Prediction of Initial Column Size for Reinforced Concrete Frame Buildings with Intermediate Drift

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SUMMARY:
A method to predict the initial column size for displacement based design of reinforced concrete frame buildings with intermediate design inter storey drift is reported here. An algorithm based on an empirical relation is used for given beam section, target inter-story drift, building plan features and common displacement based design parameters. To maintain a gradual descent of column width up height from ground, a pattern of point of contra flexure is used as a parameter. Displacement components with complex behaviour related to column shear and moment are considered in the form of their corresponding dynamic amplification factors. The method is applied to four, eight and twelve story frame buildings for two percent target inter storey drift and tested with non linear time history analysis using five standard artificial ground motions. The columns are found elastic enough to avoid column sway mechanism when used directly without changes in size.

Keywords: Column size, storey drift, point of contra flexure, displacement based design, reinforced concrete

1. INTRODUCTION

Manual repeated iteration in reinforced concrete (R.C.) frame design procedure for concluding into an appropriate member size has always been a time consuming and cumbersome process. Member size of an R.C. frame is mainly a function of flexure and shear demand via seismic weight and lateral force generated by the design method used. However, torsion may also be in case of frame buildings of irregular geometry. Therefore, iteration is unavoidable before deciding a conclusive set of member sections after fulfilling the required design constraints like demand, percentage reinforcement, column-beam moment capacity ratio, etc.

A fairly approximate method, an algorithm for calculating the width of column using computer program for displacement based design (DBD) based on given beam size, design inter-story drift ratio (IDR), building plan features, etc. is presented here. A closed symmetrical plan is considered in the study. Despite the fact that position of point of contra flexure varies directly with sectional moment of inertia and indirectly with the square of member length, an ideal pattern of point of contra flexure is decided. Provision for complex displacement components (like beam plastic rotation and joint shear) are considered in the form of dynamic amplification factor for column shear and moment as an approximate measure. Dynamic amplification of inter storey drift is considered during the design.

2. COLUMN SIZE GENERATION PROCEDURE

2.1. Background

Avoiding soft story formation i.e. column sway mechanism and to maintain beam sway mechanism is a general mandatory requirement in displacement based design of R.C. frame buildings where all the column member except bottom ground floor and top floor columns are designed to behave elastically through capacity design. Therefore, the column section should be suitable that after design, supplying the required reinforcement, it behaves elastically with Non Linear Time History Analysis (NLTHA). If θ_{yc} represents column yield rotation (Figure 1), l the distance between the point of contra flexure and the point of maximum bending moment and $\emptyset_{yc}$ the column yield curvature, the column tip displacement (Δ_f) within elastic limit due to flexure Park and Paulay (1975) is given by

$$\Delta_f = \frac{\emptyset_{yc} l^2}{3} \tag{1}$$

If ε_y represents yield strain at expected strength of rebar steel and h_{yc} the column depth in the direction of earthquake under consideration, yield curvature ($\emptyset_{yc}$) according to Priestley (2003) for rectangular column section is given by

$$\emptyset_{yc} = \frac{2.1\varepsilon_y}{h_c} \tag{2}$$

The flexural displacement of column tip (Δ_f) can be written by combining Eqn. 1 and 2 as

$$\Delta_f = \frac{0.7\varepsilon_y l^2}{h_c} \tag{3}$$

If V represents the maximum shear force applied to the member, B the column width, d the effective depth, ρPoisson's ratio and E modulus of elasticity of the material, average column displacement due to shear (Δ_s) according to Matamoros (1999) is given by

$$\Delta_s = \frac{6V}{5Bd} \times \frac{2(1+\rho)}{E} \times 10 \tag{4}$$

Effective depth (d) of the section is assumed to be 0.9 times the overall depth (h_c). If c_r is the ratio of column depth to the width i.e. $c_r = (h_c/B)$, Eqn. 4 can be written as

