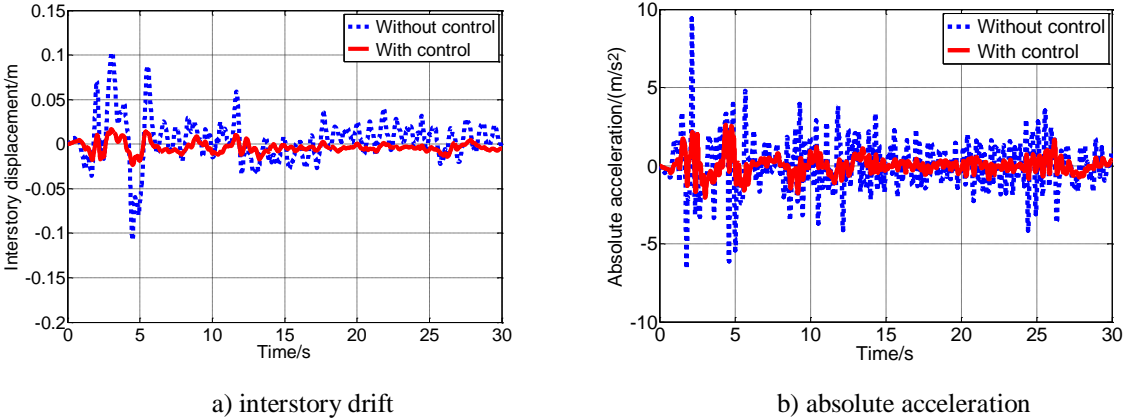
**Figure 3.6.** Response of the base layer

From the simulation results, it can be observed that the control effect of the based isolation structure is more satisfactory based on the nonlinear structure response than the case using linear structural parameters.

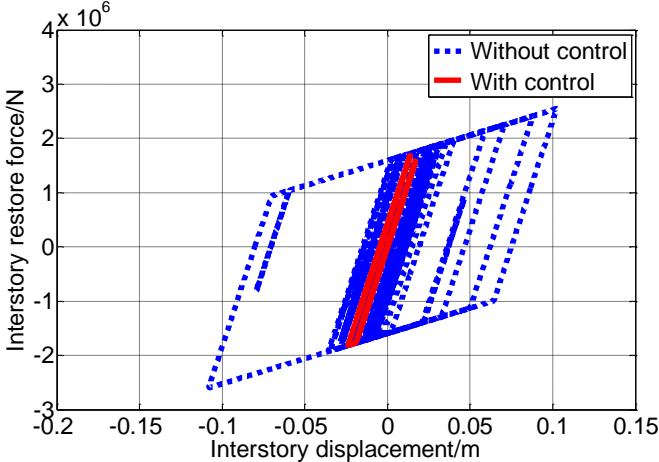
### 3.3. Active Control Based on Linear Structural Parameters

From the previous section, it can be concluded that when the passive control is considered for the nonlinear structure analysis with the base isolation system, the related nonlinear response should be used

to design the base isolation system. However, usually the nonlinear response is not easy to obtain unless there is some online system identification system which will further increase the complexity of the system and also raise the issue of convergence. Therefore, in the following we will design an active controller to compensate the nonlinear response, while the base isolation system is still designed according to the linear structural parameters. Due to the frequency difference of linear and nonlinear structure, we will use active control methodology to design the controller. Because of the stiffness degradation of the nonlinear system, the negative stiffness needs to be considered in the active control algorithms. Since the commonly used semi-active control devices can not provide the negative stiffness, it is suggested to use active control devices in this case. The control input is chosen to be the displacement and velocity of the base layer. Then the control method is to design a suitable displacement feedback gain to compensate the loss of stiffness and a suitable velocity feedback gain to increase the damping. It can be computed that the designed displacement feedback gain is  $-59929300\text{N/m}$  and the designed velocity feedback gain is  $1\text{e}6\text{N.s/m}$ . Fig. 3.7. and Fig. 3.8. shows the control results with the active control device. It can be observed that using the negative stiffness idea we can achieve a satisfactory control result.



**Figure 3.7.** Structural response of the 1<sup>st</sup> floor with base isolation device



**Figure 3.8.** Control results of hysteresis curve of the 1<sup>st</sup> floor



## 4. CONCLUSIONS

In this paper, we mainly analyze the control effect of a nonlinear structure with the base isolation system. A bi-linear hysteresis model is used to simulate the material nonlinearity of the structure. The simulation is conducted on the nonlinear structure with base isolation devices. The conclusions are drawn as following:

- 1) For the nonlinear structure with base isolation system, if linear structural parameters are considered for the base isolation system, the control effect may be unacceptable. In some cases, it is even worse than the case without base isolation system.
- 2) If the base isolation system is designed based on the nonlinear response, the control effect of both displacement and acceleration will be better than the case based on linear structural parameters. Therefore, if possible it is recommended to use nonlinear response related parameters to design the nonlinear structure with base isolation system.
- 3) Using the concept of negative stiffness, active or semi-active device can be used to compensate the nonlinear response of the structure.

## ACKNOWLEDGEMENT

This work was supported in part by the National Science Foundation of China under Award No. 50908033 and the Fundamental Research Funds for the Central Universities under Award No. DUT11RC(3)01.

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