# Influence of the Vibro Stone Column Reinforcement on the Seismic Bearing Capacity of a Surface Shallow Footing

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#### SUMMARY:

In difficult soil conditions, engineers sometimes choose to combine reinforced soil and spread footings instead of deep foundations. Among the available techniques for soil reinforcement, vibro stone columns are frequently used to reduce the liquefaction risk, improve the soil properties and improve the overall seismic performance. While there are different models available to evaluate the bearing capacity of reinforced soils under static conditions, its evaluation under seismic conditions have been much less investigated. In this communication, a new approach to estimating the bearing capacity of reinforced soils under seismic conditions is proposed. It is based on limit equilibrium theory, pseudo-static and pseudo-dynamic concepts and a specialized method for estimating reinforced soil properties. The method is validated for the pseudo-static case and the results of an extensive parametric study on the dimensions of the reinforced zone are also presented.

Keywords: vibro replacement, bearing capacity, Priebe, surface footing

## **1. INTRODUCTION**

For regions with moderate to high seismic activity, the evaluation of the bearing capacity of shallow footings under dynamic conditions is a very important task. Indeed, although it is recognized that the bearing capacity may differ under dynamic and static conditions, there are currently no specific recommendations for addressing these distinctions in Canadian codes and standards (CAN/CSA, 2006; NRC, 2005). The AASHTO (2010) recommends accounting for the degradation of geotechnical resistance for some types of soil but does not specify a specific method to evaluate the degradation. The International Building Code (Day, 2006) permits an increase in allowable bearing capacity of 33% under earthquake loading for the design of footings. However, several researchers have shown that the ultimate bearing capacity of soil, also known as the geotechnical resistance, is reduced in the case of dynamic loading (Choudhury and Subba Rao, 2005; 2006; Dormieux and Pecker, 1995; Fishman et al., 2003; Paolucci and Pecker, 1997; Pecker, 1996; Richards et al., 1993). To design a footing for seismic loading, a pseudo-static inclined and off-centered load is generally considered (Fraser Bransby, 2001). The limits of the applicability of this method are relatively unknown and, therefore, much research has recently been conducted on estimating soil bearing capacity under complex combined loads (Bienen et al., 2006; Cassidy et al., 2002; Cassidy et al., 2004a; Cassidy et al., 2004b). This research has led to the development of "failure envelopes" concept representing the three-dimensional plastic limit of soil in a load space H (horizontal load) M (moment) and V (vertical load). Most of these envelope curves are still experimental and accurately represent the behavior of the soil being studied for a pseudo-static plan loading.

In difficult soil conditions, and for economic reasons, engineers are sometimes choosing to combine reinforced soil and spread footings instead of deep foundations (Hussin, 2006). The main objective is primarily to build a reinforcing element into the soil. Among the available techniques for soil reinforcement, vibroreplacement with stone columns is well understood and can lead to significantly

improved mechanical soil properties (Kirsch, 2008). These improved soil properties must be adequately evaluated in order to adopt an efficient design for the spread footings. Vibro stone columns are also frequently used to reduce the liquefaction risk (Adalier and Elgamal, 2004; Hussin, 2006) and improve the overall seismic performance, however much uncertainty remains in the geotechnical and structural design of these foundations elements built on soils reinforced by vibro stone columns.

This problem becomes more particularly acute in Eastern Canada since the seismic loading criteria have been significantly increased in the 2005 version of the building codes (NRC, 2005). When designing a surface footing, the first step is to estimate the geotechnical properties of the soil with in situ testing, lab testing and empirical relationships such as Peck's. Sometimes a reinforcement of the soil is needed and in such a case, the modified properties are determined either by in situ testing or by using mechanical models such as Priebe's. The third and final step is to compute the bearing capacity with a plasticity based method. This paper focuses on the last step, and aims at proposing a practical procedure to evaluate the static and dynamic bearing capacity of shallow foundations on a vibro stone reinforced soil. This contribution includes a validation for the pseudo-static case and a parametric study on the plane dimensions of the zone to be reinforced (width and depth) in order to reach the optimum efficiency in terms of cost and increase in bearing capacity

