Frequency Domain Analysis of Concrete Gravity Dam-Reservoir Systems by Wavenumber Approach

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SUMMARY:
Dynamic analysis of concrete gravity dam-reservoir systems can be carried out in frequency or time domain. It is well-known that the rigorous approach for solving this problem relies heavily on employing a two-dimensional semi-infinite fluid element. The hyper-element is formulated in frequency domain and its application in this field has led to many especial purpose programs, which were demanding from programming point of view. In this study, a technique is proposed for dynamic analysis of dam-reservoir systems in the context of pure finite element programming which is referred to as the Wavenumber approach. In this technique, the wavenumber condition is imposed on the truncation boundary or the upstream face of the near-field water domain. The method is initially described. Subsequently, the response of an idealized triangular dam-reservoir system is obtained by this approach, and the results are compared against the exact response. Based on this investigation, it is concluded that this approach can be envisaged as a great substitute for the rigorous type of analysis.

Keywords: concrete gravity dams, wavenumber, absorbing boundary conditions, truncation boundary

1. INTRODUCTION

Dynamic analysis of concrete gravity dam-reservoir systems can be carried out rigorously by FE-(FE-HE) method in the frequency domain. This means that the dam is discretized by plane solid finite elements, while, the reservoir is divided into two parts, a near-field region (usually an irregular shape) in the vicinity of the dam and a far-field part (assuming uniform depth) which extends to infinity in the upstream direction. The former region is discretized by plane fluid finite elements and the latter part is modeled by a two-dimensional fluid hyper-element (Hall and Chopra 1982, Waas 1972). It is well-known that employing fluid hyper-elements would lead to the exact solution of the problem. However, it is formulated in the frequency domain and its application in this field has led to many especial purpose programs which were demanding from programming point of view.

On the other hand, engineers have often tried to solve this problem in the context of pure finite element programming (FE-FE method of analysis). In this approach, an often simplified condition is imposed on the truncation boundary or the upstream face of the near-field water domain. Thus, the fluid hyper-element is actually excluded from the model. Some of these widely used simplified conditions (Sommerfeld 1949, Sharan 1987), may result in significant errors if the reservoir length is small, and it might lead to high computational cost if the truncation boundary is located at far distances. The main advantage of these conditions is that it can be readily used for time domain analysis. Thus, they are vastly employed in nonlinear seismic analysis of concrete dams.

Of course, there have also been many researches in the last three decades to develop more accurate absorbing boundary conditions to be applied for similar fluid-structure or soil-structure interaction problems. Perfectly matched layer (Berenger 1994, Chew and Weedon 1994, Basu and Chopra 2003) and, high-order non-reflecting boundary condition (Higdon 1986, Givoli and Neta 2003, Hagstrom and Warbuton 2004, Givoli, *et al.* 2006) are among the two main popular groups of methods which researchers have applied in their attempts. It is emphasized that these techniques have become very popular in recent years due to the fact that they could be applied in time domain as well as the frequency domain. However, it should be realized that they are not that attractive in the frequency domain. This is due to the fact that they are not very simple to be used and more importantly, they are compared with the hyper-element alternative which produces exact results no matter how small the reservoir near-field length is considered.

In the present study, the FE-FE analysis technique is chosen as the basis of a proposed method for dynamic analysis of concrete dam-reservoir system in the frequency domain, which is referred to as the Wavenumber approach. The method is simply applying an absorbing boundary condition on the truncation boundary which is referred to as the Wavenumber condition. It is as simple as employing Sommerfeld or Sharan condition on the truncation boundary. The method of analysis is initially explained. Subsequently, the response of an idealized triangular dam is studied due to horizontal ground motion based on this technique. A sensitivity analysis is carried out and the parameter varied, is the length of reservoir near-field region. It is mainly concluded that the Wavenumber approach is very accurate. It is also ideal from programming point of view due to the local nature of wavenumber condition imposed on truncation boundary. Moreover, it can be envisaged as a great substitute for the rigorous FE-(FE-HE) type of analysis in the frequency domain which is heavily relying on a fluid hyper-element as its main core.

