Small Strain Shear Modulus of High and Low Plasticity Clays and Silts

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SUMMARY:
Many researchers have devoted considerable effort towards developing analytical techniques for evaluating the seismic response of soft soil deposits. However, the accuracy and reliability of seismic response analyses are highly dependent on the characterization of the dynamic properties of the soil. This paper investigates the influence of various soil parameters on the small strain shear modulus. The results are based on experimental data collected from thirteen different studies and 404 tests. The database shows that the small strain shear modulus is proportional to the void ratio, the mean effective confining pressure, and a new parameter called the relative activity. Relative activity is defined as the plasticity index over the liquid limit, and describes the change in the small strain shear modulus with soil type better than the plasticity index alone. The effects of overconsolidation ratio were found to be negligible.

Keywords: Shear modulus, relative activity, clays, silts

1. INTRODUCTION

Site response is very important in urban areas. The amount of settlement, displacement, and other failure criteria allowed for urban settings is significantly reduced due to the increased risks associated with high density, tall, closely packed structures. In addition, many large, densely populated urban areas are built at the mouth of rivers over soft soil deposits. Therefore, analytical techniques for evaluating the seismic response of soft soil deposits are essential for determining the overall safety of structures in urban areas. The seismic response of soil is highly dependent on its dynamic properties, such as the small strain shear modulus (G_{max}) and how the shear stiffness (G) and damping (D) change with shear strain (γ).

The authors collected dynamic test data from many different researchers in order to determine a simplified empirical equation for the G_{max} of cohesive soils. Much research has already been done in this area, however, the goal of this paper was to create an equation that uses as few parameters as possible and can be used for a wide variety of cohesive soil types and states. This equation can then be used for a preliminary estimate of the seismic site response. The following sections describe the soils and test types that form the database for the shear modulus at small strains, as well as the results and conclusions that were drawn from it.

2. DATABASE

Small strain shear modulus test data was collected from eleven different studies (Kokusho et al. 1982; Athanasopoulos 1993; D'Elia and Lanzo 1996; Cavallaro et al. 2000; Zhang et al. 2005; Lanzo and Pagliaroli 2006; Okur and Ansal 2007; Kallioglou et al. 2008; Yamada et al. 2008; Lanzo et al. 2009; Tika et al. 2010). The data is a mix of mostly 'undisturbed' with some reconstituted samples tested in resonant column apparatus as well as wide strain cyclic triaxial, cyclic simple shear, and cyclic torsional shear apparatus. All small strain shear modulus measurements were made at shear strains of $\gamma = 10^{-4}$ % or less. The database contains a total of 372 tests conducted on 140 different soils with USCS classifications of ML, MH, CL, and CH. Table 2.1 presents the maximum and minimum values for seven different soil and state properties and the total number of tests that reported those properties. The properties given along with the small strain shear modulus (G_{max}) are mean effective confining pressure (σ'_m), overconsolidation ratio (*OCR*), natural water content (w_n), initial void ratio (e_0), liquid limit (*LL*), plasticity index (*PI*), and clay fraction (*CF*, percent weight of fines with grain size < 2μm). Figure 2.1a is the data plotted on Casagrande's plasticity chart, and Figure 2.1b shows e_0 versus σ'_m. It is apparent that the dataset contains a good distribution of low and high plasticity clays and silts with a large range of initial void ratio and mean effective confining pressure.

Table 2.1 Soil properties of collected small strain shear modulus tests

	σ'_m (kPa)	OCR	w_n	e_0	LL	PI	CF (%)	G_{max} (MPa)
Total #	372	237	372	372	372	372	300	372
Max	1115	23	142	3.86	157	111	69	286
Min	13	1	12	0.20	21	2	1	1.5