$$\Delta_s = \frac{24V(1+\rho)c_r}{0.9h_c^2 E} \tag{5}$$

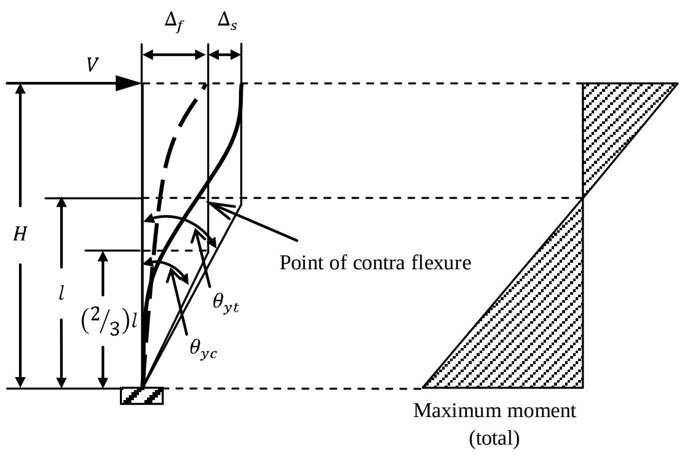

Figure 1. Assumed elastic column deformation behaviour.

Assuming that the column section are meant for elastic action only and neglecting the deformation due to slip, the overall column drift (Δ_{total}) can be considered to be composed of only flexure (Δ_f) and shear (Δ_s) component represented in Eqn. 6. Other complex components (including beam plastic rotation and joint shear) influencing moment and shear of column are considered in terms of dynamic amplification factors in Eqn. 10.

$$\Delta_{total} = \Delta_f + \Delta_s \tag{6}$$

Eqn. 6 can be written by using Eqn. 3 and 5 as

$$\Delta_{total} = \frac{0.7\varepsilon_y l^2}{h_c} + \frac{24V(1+\rho)c_r}{0.9h_c^2 E} \tag{7}$$

If θ_{yt} represents the overall yield column rotation, rotates with the same l as in Eqn. 1.

$$\theta_{yt} l = \frac{0.7\varepsilon_y l^2}{h_c} + \frac{24V(1+\rho)c_r}{0.9h_c^2 E} \tag{8}$$

$$\Longrightarrow \theta_{yt} l h_c^2 - 0.7\varepsilon_y l^2 h_c - \frac{24V(1+\rho)c_r}{0.9E} = 0 \tag{9}$$

According to Paulay and Priestley (1992), for one way frame having a design over strength factor of $\emptyset_o$, columns should be designed for $1.3 \times \emptyset_o$ times the design column shear, and $\omega_m \times \emptyset_o$ times the design column moment; Where, $1.3 \leq \omega_m \leq 1.8$, $\omega_m = 0.5T + 0.85$ and T is the natural period of the structure. Therefore, Eqn. 9 can be written as

$$\theta_{yt} l h_c^2 - 0.7\varepsilon_y l^2 h_c \emptyset_o \omega_m - \frac{31.2V(1+\rho)c_r\emptyset_o}{0.9E} = 0 \tag{10}$$

The position of the point of contra flexure changes with the ratio of moment of inertia of the section and square of the member length. To avoid complexities, a pattern of average position of point of contra flexure for column members is introduced. According to Eqn. 10, l is the parameter deciding the variation h_c value with different storey levels. Generally, for capacity design the position of the point of contra flexure is assumed to be located at the mid height of the column except for the ground floor column which is assumed as 0.6 times the column height. An analysis for l value (Figure 2, 3 and 4) is done using 45 frames of heights 4, 8 and 12 storey designed for 1%, 2% and 3% design IDRs using the minimum column section just to accommodate a maximum reinforcement of 4% where l value is found very sensitive specially at the lower storey columns in all cases. If n represents the total number of floors, the relative distance (z i.e. $l = z\,\times$column height) between the point of contra flexure and the point of maximum bending moment can be represented for 1st and 2nd floor as shown in Eqn. 11.1 and 11.2 respectively. For simplicity, yet reliable, z above 2nd floor is assumed to be 0.5.

For 1st floor

$$z = 0.00222n^2 - 0.0183n + 0.6495 \tag{11.1}$$

For 2nd floor

$$z = 0.005n + 0.48 \tag{11.2}$$

2.2. Algorithm

An algorithm is used to solve Eqn. 10, presented stepwise as follows

(A) The input parameters required are listed below
   - (i) Description/detail of a closed symmetric plan
   - (ii) Beam size
   - (iii) Target drift
   - (iv) Material properties
   - (v) A trial bottom column depth, h_{ci}

(B) Derive the trial column width for other floors as 0.9 times that of the lower floor until 3rd storey. The rest are set the same as that of the 3rd storey. This is being done just to reduce the number of iterations. The significance of the multiplier 0.9 vanishes with consecutive iterations.