# 2. LITTERATURE REVIEW

# 2.1. Evaluation of the Dynamic Bearing Capacity

## 2.1.1 Limit equilibrium based method with equivalent pseudo-static loads

There are currently no specific recommendations or guidelines to determine bearing capacity under dynamic conditions in the codes and standards of Canada (CAN/CSA, 2006; NRC, 2005). In order to estimate the bearing capacity for a seismic loading, a combined pseudo-static loading is applied to the foundation (H, M, V), and plasticity based methods for static conditions are then used (AASHTO, 2010). However, it should be noted that under dynamic conditions, the bearing capacity may be less than (especially in saturated soil) or greater than (in unsaturated soil) the bearing capacity under static conditions (AASHTO, 2010). In most cases, the design for a seismic load is actually carried out with equivalent static conditions. However, the duration of vibrations and the number of cycles generated by earthquakes are relatively short and therefore the risk of soil failure may not be significant (AASHTO, 2010) and the main problem becomes the occurrence of permanent displacements and/or rotations of the foundation leading to a redistribution of moments and shear forces in the structure (AASHTO, 2010). This case would no longer be part of footing design, but its proper evaluation requires a soil structure interaction analysis. The liquefaction potential of the soil must also be considered in the design of shallow foundations (AASHTO, 2010; CAN/CSA, 2006; NRC, 2005).

The seismic loading on a structure acts as a horizontal load H and an overturning moment M at the foundation level. The overturning moment applied to the footing is equivalent to an off-centered vertical load V. For the seismic design at the ULS, S6-06 guidelines allows an off-centered load with an offset up to 0.4B (CAN/CSA, 2006). If this condition is satisfied, the footing dimensions are established by considering a uniform pressure distribution on the reduced footing dimensions B' and L' as defined by Meyerhof. The center of this surface is equivalent to the point of application of the off-centered vertical load. The uniform pressure due to factored loads must be less than the factored ultimate geotechnical strength ( $\phi$ ',  $q_{ult}$ ). The S6-06 and the Canadian Foundation Engineering Manual (CFEM) standards offer similar equations for determining the capacity  $q_{ult}$  (CGS, 2006). This approach will be referred to as the CFEM method later in the text.

#### 2.1.2 Pseudo-dynamic approach

For some cases (specially for-saturated soils) the bearing capacity may decrease under seismic conditions (AASHTO, 2010). This degradation is a complex phenomenon and is partly due to the presence of inertial forces, straining and pore pressure effects in the soil. Several researchers have worked on determining "dynamic" bearing capacity factors from the Coulomb failure mechanism,

among them Richards et al. (1993) and Choudhury and Subba Rao (2006). These methods use dynamic/static bearing capacity factor ratios ( $N_{\gamma E}/N_{\gamma}$ , for example). These authors emphasize that their approach is probably conservative but it does allow for the development of safer design procedures for areas subjected to seismic hazards. For non-cohesive unsaturated soils, the usual pseudo-static methods are sufficiently safe in most cases. For cohesive soils or saturated sand the risk of degradation of soil properties should be taken into account (AASHTO, 2010).

The dynamic/static ratio, presented in Fig 2.1, depends on the horizontal and vertical acceleration coefficients of the earthquake ( $k_h$  and  $k_v$ , respectively). As illustrated in Fig 2.1, the bearing capacity factors decrease rapidly with increasing tan $\theta$  where tan $\theta = k_h/(1-k_v)$ . Richards et al. state that for a horizontal acceleration coefficient  $k_h = 0.25$ , the ultimate bearing capacity is divided by 3.



The advantage of this type of method is the simplicity of its implementation, which allows its application to be considered in combination with conventional methods described in standards and design guidelines. However, an experimental validation remains to be carried out. Numerous other studies have achieved results very similar to those obtained with Richards' method (Choudhury and Subba Rao, 2005, 2006; Dormieux and Pecker, 1995; Subba Rao and Choudhury, 2005).

#### 2.2 Vibro Reinforced Soil Parameters Estimation with Priebe's Method

Priebe's method consists in the calculation of an improvement factor '*n*' (Priebe, 1995) to evaluate the modified properties. This mechanical model is now a German Standard (DIN 4017) and is commonly used for the design of shallow foundation resting on a reinforced medium. The author specifies 3 different improvement ratios:  $n_0$ ,  $n_1$  and  $n_2$ . The first one,  $n_0$ , is the "basic improvement factor" and does not account for some hypotheses. The second one ( $n_1$ ) takes into account the column compressibility. The last one ( $n_2$ ) adds the consideration of the overburden. These improvement factors are all function of the area replacement ratio  $A_c/A$ , where  $A_c$  is the area of soil replaced and A the total area of the zone reinforced. The equations [2.1] to [2.3] allow the user to compute the improvement factor  $n_0$  plotted in Fig 2.2 as a function of the area replacement ratio and the internal friction angle of the reinforced soil.