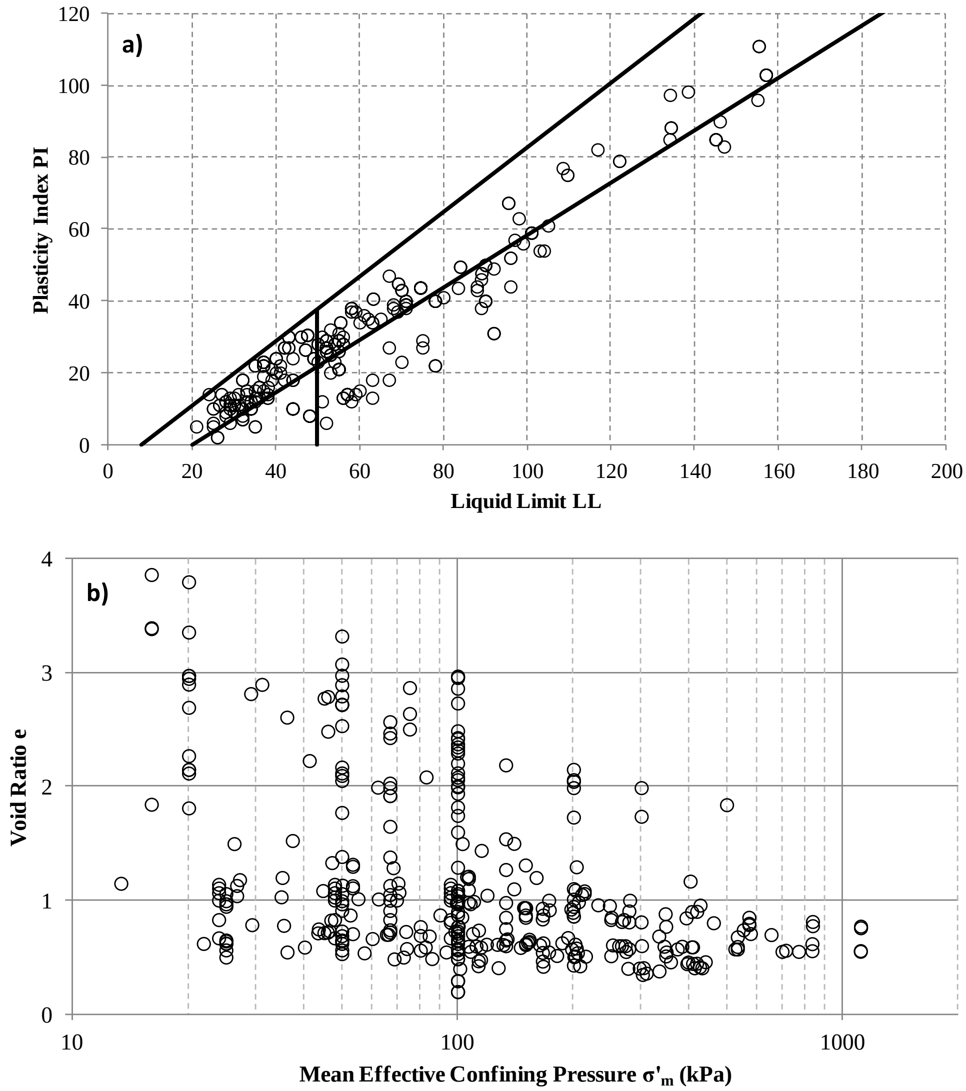

Figure 2.1. Soil (a) and state (b) parameters of collected G_{max} data

3. PREVIOUS OBSERVED TRENDS

A thorough review of the literature revealed many different parameters that are thought to influence the small strain stiffness of cohesive soils. The seven most studied parameters are summarized in Table 3.1. An upward facing arrow indicates that as the parameter in the left column increases, G_{max} increases, a downward facing arrow indicates that G_{max} decreases, and NE means that the parameter has a negligible effect on G_{max}. The number of arrows indicates the relative influence of that parameter on G_{max}. The right side of Table 3.1 shows the cross correlations between parameters. For example, as *OCR* increases, studies have shown that the influence of the mean effective confining pressure on G_{max} increases.

Table 3.1. Effect and relative importance of parameters that control G_{max} of cohesive soils

Increasing Parameter	G_{max}	Cross Correlations	
		OCR	*PI*
σ'_m	↑↑↑	↑	
e_0	↓↓↓		
t_g	↑↑	↑	↑
OCR	↑		↑
f	↑		
PI	NE		
N	NE		

Numerous studies have found that the mean effective confining pressure and void ratio have the greatest effect on the small strain shear modulus (e.g. Kim and Novack 1981; Jamiolkowski et al. 1991; Vucetic and Dobry 1991; Kagawa 1992; Darendeli 2001; Teachavorasinskun et al. 2002; Kallioglou et al. 2008; Lanzo et al. 2009). The confinement time/geologic age (t_g) of the soil is thought to be the next most important parameter (Afifi and Richart 1973; Anderson and Stokoe 1978; Kim and Novack 1981; Kokusho et al. 1982; Kagawa 1992; Darendeli 2001; Kallioglou et al. 2008). The effect of confinement time on G_{max} was not considered in this study because it was not available for most of the data collected.