(C) Calculate the seismic weight for column depth h_{ci}

(D) Calculate the floor wise design lateral force according to the general DBD procedure and hence individual column shear force.

(E) Calculate the corresponding column depth h_{ci+1} from Eqn. 10 using Newton-Raphson method as follows

$$f(h_{ci}) = \theta_{yt} l h_{ci}^2 - 0.7\varepsilon_y l^2 h_{ci} \emptyset_o \omega_m - \frac{31.2V(1+\rho)c_r\emptyset_o}{0.9E} = 0 \tag{12.1}$$

$$f'(h_{ci}) = 2\theta_{yt} l h_{ci} - 0.7\varepsilon_y l^2 \emptyset_o \omega_m = 0 \tag{12.2}$$

$$h_{ci+1} = h_{ci} - \frac{f(h_{ci})}{f'(h_{ci})} \tag{12.3}$$

The tolerance of convergence is taken as 1×10^{-6} during the application.

(F) Check for $(h_{ci+1} - h_{ci}) < 0.0001$, assuring that the resulting shear force from the iterative DBD matches for two consecutive iterations and hence the column depth for each floor column. If so, the last value of column depth for each floor gives the required depth for the respective floor. Else, go to step (C).

Based on this algorithm, a computer program is written which is used throughout the column size generation in this report.

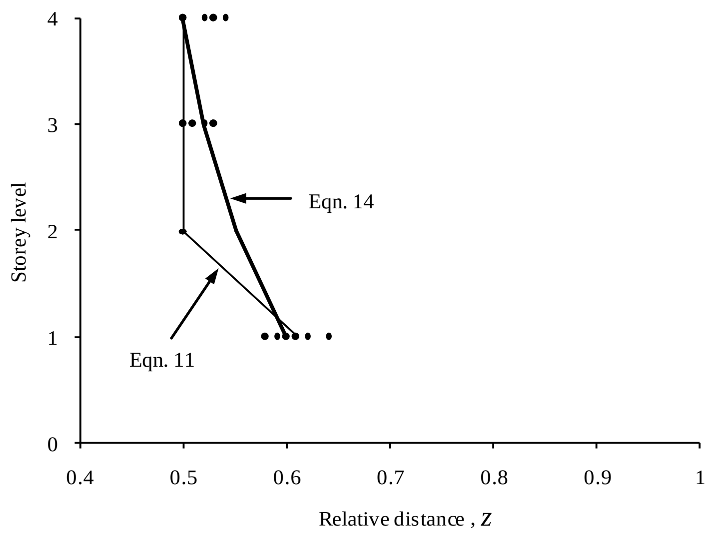

Figure 2. z value for 4-storey buildings.

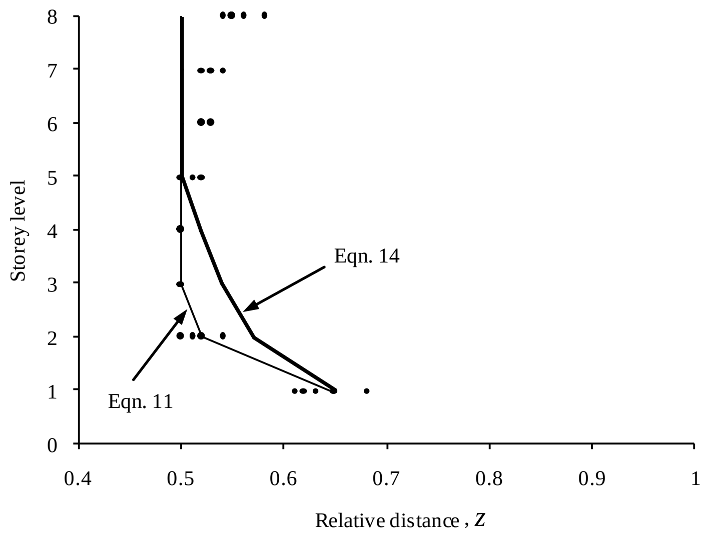

Figure 3. z value for 8-storey buildings.