$$n_{0} = 1 + \frac{A_{c}}{A} \left[ \frac{0.5 + f(\mu_{s}, A_{c} / A)}{K_{aC} \cdot f(\mu_{s}, A_{c} / A)} - 1 \right]$$
(2.1)

$$f\left(\mu_{s},\frac{A_{c}}{A}\right) = \frac{\left(1-\mu_{s}\right)\left(1-\frac{A_{c}}{A}\right)}{1-2\mu_{s}+\frac{A_{c}}{A}}$$
(2.2)

$$K_{ac} = \tan^2 \left( 45^\circ - \frac{\varphi_c}{2} \right) \tag{2.3}$$

Where  $\mu_s$  is the Poisson's ratio of the soil and  $\varphi_c$  the internal friction angle of the ballast.

Priebe gives two equations to evaluate the internal friction angle and the effective cohesion of the composite system. Those two equations are given below [2.4 and 2.5]. Using those parameters for the composite system, it is possible to evaluate the bearing capacity for a homogeneous equivalent soil.



Figure 2.2. Improvement factor  $n_0$  as a function of the area replacement ratio and internal friction angle. Adapted from Priebe 1995.

$$tan\bar{\varphi} = m' \cdot tan\varphi_c + (1 - m')tan\varphi_s \tag{2.4}$$

$$c = \left(1 - \left(\overline{A_c/A}\right)\right)c_s \tag{2.5}$$

Where m'=(n-1)/n,  $\bar{\varphi}$  is the internal friction angle of the composite system,  $\varphi_c$  is the friction angle of the ballast,  $\varphi_s$  is the friction angle of the soil, and  $\overline{A_c/A}$  is different from  $A_c/A$  and is defined in Priebe 1995. Baez Satizabal (1995) proposed another method to evaluate the properties of the soil after reinforcement. This method is not presented herein but a short review and a comparison with Priebe's method is available elsewhere (Galy et al.,2012).

# **3.** A PRATICAL PROCEDURE FOR EVALUATING THE SEISMIC BEARING CAPACITY OF SHALLOW FOOTINGS ON STONE REINFORCED SOILS

## 3.1 Presentation of the Proposed Procedure

As listed in Section 2, different methods were developed in order to take into account the inertial forces occurring in the soil during an earthquake and the soil improvement properties induced by the installation of vibro stone columns. This paper proposes a practical procedure to evaluate the ultimate bearing capacity that includes the dynamic effects and the soil reinforcement. This procedure is summarized in Fig 3.1. The first step is to compute the improvement factor  $n_1$  or  $n_2$  according to Priebe's equations or to compute the N<sub>1/60</sub> after reinforcement according to Baez's model. In step 2, from either *n* or N, the user is able to compute the modified geotechnical properties of the reinforced soil (composite system in Priebe's case). In step 3 the user can compute the static and/or pseudo-dynamic bearing capacity factors. In the final step, the user computes the bearing capacity as usual with the CFEM method, using the reinforced geotechnical properties estimated for the composite system and the bearing capacity factors computed in step 3.

In this approach, the Richards's method, which has been supported by numerous studies (as indicated in Section 2.1), is combined with the CFEM method by applying the dynamic/static ratios to the

bearing capacity factors of Fig 2.1. By its nature, this method is only approximate for complex geometries and loadings (Fraser Bransby, 2001). Nevertheless, the proposed procedure could give to the designer a good indication on the safety of the foundation. In Fig 3.1, Peck's equations relating the internal friction angle to the SPT results is used (Peck et al., 1974).



Figure 3.1. Evaluation of the bearing capacity for a reinforced soil in static conditions (Method A) and pseudodynamic conditions (Method B).

Priebe's and Baez Satazabal's methods have been proved to be quite accurate for static conditions (Baez Satizabal, 1995; Kirsch, 2008). However, the estimation of the ultimate bearing capacity is not the primary goal of these different methods and the procedure proposed in this paper requires a validation. Finally there is also a need for a validation of the estimation of the ultimate bearing capacity of reinforced soil in a dynamic condition. The dynamic validation (using the pseudo-dynamic bearing capacity factors from Richards et al.(1993)) is actually under study and will not be presented in this contribution.