2. METHOD OF ANALYSIS

As mentioned, the analysis technique utilized in this study is based on the FE-FE method, which is applicable for a general concrete gravity dam-reservoir system. The coupled equations can be obtained by considering each region separately and then combine the resulting equations.

2.1 Dam Body

Concentrating on the structural part, the dynamic behavior of the dam is described by the well-known equation of structural dynamics (Zienkiewicz and Taylor 2000):

$$\mathbf{M}\,\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\,\mathbf{r} = -\mathbf{M}\,\mathbf{J}\mathbf{a}_{\mathrm{g}} + \mathbf{B}^{\mathrm{T}}\,\mathbf{P} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ in this relation represent the mass, damping and stiffness matrices of the dam body. Moreover, $\mathbf{r}$ is the vector of nodal relative displacements, $\mathbf{J}$ is a matrix with each two rows equal to a 2×2 identity matrix (its columns correspond to a unit horizontal and vertical rigid body motion), and $\mathbf{a}_{\mathrm{g}}$ denotes the vector of ground accelerations. Furthermore, $\mathbf{B}$ is a matrix which relates vectors of hydrodynamic pressures (i.e., $\mathbf{P}$), and its equivalent nodal forces.

Let us now consider harmonic excitation with frequency ω, and limit the present study to the horizontal ground motion only. It is well known that the response will also behave harmonic (i.e., $\mathbf{r}(t) = \mathbf{r}(\omega)\,\mathrm{e}^{\mathrm{i}\,\omega t}$). Thus, Eq. (1) can be expressed as:

$$\left(-\omega^2\,\mathbf{M} + (1+2\beta\ \mathrm{i})\,\mathbf{K}\right)\mathbf{r} = -\mathbf{M}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} + \mathbf{B}^{\mathrm{T}}\,\mathbf{P} \tag{2}$$

In this relation, it is assumed that the damping matrix of the dam is of hysteretic type. This means:

$$\mathbf{C} = (2\beta\,/\,\omega)\,\mathbf{K} \tag{3}$$

Moreover, it should be emphasized that the superscript h on the acceleration vector refers to the horizontal type of excitation. That is:

$$\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} = \begin{pmatrix} \mathrm{a}_{\mathrm{g}}^{\mathrm{x}} \\ 0 \end{pmatrix} \tag{4}$$

2.2 Water Domain

Assuming water to be linearly compressible and neglecting its viscosity, its small irrotational motion (Fig. 1) is governed by the wave equation (Chopra 1967, Chopra, *et al*. 1980):

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}-\frac{1}{c^2}\ddot{p}=0 \quad \text{in } \Omega \text{ and } D \tag{5}$$

where p is the hydrodynamic pressure and, c is the pressure wave velocity in water.

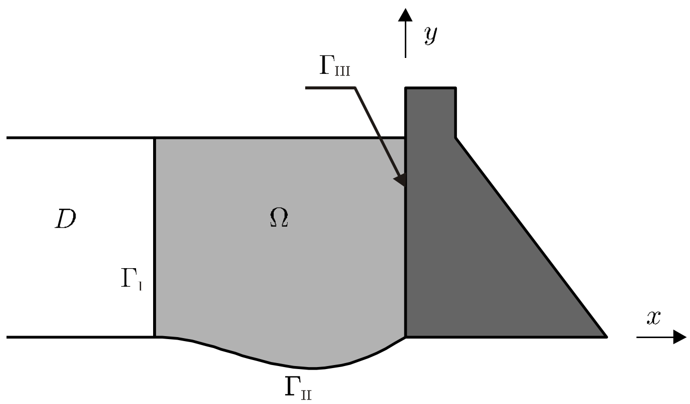


Figure 1. Schematic view of a typical dam-reservoir system. The near-field reservoir domain Ω, the truncation boundary Γ_I and the far-field region *D* (excluded in the FE-FE type of analysis)