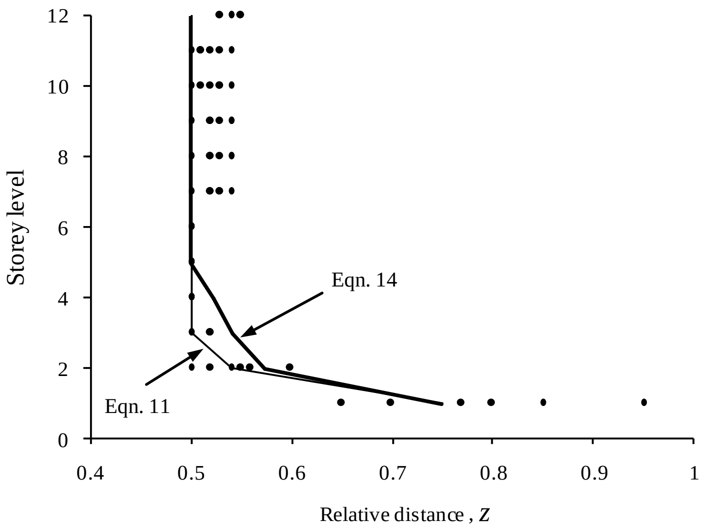

Figure 4. z value for 12-storey buildings.

3. DESIGN AND VALIDATION

The method is applied for the code assumed drift limit of 2% target IDR to a closed symmetrical plan having four beam spans of 5 meters each in the long direction and three beam spans of 6 meters each in the short direction shown in Figure 5. A constant floor height of 3.3 meters is used throughout in buildings of 4-storey, 8-storey and 12-storey.

Design spectrum of 0.45g acceleration level for type B soil is considered as per Eurocode 8 (CEN, 2002). Design displacement spectra set is shown in Figure 6. A corner period extension of 5 seconds is applied to tackle larger displacement demand for higher storey buildings and also to incorporate the significance of magnitude on the corner period as per Faccioli *et. al.* (2004). Materials used in the design include concrete of cube strength 30 MPa and reinforcing rebar steel with yield strength 415 MPa, for both expected material strengths as per FEMA-356 are used. Buildings are modelled using SAP2000 (2006). The design procedure is similar to that of Pettinga and Priestley (2005, 2007) without considering dynamic amplification of column shear and column moment, and no similar effort for bottom storey column is taken to assure beam sway mechanism. However, dynamic amplification of IDR is considered according to the capacity design.

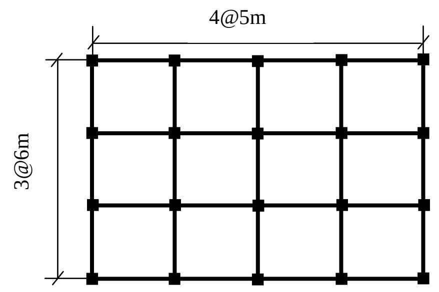

Figure 5. Building plan.

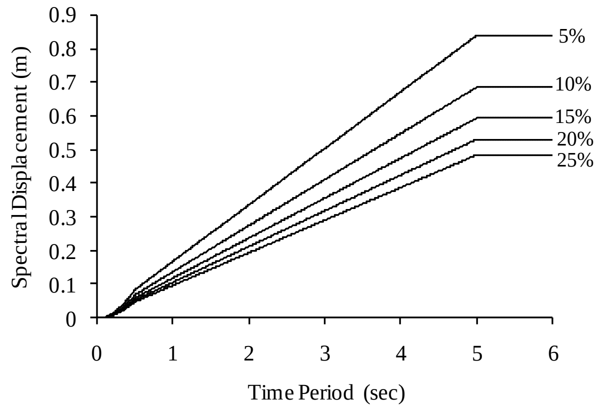

Figure 6. Design displacement spectra

The basic concept of capacity design is to avoid column sway mechanism by making the designed column moment capacity greater than those of the adjacent beams. If, $\sum M_{Rc}$ represents the sum of the design moments of resistance of the columns framing into the joint and $\sum M_{Rb}$ is the corresponding sum of the design moments of resistance of the beams, capacity design criteria suggested as per Eurocode 8 (CEN, 2002) is given in Eqn. 13. Plastic bottom column hinges at the ground level is not made compulsory in this study though it may lead to uneven distribution of %IDR and plastic hinges in the frame members.

$$\sum M_{Rc} \leq 1.3 \sum M_{Rb} \tag{13}$$

Trial column sizes are generated and designed where the reinforcement of some of the lower storey columns above the ground floor for most of the buildings are getting above 4% which is not an ideal situation meaning that those column sections cannot accommodate the required reinforcement below 4%. To handle this, the pattern of point of contra flexure (Eqn. 11.1 and 11.2) up to 4th floor is modified based on experience as given in Eqn. 14.1, 14.2, 14.3 and 14.4 also shown in Figure 2, 3 and 4 for 4, 8 and 12 heights, while the z values for above floor columns remain as 0.5.