## 3.2 Validation of the Proposed Procedure with a Finite Difference Model

This section presents a comparison between the bearing capacity obtained with the Method A of the proposed procedure (Fig. 3.1) and a 2D Finite Difference Model using FLAC software (Itasca, 2006). The techniques adopted in order to compute the bearing capacities are presented in Fig 3.2. The two numerical models (a) and (b) used in the software FLAC are detailed in this section.



Figure 3.2. Computation of the bearing capacities with different techniques.

Two different numerical models were used in order to represent the differences of the methods proposed by Baez and Priebe respectively. Baez's equations allow the designer to compute the estimated results to a SPT or CPT after reinforcement for the soil located *between* the stone columns. Therefore, the post reinforcement soil parameters should give a lower bound bearing capacity for purely un-cohesive soils. Priebe's method gives an internal friction angle for the *composite system* for

the soil *and* the stone columns. This has been modeled as a homogeneous equivalent soil in the finite difference model. The two numerical models are presented in Fig 3.4.

The footing considered has a width of 0.45 m and is a strip footing. The model uses the symmetry of the problem and thus only a half of the foundation and the soil are represented. The model has a width and a height of 5 m. The grid is square and homogeneous for the far field and has a width of 0.25 m. Underneath the foundation (2m deep and 1.5 wide) the grid is still square and homogeneous but it is five times smaller and has a width of 0.05m. Numerous sensitivity analyses have been performed in order to ensure that the dimensions of the grid do not impact the results. It has been found that homogeneous grids give good results in order to estimate the bearing capacity with FLAC as it has been emphasized by Frydman and Burd (1995) and Yin et al. (2001). Although this foundation is relatively small, larger widths have been studied and the results are very similar to those obtained with this model. A Mohr-Coulomb model represents the soil.

The foundation is supposed to be a smooth footing therefore the nodes are only constrained in the vertical direction under the footing (Frydman and Burd, 1995). The internal friction angle values of the stone column are generally included between 40 and 50° but a relatively low value is generally considered (mostly 40 or  $42^{\circ}$ ) in order to have conservative estimates (McCabe et al., 2007).

Model (a) is made of a homogeneous equivalent reinforced soil underneath the foundation  $(2 \times 1,5m)$  and homogeneous fluvio-glacial sand for the unreinforced part. Since we only take interest in the ultimate bearing capacity, the plastic components (E and v) do not influence the results and we can just modify the internal friction angle in order to represent the reinforced soil. Model (b) is made of reinforced soil between the columns (using the  $\phi'$  estimated with Baez's method and Peck's equation (Peck et al. 1974)).



Figure 3.4. 2D FDM FLAC models used in this study.

The two FDM give very similar results: the maximum difference between the bearing capacities computed with FLAC is 12,4%. The CFEM Priebe method is the one that gives the best results, in comparison with the FDM, and the maximum difference between the CFEM Priebe and FLAC Priebe methods is 5,5%. Those results show that Method A appears to be efficient for estimating the bearing capacity of a vibro stone reinforced soil in pseudo-static conditions.

# 4. INFLUENCE OF THE REINFORCED ZONE DIMENSIONS ON THE BEARING CAPACITY

#### 4.1 Scope and Limitations of the Parametric Study

The parametric study includes the two case scenario presented in Fig 4.1. The first case corresponds to a full soil treatment before the installation of the shallow footing and is referred to as scenario "1" [Fig 4.1 (a)]. The second scenario ("2") corresponds to partial soil improvement. Scenario "2" is encountered most frequently in rehabilitation projects as the access to the soil below the footing is more difficult and more costly [Fig 4.1 (b)].



Figure 4.1. "Full" (a) and "partial" (b) soil improvement scenario

The results for this parametric study were obtained with the FDM Model (a) (Priebe) presented in Fig 3.4 varying the plane dimensions of the reinforced area, width (W) and depth (D), both function of the footing width B. A total of 288 models (Priebe) were used to conduct this parametric study and 42 Baez models were used in order to validate the results. The maximum difference between the results obtained with those two series of models was 8%.

## 4.2 Results

The computed bearing capacities are function of three parameters: the reinforcement width W, the reinforcement depth D, and the area replacement ratio  $A_r$ . For concision only the results from the "partial improvement" scenario are presented in this paper. As underlined in Section 4.1, W and D are a function of the footing width B, meaning that when D=2 on the graph, the treatment depth is in fact two times B. The computed bearing capacities are presented in Fig 4.2 as surface planes for three values of the area replacement ratio Ar, 10%, 20% and 30%.