One can apply the weighted residual approach to this equation and impose all boundary conditions in explicit form except for the truncation boundary (i.e., Γ_I), to obtain the finite element equation of the fluid domain, which may be written as:

$$\mathbf{G}^e\ddot{\mathbf{P}}^e+q\mathbf{L}^e_{II}\dot{\mathbf{P}}^e+\mathbf{H}^e\mathbf{P}^e=\mathbf{R}^e_I(t)-\mathbf{B}^e\ddot{\mathbf{r}}^e-\mathbf{B}^e\mathbf{J}^e\mathbf{a}^h_g \tag{6}$$

This is the well-known equation of the fluid domain and the exact definitions of the matrices involved can be found elsewhere in details (Lotfi and Samii 2011). It is worthwhile to emphasize that the superscript (^{e}) states that these matrices are related to the element level. Moreover, $\mathbf{L}^e_{II}$ is a matrix, which corresponds to the absorption of energy at reservoir's bed (i.e., Γ_{II}). It is reminded that the admittance or damping coefficient q which is factored out from this matrix may also be related to a more meaningful wave reflection coefficient α (Fenves and Chopra 1985):

$$\alpha=\frac{1-q\,c}{1+q\,c} \tag{7}$$

which is defined as the ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of a vertically propagating pressure wave incident on the reservoir bottom. For a fully reflective reservoir bottom condition, α is equal to 1 which leads to $q=0$.

Furthermore, $\mathbf{R}^e_I$ is part of the right hand side vector corresponding to the truncation boundary. This boundary integral has the following original form:

$$\mathbf{R}^e_I=\frac{1}{\rho}\int_{\Gamma_I}\mathbf{N}\,(\partial_n p)\,\mathrm{d}\Gamma^e \tag{8}$$

with $\mathbf{N}$ being the vector of fluid element's shape functions, ρ is the water density and n denotes the outward (with respect to fluid region) perpendicular direction. It is emphasized that the directional derivative $\partial_n p$ needs to be defined by imposing an appropriate condition.

The equivalent form of equation (6) in the frequency domain would be:

$$-\omega^2 \mathbf{G}^e\, \mathbf{P}^e + \mathrm{i}\omega q\, \mathbf{L}_{\mathrm{II}}^e\, \mathbf{P}^e + \mathbf{H}^e\, \mathbf{P}^e = \mathbf{R}_{\mathrm{I}}^e(\omega) + \omega^2\, \mathbf{B}^e\, \mathbf{r}^e - \mathbf{B}^e \mathbf{J}^e \mathbf{a}_{\mathrm{g}}^{\mathrm{h}} \tag{9}$$

By assembling the element equations, one would obtain the overall FE equation of the fluid domain:

$$-\omega^2 \mathbf{G}\,\mathbf{P} + \mathrm{i}\omega q\, \mathbf{L}_{\mathrm{II}}\, \mathbf{P} + \mathbf{H}\,\mathbf{P} = \mathbf{R}_{\mathrm{I}} + \omega^2 \mathbf{B}\,\mathbf{r} - \mathbf{B}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} \tag{10}$$

In this equation, $\mathbf{R}_{\mathrm{I}}$ is obtained by assembling the boundary integrals of Eq. (8) on Γ_{I}.

2.3 Dam-reservoir system

The necessary equations for both dam and reservoir domains were developed in the previous sections.

Thus, combining the main relations (2) and (10) would result in the FE equations of the coupled dam-reservoir system in its initial form for the frequency domain:

$$\begin{bmatrix} -\omega^2 \mathbf{M} + (1+2\beta \mathrm{i})\mathbf{K} & -\mathbf{B}^{\mathrm{T}} \\ -\omega^2 \mathbf{B} & (-\omega^2 \mathbf{G} + \mathrm{i}\omega q\, \mathbf{L}_{\mathrm{II}} + \mathbf{H}) \end{bmatrix} \begin{Bmatrix} \mathbf{r} \\ \mathbf{P} \end{Bmatrix} = \begin{Bmatrix} -\mathbf{M}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} \\ (-\mathbf{B}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} + \mathbf{R}_{\mathrm{I}}) \end{Bmatrix} \tag{11}$$

It is noted from the above equation that the vector $\mathbf{R}_{\mathrm{I}}$ still needs to be explicitly defined. This is related to the truncated boundary Γ_{I} which will be discussed below.