For 1st floor

$$z = 1.56 \times 10^{-3} n^2 - 6.25 \times 10^{-3} n + 0.6 \tag{14.1}$$

For 2nd floor

$$z = 5.6 \times 10^{-5} n^3 - 1.9 \times 10^{-3} n^2 + 2.17 \times 10^{-2} + 0.4904 \tag{14.2}$$

For 3rd floor

$$z = 7.0 \times 10^{-5} n^3 - 2.26 \times 10^{-3} n^2 + 2.43 \times 10^{-2} + 0.4546 \tag{14.3}$$

For 4th floor

$$z = 5.31 \times 10^{-5} n^3 - 1.83 \times 10^{-3} n^2 + 2.1 \times 10^{-2} + 0.4418 \tag{14.4}$$

Dynamic amplification of IDR is considered by reducing the design IDR from the actual target IDR. For the 4, 8 and 12 storey building, based on trial analysis, their target IDRs are reduced by 0.8, 0.75 and 0.8 times i.e. they are designed for 1.6%, 1.5% and 1.6% drift respectively. Beam sizes of $350mm \times 600mm$ and $400mm \times 720mm$ are used in long and short direction respectively for the 4-storey building, while for both 8-storey and 12-storey building, $350mm \times 650mm$ and $400mm \times 780mm$ beam sizes are used respectively in long and short direction. Constant beam size is used throughout the building height. The floor wise column sizes are generated (Table 1) for the design IDR corresponding to 2% target IDR.

Table 1. Generated column size

% IDR		Heights	Floor wise column depth in millimeter (square column)											
Target	design		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
2.0	1.6	4-storey	591	544	513	489	-	-	-	-	-	-	-	-
2.0	1.5	8-storey	786	696	661	636	611	607	602	597	-	-	-	-
2.0	1.6	12-storey	941	726	689	666	638	637	636	634	632	630	628	625

Table 2. Details of spectrum compatible ground motions

Sl. No.	Name	Background Earthquake	Record No.	Duration (sec)
1	GM1	Duzce 1999	Duzce, 270 (ERD)	25.9
2	GM2	El Centro 1940	N-S Component	31.8
3	GM3	Gazli 1976	Karakyr, 090	16.3
4	GM4	Kocaeli 1999	Sakarya, 090 (ERD)	30.0
5	GM5	N. Palm Spring 1986	0920, USGS station 5070	20.0

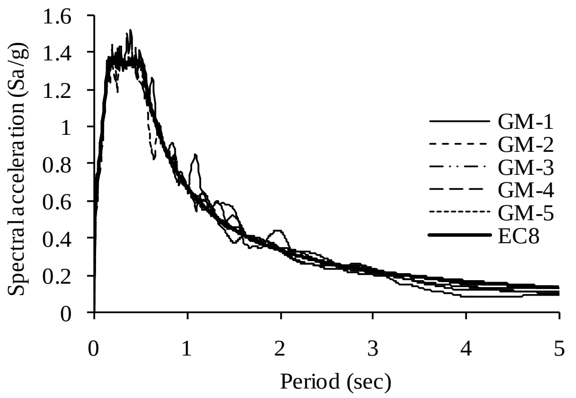

Figure 7. Comparison of Design spectrum with SCGM Response spectrum.

The spectrum compatible ground motions (SCGM) shown in Table 2 are generated by using the software developed by Kumar (2004). The phase angle, frequency content and duration of the background earthquake are incorporated in the SCGM. Figure 7 shows the matching comparison between design spectrum and response spectrum for 0.45g acceleration level and 5% damping. Effective member section properties are used according to Priestley (2003) for non linear analysis. The yield moments are derived from resulting design steel. If E is modulus of elasticity, $I_{eff,beam}$ and $I_{eff,column}$ the effective moment of inertia of beam and column, M_{yb} and M_{yc} the beam and the column yield moment, and $\emptyset_{yb}$ and $\emptyset_{yc}$ the beam (Eqn. 15.2) and column (Eqn. 2) yield curvature, the effective beam and column flexural rigidity is given by Eqn. (15.1) and (15.3) respectively.