Several observations can be drawn from Fig 4.2. First, as expected, it can be seen that the bearing capacity increases when the dimensions of the reinforced zone increase (increase in D and W). Increasing the treatment depth D from B to 3.5B increases the bearing capacity of approximately 5% for a fixed treatment width. When the treatment width is increased from 0.25B to 2.5B an increase of 10 to 20% can be observed in the bearing capacity (depending on the area replacement ratio chosen). It also can be noted that past a 1.5B width of treatment the increase in the bearing capacity is negligible. Those conclusions are valid for the increase in bearing capacity post reinforcement and not for the liquefaction mitigation effects by vibroreplacement a case that may need a larger treated zone. Very similar observations can be drawn for the "full improvement" scenario.



**Figure 4.2.** Bearing capacity q<sub>u</sub> as a function of the dimensions of the reinforced zone (W and D) and the area replacement ratio Ar for a "partial improvement" scenario.

The obtained bearing capacities have been compared to those computed with Method A. The ratio of the bearing capacities is called  $Rq_u$  and equals the bearing capacity computed with FLAC divided by the bearing capacity computed with Method A. This ratio can be used by the user to refine the bearing capacity estimation with Method A. This ratio is plotted in Fig. 4.4 as a function of W, D and Ar. The equations of the parametric curves presented in Fig 4.3 are obtained from Equation 4.1 and Table 4.1.

$$q_{\nu}(W,D) = C00 + C10 \cdot W + C01 \cdot D + C20 \cdot W^{2} + C11 \cdot W \cdot D + C30 \cdot W^{3} + C21 \cdot W^{2} \cdot D \quad (4.1)$$

The constant values CXX are given in Table 4.1 for both scenarios. The adjusted coefficient of determination  $R^2$  is above 0.98 for all the cases studied. Some comments can be made on Fig 4.3. First it can be observed that Method A overestimates the bearing capacity (Rq<sub>u</sub> smaller than 1.0) and that the use of a correction coefficient (calculated with Equation 4.1) is necessary. The highest area replacement ratio (Ar=30%) gives the highest overestimation of the bearing capacity for a partial improvement case (Rq<sub>u</sub>=0,55). It should be noted that the corresponding curves for the "full improvement" scenario, not presented herein, are quite different from those in Fig 4.3 for the "partial improvement" scenario.

		C00	C10	C01	C20	C11	C30	C21	R <sup>2</sup> (adjusted)
"Full improvement"	Ar=10%	0,5127	0,4619	0,002344	-0,128	0,01832	0,01161	-0,00219	0,9888
	Ar=20%	0,3159	0,5924	-0,002823	-0,1585	0,01933	0,01408	-0,002247	0,9904
	Ar=30%	0,2184	0,6464	-0,004997	-0,1692	0,01912	0,01487	-0,002206	0,9913
"Partial improvement"	Ar=10%	0,7382	0,2173	0,008664	-0,1188	0,009061	0,02163	-0,0026	0,9985
	Ar=20%	0,6159	0,2523	0,004488	-0,1264	0,009968	0,02103	-0,002889	0,9984
	Ar=30%	0,5469	0,2675	0,004309	-0,1219	0,005555	0,01803	-0,001393	0,9976

Table 4.1. Constant parameters values for the parametric curves



Figure 4.3. Bearing capacity ratios Rq<sub>u</sub> as a function of the dimensions of the reinforced zone (W and D) and the area replacement ratio Ar for a "partial improvement" scenario.

#### **5. CONCLUSIONS AND FUTURE WORK**

A new practical procedure to compute the ultimate bearing capacity of a vibro stone reinforced soil for static and pseudo-dynamic conditions has been introduced in this contribution. It combines the well know plasticity based approach presented in the CFEM, Richard's method to estimate the dynamic bearing capacity and Priebe's or Baez's method to estimate the vibro stone reinforced soil parameters. The static case was validated using FDM. The results indicate that the proposed practical procedure differs only by 5,5% from the results obtained with the FDM. The parametric study presented indicates that a 1.5B treatment width on each side of the footing is sufficient to increase the original bearing capacity by 25 to 50% (depending on the Ar) in the case of a "partial improvement" scenario presented here. Furthermore, the parametric surfaces obtained allow the user to compute a correction factor in order to adjust the bearing capacity computed with Method A. A validation of Method B is currently under study and will be the object of another paper.

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