2.4 Modification due to truncation boundary contribution

The effect of truncation boundary will be treated in this section. For this purpose, let us now assume that this boundary (i.e., Γ_{I}) is vertical (i.e. along y-direction) and consider a harmonic plane wave with unit amplitude and frequency ω propagating along a direction which makes an angle θ with negative x-direction. This may be written in many different forms such as:

$$p = e^{\mathrm{i}(k' x + \lambda y + \omega t)} \tag{12a}$$

$$p = e^{(\mathrm{i}\,\omega/c)\left[(\cos\theta)x + (\sin\theta)y + c\,t\right]} \tag{12b}$$

With the following relations being valid:

$$k' = \frac{\omega}{c}\cos\theta \tag{13a}$$

$$\lambda = \frac{\omega}{c}\sin\theta \tag{13b}$$

$$k'^2 + \lambda^2 = \frac{\omega^2}{c^2} \tag{13c}$$

It is easily verified that the following condition is appropriate for the truncated boundary based on the assumed traveling wave (i.e, Eq. (12a)):

$$\frac{\partial p}{\partial x} - \mathrm{i}\,k'\,p = 0 \tag{14}$$

Employing (14) in (8), it yields:

$$\mathbf{R}_{\mathrm{I}}^{e} = -(\mathrm{i}\,k')\,\mathbf{L}_{\mathrm{I}}^{e}\,\mathbf{P}^{e} \tag{15}$$

with the following definition:

$$\mathbf{L}_{\mathrm{I}}^{e} = \frac{1}{\rho}\int_{\Gamma_{\mathrm{I}}^{e}} \mathbf{N}\,\mathbf{N}^{\mathrm{T}}\mathrm{d}\Gamma^{e} \tag{16}$$

Assembling $\mathbf{R}_{\mathrm{I}}^{e}$ for all fluid elements adjacent to truncation boundary leads to:

$$\mathbf{R}_{\mathrm{I}} = -(\mathrm{i}\,k')\,\mathbf{L}_{\mathrm{I}}\,\mathbf{P} \tag{17}$$

This can now be substituted in (11) to obtain the FE equations of the coupled dam-reservoir system in its final form for the frequency domain:

$$\begin{bmatrix} -\omega^{2}\mathbf{M} + (1+2\beta\mathrm{i})\mathbf{K} & -\mathbf{B}^{\mathrm{T}} \\ -\mathbf{B} & \omega^{-2}(-\omega^{2}\mathbf{G} + \mathrm{i}\,k'\,\mathbf{L}_{\mathrm{I}} + \mathrm{i}\omega q\,\mathbf{L}_{\mathrm{II}} + \mathbf{H}) \end{bmatrix} \begin{Bmatrix} \mathbf{r} \\ \mathbf{P} \end{Bmatrix} = \begin{Bmatrix} -\mathbf{M}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} \\ \omega^{-2}(-\mathbf{B}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}}) \end{Bmatrix} \tag{18}$$

It is also noticed that the lower matrix equation of (18) is multiplied by ω^{-2} in this process to obtain a symmetric dynamic stiffness matrix for the dam-reservoir system.

2.5 Defining parameter k'

The major remaining concept is the determination of parameter k' which will be discussed in this section. There are different available options (Lotfi and Samii 2011). However, before explaining about the adopted strategy, it is worthwhile to review some salient aspects on the exact analytical solution available for the domain *D* (Fig. 1). It is reminded that this domain is actually eliminated from our problem.