$$EI_{eff,beam} = \frac{M_{yb}}{\emptyset_{yb}} \quad (15.1)$$

$$\theta_{yb} = 1.7\frac{\varepsilon_y}{h_b} \quad (15.2)$$

$$EI_{eff,column} = \frac{M_{yc}}{\emptyset_{yc}} \quad (15.3)$$

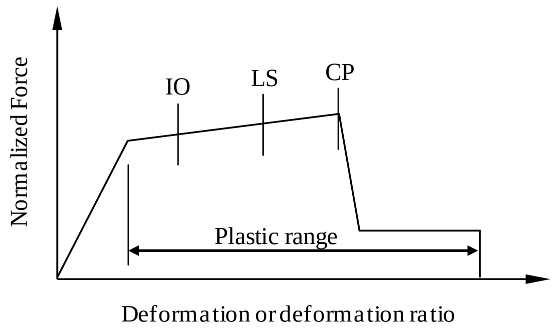

Figure 8. Force-Deformation behavior (FEMA-356).

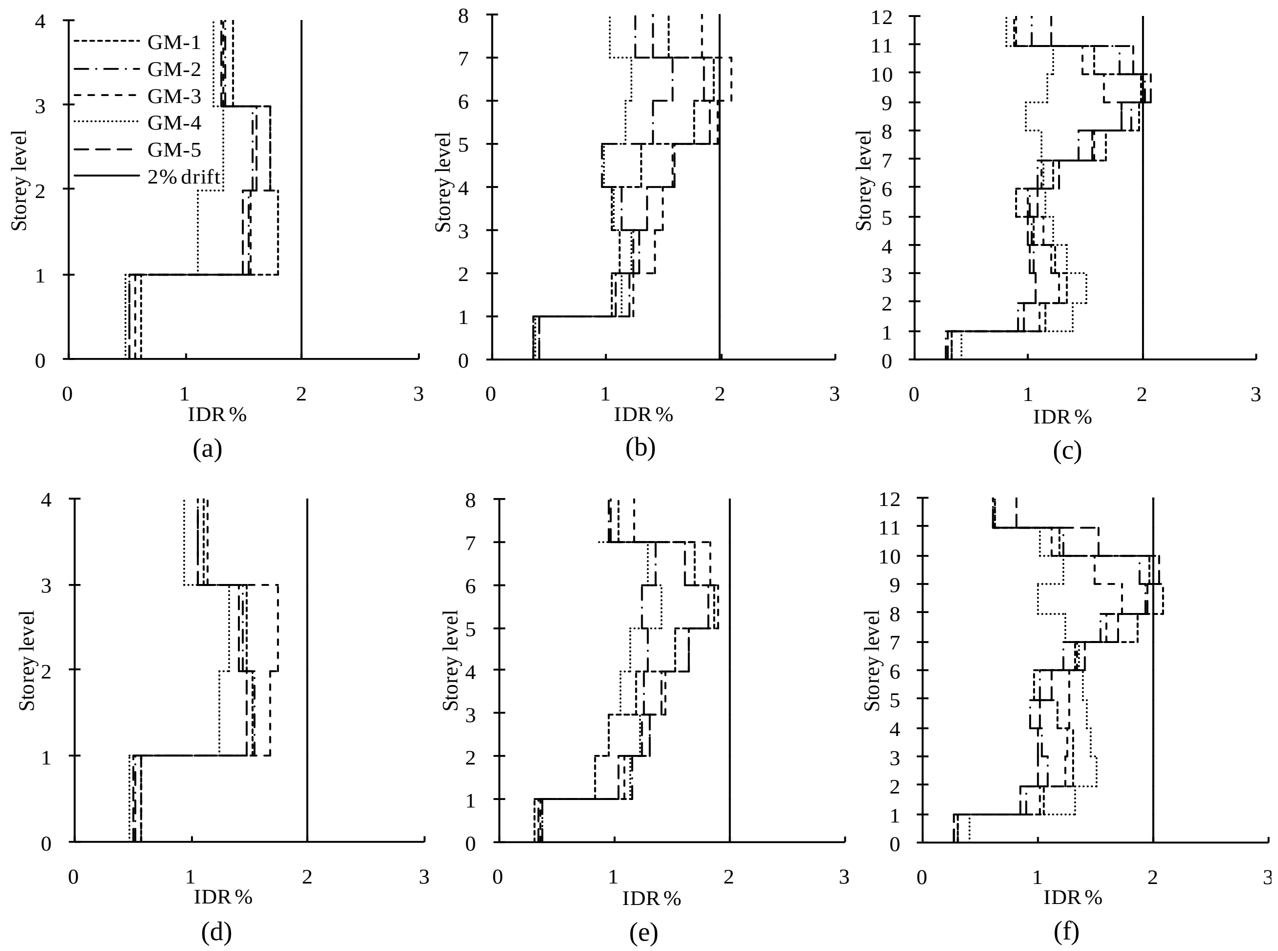

Figure 9. Inter storey drift diagram in long and short directions; (a and d) 4, (b and e) 8, (c and f) 12 storey.

The post-elastic force-deformation behaviour for the members (Figure 8) is adopted as per FEMA-356 (2000). The buildings are subjected to NLTHA under SCGM (Table 2) shown in Figure 7 which assures that the energy levels of the ground motions are set to the level of design spectrum for NLTHA.

The resulting IDR diagrams are shown in Figure 9 are satisfactory. The original target % IDRs of the buildings is achieved with a fair precision. The maximum deviation above the original target % IDR is less than 9.5% in case of 12 storey building shown in Figure 9(f). No columns develop plastic hinges avoiding column sway mechanism (Figure 10). The maximum angle of plastic rotation allowed for beams controlled by flexure in radian for immediate occupancy (IO) , life safety (LS) and collapse prevention (CP) levels are 0.00625, 0.01125 and 0.01875 respectively (taken as average from FEMA-356). Hinges formation on beams at last step in time history analysis are found to be within LS level shown by blue hinges in Figure 10(a), 10(b), 10(c), 10(d) and 10(e) while majority hinges are of IO represented by pink hinges.