The overconsolidation ratio of the soil has only a small effect on G_{max} (Hardin and Drnevich 1972; Vucetic and Dobry 1991; Rampello and Viggiani 1995; Darendeli 2001). Darendeli (2001) found that the frequency/strain rate (*f*) of the test has a small effect on G_{max}, but the number of shearing cycles (*N*) has a negligible effect due to the fact that the strain amplitude is small. There is considerable debate over the effect of plasticity index on the small strain shear modulus. Several studies have shown that G_{max} increases with increasing *PI* as a function of the OCR (Hardin and Drnevich 1972; Kim and Novack 1981; Vucetic and Dobry 1991), while others show G_{max} decreasing with increasing *PI* (Kagawa 1995; Yamada et al. 2008; Kallioglou et al. 2008) or that *PI* has a negligible effect (Okur and Ansal 2007). The reason for this apparent discrepancy will be shown in section 4.

4. RESULTS

A non-linear regression analysis was conducted to determine an empirical prediction equation to estimate the small strain shear modulus with as few parameters as possible. Since many previous authors have shown that the mean effective confining pressure and void ratio have the greatest effect on the small strain shear modulus, Eqn. 4.1 was first proposed using just these two parameters.

$$\frac{G_{max}}{\sigma_{ref}} = G_b \left(\frac{\sigma'_m}{\sigma_{ref}}\right)^n \left(\frac{1}{e_0}\right)^m \tag{4.1}$$

Where G_b = 200 to 500 with an average value of 350 and a standard deviation of 110, n = 0.5, m = 1.3, and σ_{ref} is a reference stress in the same units as the mean effective stress and G_{max}. The predicted response versus the measured G_{max} for the collected database is good, with an R^2 = 0.80 for G_b = 350. The standard deviation of the natural logarithm (σ_{ln}) of the ratio of the predicted G_{max} to the measured G_{max} (LN($G_{max,predicted}/G_{max,measured}$)) is 0.323.

Pestana and Salvati (2006) found that the G_{max} of most sands could also be described by Eqn. 4.1 using the same values for n and m, and by varying G_b from 400 to 800 with an average value of 600. This lead to the conclusion that soil type must affect the value of G_b. To investigate the effect of soil type on G_b, 32 small strain shear modulus tests on clean Toyura, Quiou, and Ticino sands (Jamiolkowski et al. 1991; Lo Presti et al. 1997) were added to the database, bringing the total number of tests to 404. The G_b for each soil was then determined by Eqn. 4.2 and non-linear regression analyses were conducted to determine the relation between G_b and LL, PI, plastic limit (PL), activity (A), and CF.

$$G_b = \frac{G_{max,measured}}{\left(\frac{\sigma'_m}{\sigma_{ref}}\right)^n \left(\frac{1}{e_0}\right)^m} \tag{4.2}$$

The results for PI are plotted in Figure 4.1. It is evident that as PI increases a small correlation exists predicting that G_b decreases. However, there is much scatter, especially at small values of PI. The reason for the discrepancy between studies over the effect of PI on G_{max} is probably due to this large scatter and the fact that many studies only look at narrow ranges of PI. The R^2 values of the best fit curves and the standard deviations of the natural logarithm (σ_{ln}) of the ratio of the predicted G_b using the best fit curves to the measured G_b given by Eqn 4.2 (LN($G_{b,bestfitcurve}/G_{b,Eqn4.2}$)) for all five soil type parameters are given in Table 4.1. As is evident, PI shows a better correlation with G_b than the others, but all five have low R^2 values.

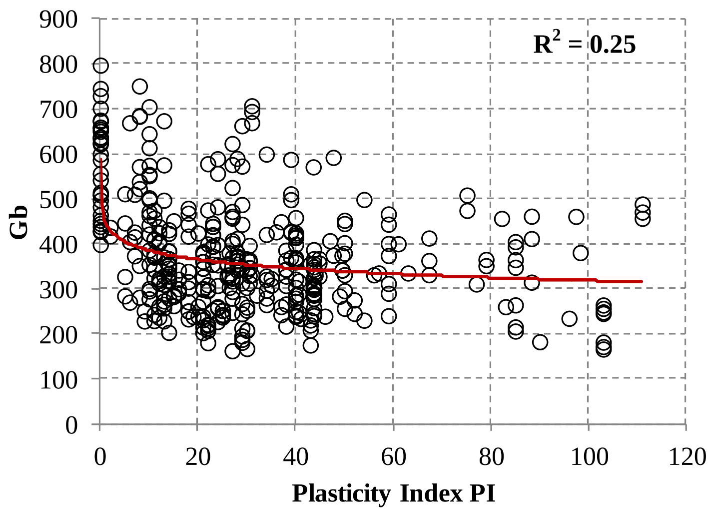

Figure 4.1. Correlation between G_b and plasticity index

A new parameter, called the relative activity (RA), was developed in order to better define the affect of soil type on parameter G_b and ultimately the effect of soil type on the small strain shear modulus. The relative activity of a soil is defined as its plasticity index normalized by its liquid limit (Eqn. 4.3). Physically, the relative activity describes the fraction of the liquid limit that the soil behaves plastically. Therefore, two soils with different plasticity indices or liquid limits could have the same relative activity.