This is a regular semi-infinite region with constant depth H extending to infinity in the upstream direction. It is also assumed presently that the base of this region is completely reflective (i.e, $\alpha = 1$ or $q = 0$) and we are considering merely horizontal ground excitation. Under these circumstances, the exact solution for this region may be written as follows (Chopra, *et al*. 1980):

$$p(x, y, t) = \sum_{j=1}^{\infty} B_j \cos(\lambda_j y) e^{\mathrm{i}(k'_j x + \omega t)} \tag{19}$$

It is noted that the solution is composed of different modes, and amplitude B_j depends on the existing conditions on the downstream face of that region. Moreover, parameters λ_j and k'_j are defined as:

$$\lambda_j = \frac{(2j-1)\pi}{2\mathrm{H}} \tag{20a}$$

$$k'^{2}_{j} + \lambda_j^{2} = \frac{\omega^{2}}{c^{2}} \tag{20b}$$

Herein, H represents the water depth. Based on (20b), one can actually determine k'_j (referred to as the *j*-th wavenumber) as follows:

$$k'_j = \pm\,\mathrm{i}\,\sqrt{\lambda_j^{2} - \frac{\omega^{2}}{c^{2}}} \tag{21}$$

Although, there are two options in this definition, the negative sign is merely admissible for a semi-infinite region extending to infinity in the negative x-direction as in our present case. This is due to the fact we are only interested with evanescent modes (imaginary wavenumber) or traveling modes (real wavenumber) which are decaying or propagating towards the upstream direction. It is also noted that the *j*-th wavenumber (k'_j) becomes zero at a cut-off frequency referred to as the *j*-th natural frequency of the reservoir (i.e., ω_j^r). This is obtained by substituting (20a) in (21) under that condition which results in:

$$\omega_j^r = \frac{(2j-1)\pi c}{2\mathrm{H}} \tag{22}$$

Eq. (21) with the admissible negative sign, may also be written as:

$$k'_j = \frac{\omega}{c}\left(\frac{-\mathrm{i}(2j-1)}{\Omega}\sqrt{1-\frac{\Omega^2}{(2j-1)^2}}\right) \tag{23}$$

with the help of dimensionless frequency Ω:

$$\Omega = \frac{\omega}{\omega_1^r} \tag{24}$$

Let us now describe the strategy adopted in the Wavenumber approach for the selection of parameter k' in Eq. (18):

The proposed method is to define k' based on different wavenumbers (i.e., by utilizing Eq. (23)) for various frequency ranges. In particular, use the following plan:

$$k' = k'_1 \quad ; \quad for \ \left[0 \le \Omega \le 3 \ \ or\, (0 \le \omega \le \omega_2^r)\right] \tag{25a}$$

$$k' = k'_j \quad ; \quad for \ \left[(2j-1) \le \Omega \le (2j+1) \ \ or\, (\omega_j^r < \omega \le \omega_{j+1}^r)\right] and \ j \ge 2 \tag{25b}$$

In other words, employ the most appropriate wavenumber for each frequency range. This is shown to be true in details elsewhere (Lotfi and Samii 2011).

3. MODELING AND BASIC PARAMETERS

The introduced methodology is employed to analyze an idealized dam-reservoir system. The details about modeling aspects such as discretization, basic parameters and the assumptions adopted are summarized in this section.

3.1 Models

An idealized triangular dam with vertical upstream face and a downstream slope of 1:0.8 is considered on a rigid base. The dam is discretized by 20 isoparametric 8-node plane-solid finite elements.

As for the water domain, two strategies are adopted (Fig. 2). For the FE-FE method of analysis which is our main procedure, only the reservoir near-field is discretized and the Wavenumber condition is employed on the upstream truncation boundary according to the strategy discussed. The length of this near-field region is denoted by L and water depth is referred to as H. Three cases are considered. These are in particular the L/H values of 0.2, 1 and 3 which represent low, moderate and high reservoir lengths. This region is discretized by 5, 25 and 75 isoparametric 8-node plane-fluid finite elements for three above-mentioned L/H values, respectively.