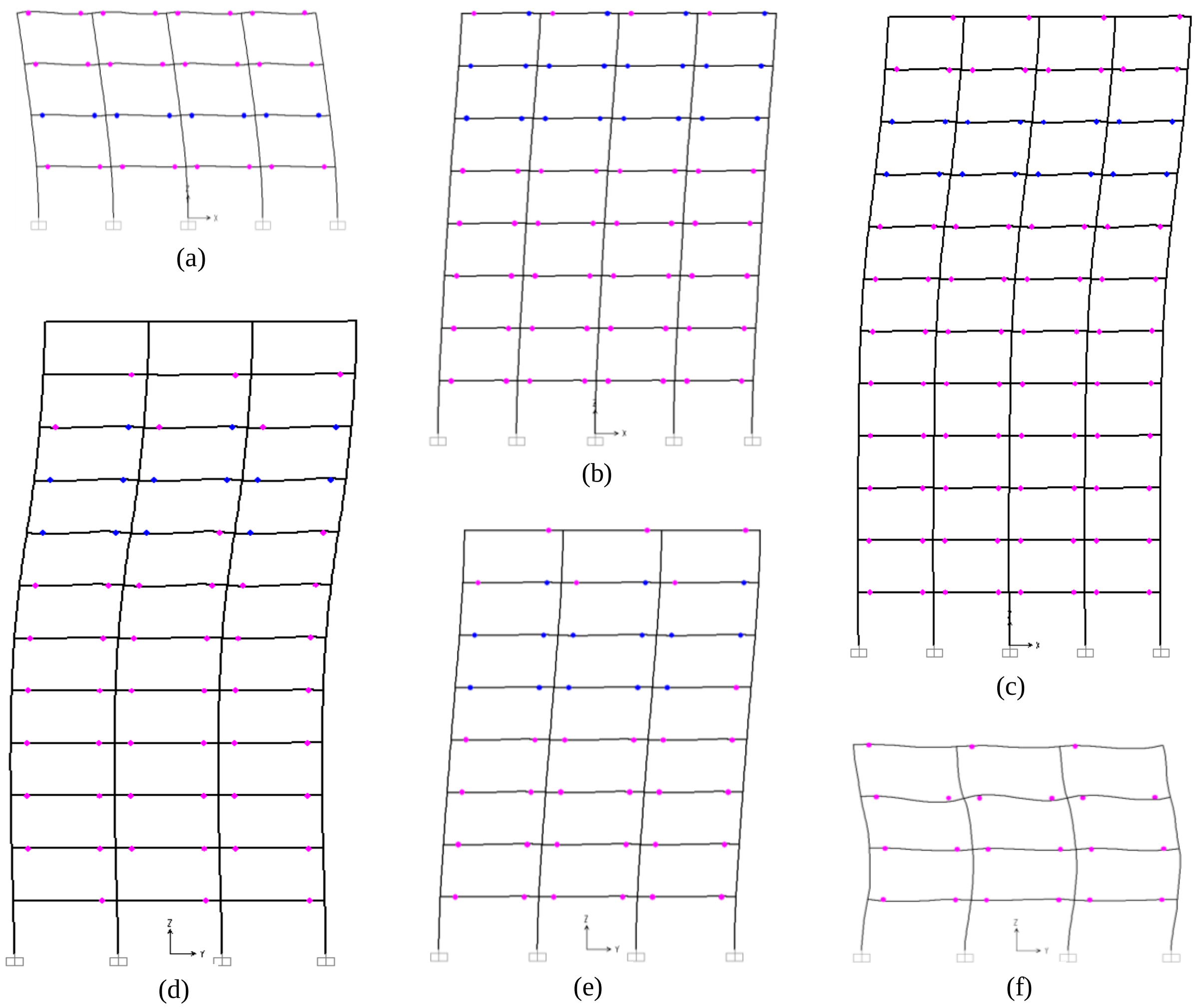

Figure 10. Hinges in typical frames in long and short direction; (a and f) 4 storey under GM1, (b and e) 8 storey under GM5, (c and d) 12 storey under GM3

4. CONCLUSION

This article reports an approach to fairly predict the floor wise column size of reinforced concrete frame buildings for displacement based design meant for intermediate target drift. An algorithm based on empirical relations is used which requires given beam section, design inter-story drift, building plan features and regular displacement based design parameters as the inputs.

To maintain a smooth descending column width with building height, a pattern of point of contra flexure is introduced as a parameter required in the process. The pattern is later modified to maintain reasonable percentage of design reinforcement. Necessary adjustments of column beam moment capacity ratio and design inter storey drift are made while designing.

The generated column size using the method may or may not undergo minor changes in the dimension according to the designer's wish, can be effectively used for designing reinforced concrete frame buildings of intermediate target drift which is meant for codal designed buildings using direct displacement based design with some usual adjustments for inter storey drift and proper capacity design in the design method.

REFERENCES

CEN, European Prestandard ENV 1998-1-4. (May 2002). Eurocode 8 — Design Of Structures for Earthquake Resistance, Part 1: General Rules, Seismic Actions and Rules for Buildings. *Comit´eEurop`een de Normalisation, Brussels* Draft No. 5, Doc CEN/TC250/SC8/N317.