$$RA = \frac{PI}{LL} = \frac{LL - PL}{LL} \tag{4.3}$$

From Figure 4.2 and Table 4.1 it is evident that relative activity does a better job of describing the change of G_b due to soil type than the five previously mentioned parameters. While the correlation between G_b and *RA* is still low, with R^2 = 0.35 and σ_{ln} = 0.253, this is a significant improvement over using just *PI* while at the same time it does not require any additional information.

Table 4.1. R^2 and σ_{ln} values for correlations between G_b and soil type parameters

Parameter	R^2	σ_{ln}
Relative Activity	0.35	0.253
Plasticity Index	0.25	0.285
Clay Fraction	0.24	0.291
Liquid Limit	0.22	0.292
Activity	0.21	0.310
Plastic Limit	0.20	0.297

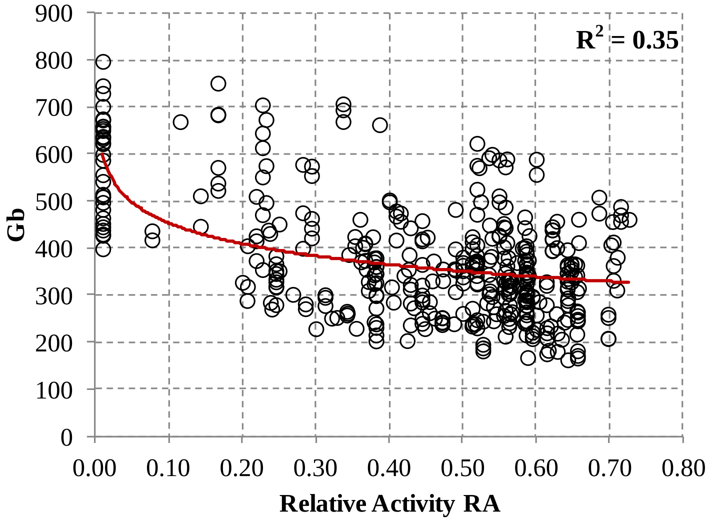

Figure 4.2. Correlation between G_b and relative activity

Accounting for soil type and using the correlation between G_b and *RA*, Eqn. 4.1 becomes:

$$\frac{G_{max}}{\sigma_{ref}} = \frac{G_b}{(1 + t * RA^k)}\left(\frac{\sigma'_m}{\sigma_{ref}}\right)^n \left(\frac{1}{e_0}\right)^m \tag{4.4}$$

Where t = 1.2, k = 0.5, G_b = 630, and all other variables are as previously defined. Figure 4.3a shows the predicted G_{max} using Eqn. 4.4 versus the measured G_{max}. Equation 4.4 has an R^2 = 0.89 and σ_{ln} = 0.275, which is a significant improvement over Eqn. 4.1. Figures 4.3b and 4.3c show the natural logarithm of the ratio of the predicted G_{max} using Eqn. 4.4 to the measured G_{max} versus OCR, and the measured G_{max} respectively. From Figure 4.3b it is evident that the data does not support any correlation between OCR and G_{max}. In addition, Figure 4.3c shows that Eqn. 4.4 predicts equally well for low and high values of G_{max}. All of the values plot between -0.7 and 0.7, which means that the data is within a factor of ½ to 2. A σ_{ln} = 0.275 means that 68.2% of the time Eqn. 4.4 will predict G_{max} to within a ratio of 0.76 to 1.31 of the correct value, and 95.6% of the time to within a ratio of 0.58 to 1.73. This is an acceptable level of uncertainty since G_{max} cannot be determined more accurately in the laboratory than plus or minus 20%. The additional uncertainty in the prediction is likely due to sample disturbance and the fact that the data does not consider aging effects (t_g), which can increase G_{max} by up to 100%.