For the FE-(FE-HE) method of analysis, the reservoir domain is divided into two regions. The near-field region is discretized by fluid finite elements, and the far-field is treated by a fluid hyper-element. Of course, it should be emphasized that this option is merely utilized to obtain the exact solution (Lotfi 2001). Moreover, it is well-known that the results are not sensitive in this case to the length of the reservoir near-field region or L/H value.

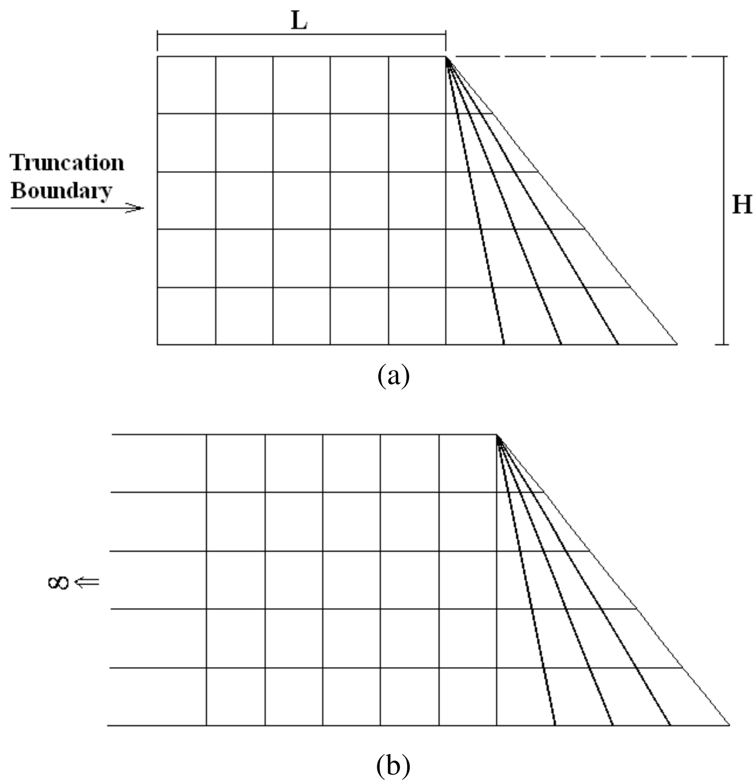


Figure 2. The dam-reservoir discretization for L/H=1; (a) FE-FE Model, (b) FE-(FE-HE) Model

3.2 Basic Parameters

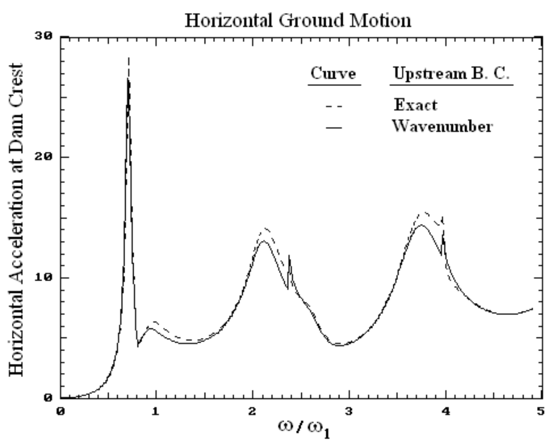
The dam body is assumed to be homogeneous and isotropic with linearly viscoelastic properties for mass concrete:

Elastic modulus (E_d)	$= 27.5$ GPa
Poison's ratio	$= 0.2$
Unit weight	$= 24.8$ kN/m^3
Hysteretic damping factor (β_d)	$= 0.05$

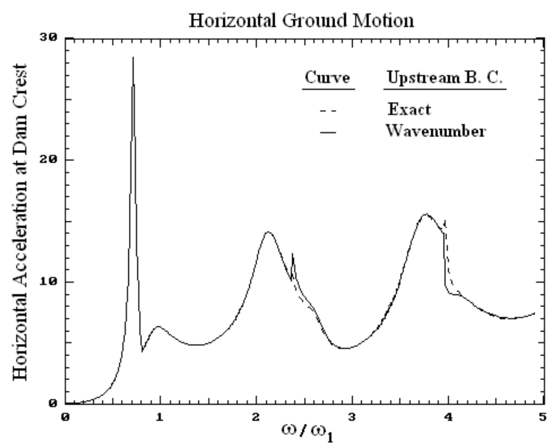
The impounded water is taken as inviscid and compressible fluid with unit weight equal to 9.81 kN/m^3 , and pressure wave velocity $c = 1440$ m/sec.