Faccioli, E., Paolucci, R. and Rey, J. (2004). Displacement spectra for long periods. *Earthquake Spectra* **20:2**, 347–376.

FEMA-356 (2000). Prestandard and Commentary for the Seismic Rehabilitation of Buildings. *US Federal Emergency Management Agency, Washington D.C.*

Kumar, A. (2004). Software for generation of spectrum compatible time history. *Proceedings of 13th World Conference on Earthquake Engineering, Canada*, 2096**.**

Matamoros, A. B. (1999). Drift Limits of High-Strength Concrete Columns. *PhD. Thesis, Department of Civil Engineering, University of Illinois at Urbana Champaign*.

Park, R., and Paulay, T. (1975). Reinforced Concrete Structures. John Wiley and Sons, New York.

Paulay, T. and Priestley, M. J. N. (1992). Seismic design of reinforced concrete and Masonry buildings. John Wiley and Sons, Inc., New York.

Pettinga, J.D. and Priestley, M.J.N. (2005). Dynamic behavior of reinforced concrete frames Designed with Direct Displacement-Based Design. *Journal of Earthquake Engineering* **9:2**, 309-330.

Pettinga, J.D. and Priestley, M.J.N. (2007). Accounting for p-delta effects in structures when using direct displacement-based design. *Research Report ROSE (European School for Advanced Studies in Reduction of Seismic Risk), IUSS Press, Pavia.*

Priestley, M.J.N. (2003). Myths and fallacies in earthquake engineering, revisited. *European School for Advanced Studies in Reduction of Seismic Risk*, 9th Mallet-Milne Lecture.

SAP2000 V. 10.12 (2006). Structural analysis program. *Computer and Structures Inc., Berkley, CA.*