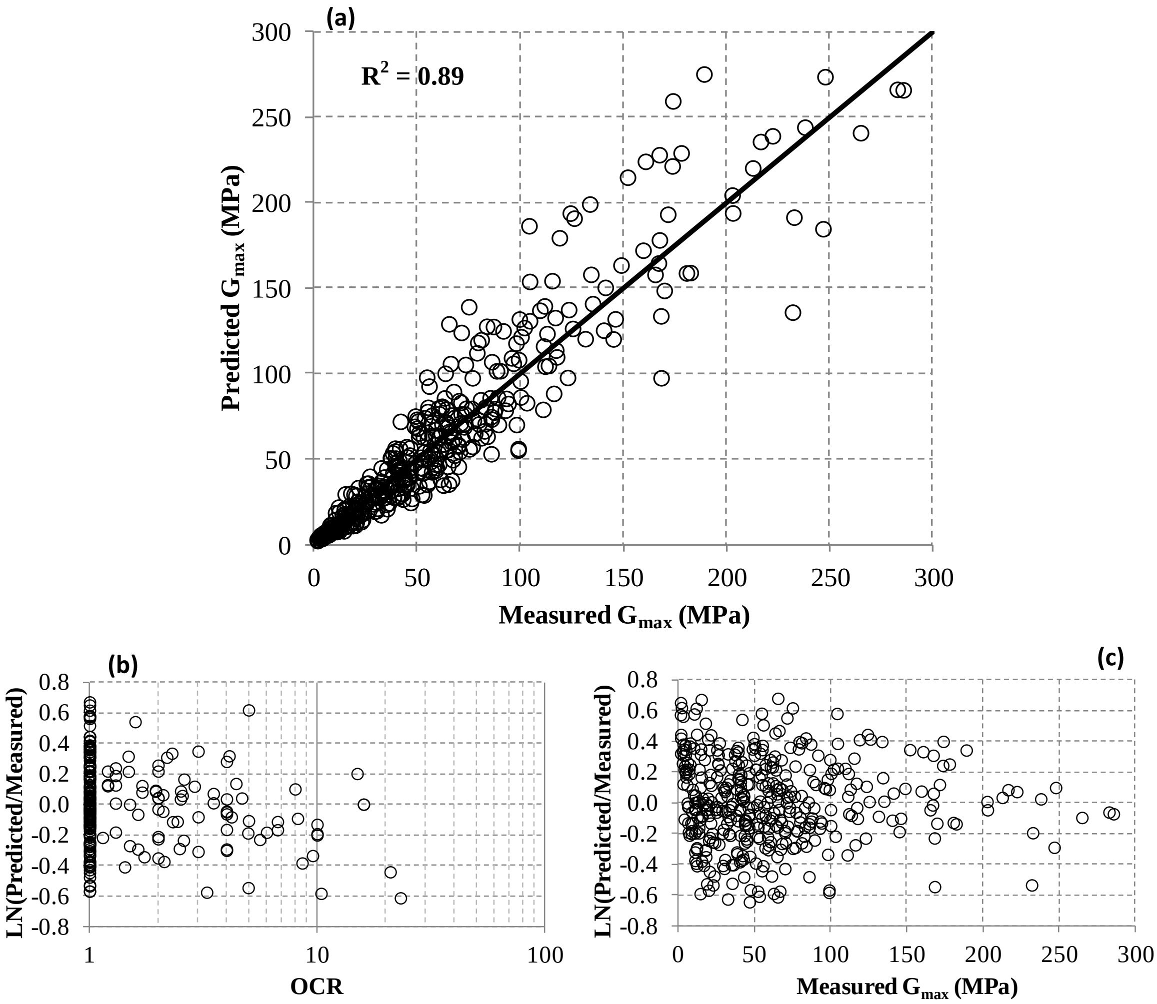

Figure 4.3. a) Predicted G_{max} using Eqn 4.4 versus the measured G_{max}. The natural logarithm of the predicted G_{max} divided by the measured G_{max} versus b) overconsolidation ratio and c) the measured G_{max}.

5. CONCLUSION

Site response is critical for earthquake engineering, and since many cities are built on soft soil deposits, it is crucial to understand the dynamic properties of these soils. Equation 4.4 is a simple, robust equation that can be used to predict the small strain shear modulus of cohesive soils with only three parameters, all of which are easily determined or estimated. The newly introduced parameter, relative activity, is a better indicator of the soil type than the plasticity index because it normalizes the *PI* by the liquid limit. The addition of the relative activity parameter increases the predictive power of Eqn. 4.4 from an R^2 value of 0.80 to 0.89, and reduces the standard deviation of the natural logarithm of the ratio of the predicted G_{max} to the measured G_{max} from 0.323 to 0.275. Relative activity also provides a smooth transition between the prediction of the small strain shear modulus for cohesive soils and cohesionless soils. Equation 4.4 could therefore be used to predict the small strain shear modulus for clean sands as well as high plasticity clays. Further research is being conducted to determine relationships for the shear modulus at medium and large strains, as well as the damping ratio.

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