4. RESULTS

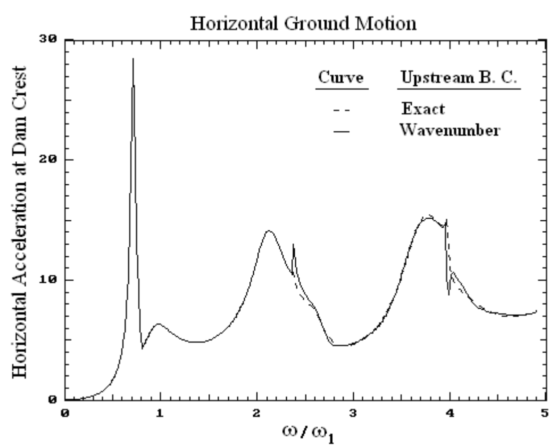
It should be emphasized that all results presented herein, are obtained by the FE-FE method discussed, by imposing wavenumber condition on the truncation boundary. Moreover, the response for each case is compared against the exact result obtained by FE-(FE-HE) analysis technique.
As mentioned previously, three cases are considered. These are in particular the L/H values of 0.2, 1 and 3 which represent low, moderate and high reservoir lengths.
The results for these three cases are presented in Fig. 3. The response illustrated in each case is the transfer function for the horizontal acceleration at dam crest with respect to horizontal ground acceleration. It is noted that they are plotted versus the dimensionless frequency. The normalization of excitation frequency is carried out with respect to ω_1, which is defined as the natural frequency of the dam with an empty reservoir on rigid foundation.



a) L/H=0.2



b) L/H=0.2



c) L/H=3

Figure 3. Horizontal acceleration at dam crest due to horizontal ground motion

Let us first consider the very low reservoir length (i.e., L/H=0.2), which is a challenging test for examining any type of absorbing boundary condition. It is observed that the response is relatively close to the exact response in most frequency ranges (Fig. 3a). However, there exist errors in the range of 5% at the major peaks of the response. Of course, this is still believed to be a remarkable result for such a challenging test.

For the moderate and high reservoir lengths considered (i.e., L/H=1, 3), it is noticed that the response agrees very well with the exact response for both cases (Figs. 3b and 3c). Moreover, there are no signs of distortions in the response for the high reservoir length, contrary to what is noticed for other usual alternative absorbing boundary conditions such as Sommerfeld and Sharan conditions (Lotfi and Samii 2011).

5. CONCLUSIONS

The formulation based on FE-FE procedure for dynamic analysis of concrete dam-reservoir systems, was reviewed. Moreover, the Wavenumber approach was discussed in that context. A special purpose finite element program was enhanced for this investigation. Thereafter, the response of an idealized triangular dam was studied due to horizontal ground motion for that approach.
Overall, the main conclusions obtained by the present study can be listed as follows:

- It is noticed that the response agrees very well in comparison with the exact response for the moderate as well as high reservoir lengths. However, errors in the range of 5% are noticed at the major peaks of the response for the low reservoir length ($L/H=0.2$), which is still believed to be a remarkable result for such a challenging test.
- There are no signs of distortions in the response under any circumstances.
- Obviously, the main disadvantage of this condition is that it cannot be utilized in time domain, and it is only suitable for frequency domain.
- The Wavenumber approach is ideal from programming point of view due to the local nature of Wavenumber condition imposed on truncation boundary. It can also be envisaged as a great substitute for the rigorous FE-(FE-HE) type of analysis which is heavily relying on a hyper-element as its main core. It is undeniable that the rigorous approach is significantly more complicated from programming point of view and also much more computationally expensive.

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