Application of high-order absorbing boundary condition in dynamic analysis of fluid-structure systems

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SUMMARY:
One of the most efficient techniques for considering the unbounded media in a mathematical model is the local non-reflecting boundary conditions (NRBCs). A relatively recent approach in this context is the one proposed by Hagstrom and Warburton (H-W boundary condition). This approach is available for scalar wave equation. This equation governs the hydrodynamic pressure distribution inside a reservoir and hence, the H-W boundary condition may be used to solve the corresponding problems. A well-known case in which one has to deal with the dynamic analysis of unbounded reservoir is the dam-reservoir interaction problem. In this study, the H-W NRBC has been applied to this problem and its harmonic response is calculated. By comparing the results with the exact solution, the performance and accuracy of this NRBC is examined. The numerical results confirm the very good behavior of the NRBC in the frequencies above the fundamental frequency of the reservoir. However, below this frequency range, this boundary condition does not perform very well, especially when it is applied in close distances from the dam.

Keywords: Dam-reservoir interaction, High order Non-reflecting boundary condition, H-W boundary condition

1. INTRODUCTION

Unbounded domains are encountered in a wide range of engineering problems. In order to analyze this kind of problems by mathematical modelling, one has to utilize special techniques to include the effects of this unboundedness into the model. Considering Fig. 1, the common feature in most of these techniques is setting up a truncation boundary (Γ_I) in the semi-infinite domain and solving the wave equation in the enclosed domain Ω. In order to contain the effects of omitted part D in the solution of the problem, one can either perform a pre-analysis of the unbounded domain or simply apply a transmitting boundary condition on the truncation boundary. The first approach is based on analyzing the problem in D, and finding a differential relation, which states the variation of the problem's solution and its derivatives on boundary Γ_I. This relation is then used as the boundary condition for solving the problem in Ω. Some of the most famous techniques in this category are DtN maps, boundary integral methods, thin layer method (Lysmer & Waas (1972)) and absorbing layers; while, in some of these methods, the analysis of D is carried out along with the analysis of Ω, in others, it is necessary to perform these two phases of analysis, separately.
In another approach, the boundary condition on Γ_I is not defined based on a pre-analysis of infinite domain. The first boundary condition of this type is the well-known Sommerfeld BC. This boundary condition is the radiation condition at infinity; however, by taking a sufficient distance from the scatterer, it may be applied on the truncation boundary with an acceptable error. The asymptotic boundary condition, proposed by Engquist & Majda (1977, 79), is another example in this category. This boundary condition is based on the Pade approximation of dispersion relation. It is also shown that this boundary condition would result in a well-posed problem. A similar approach which is proposed by Higdon (1986, 94), is the multidirectional boundary condition. By applying this boundary condition on Γ_I, inward plane waves, which are propagating in a certain set of directions, will be eliminated from the solution of the problem. There can be found other boundary conditions, which are

based on infinite products of first order differential operators. However, none of these methods were applied for orders higher than 2 or 3, before the mid 1990s.

Collino (1992) proposed the first implementation of asymptotic boundary condition for higher orders in finite difference method. In order to improve the stability of the solution, a set of corner compatibility conditions was also included in this study. Givoli and Neta (2003a, b) used a sequence of auxiliary variables to employ the higher orders of Higdon NRBC in finite difference and finite element methods to solve the waveguide problem. van Joolen et al (2005) also used the same technique to solve a fully exterior two dimensional problem; however, in the absence of any special corner treatments, some long time instabilities were observed in the solution. In order to solve this problem, Vacus (2004) proposed a technique to produce a set of compatibility equations in the corners of the medium, which could highly enhance the stability of the results. Hagstrom and Warburton (2004) proposed a new high order boundary condition, which was based on a modification of Givoli-Neta auxiliary variable NRBC. Givoli et al. (2006) implemented this boundary condition in finite element method and performed an analytical comparison between Givoli-Neta (G-N) and Hagstrom-Warburton (H-W) NRBCs. An important issue about the above techniques is their inability to absorb evanescent waves. In order to resolve this issue, Hagstrom et al (2008) proposed a modification to the HW boundary condition by adding an extra set of auxiliary variables which characterize the evanescent waves at the truncation boundary.

One of the problems, in which the unbounded media are involved, is the dam-reservoir interaction problem. In this problem, the reservoir may be considered as a semi-infinite waveguide, and divided into two domains: near-field and far-field. While the near-field and dam body are modeled by finite element method, it is necessary to employ an appropriate technique to take account of the reservoir's far field. On the other hand, some of the special features of reservoir's boundary conditions, such as vertical base excitation at the reservoir's bottom are not commonly addressed in general studies about the transmitting boundaries; yet, there may be found several techniques for this purpose in the literature. While, some of these techniques (Hall & Chopra (1982) and Tsai & Lee (1990)) are based on global procedures, others are based on local boundary conditions like the ones, proposed by Sharan (1987) and Weber (1994).

In this paper, the H-W boundary condition has been utilized to construct an appropriate boundary condition for the dam-reservoir interaction problem. Since the exact solution of this problem exists in the frequency domain, the results of this study are also presented in the frequency domain. However, by applying this method in the time domain, no special complexities would arise. Finite element method is used to discretize the solid and fluid domains. The results of the analyses are presented in terms of the variation of displacement at dam crest and total hydrodynamic force, for different excitation frequencies.

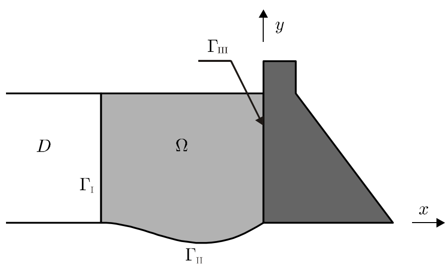

Figure 1. Schematic view of a 2D dam-reservoir system.

2. GOVERNING EQUAITON OF THE DAM-RESERVOIR SYSTEM

The governing equation of the dam-reservoir system may be established by coupling the corresponding equations of the solid and fluid domains. The coupling relation, which is used here, describes the relationship between dam accelerations and water hydrodynamic pressures, on the dam reservoir interface. In order to obtain the dynamic response of the dam to external actions, one can use the following equation (Taylor & Zienkiewicz (2000)):

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} = -\mathbf{M}\mathbf{J}\mathbf{a}_\mathrm{g} + \mathbf{B}^\mathrm{T}\mathbf{P} \tag{2.1}$$

Where, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the dam's mass, damping and stiffness matrices. $\mathbf{r}$ is the vector of nodal displacements. $\mathbf{a}_\mathrm{g}$ is the vector of ground acceleration and will be applied on the corresponding DOFs of the dam by means of matrix $\mathbf{J}$. $\mathbf{B}$ is the interaction matrix, which integrates the hydrodynamic pressures of the reservoir's water to calculate the hydrodynamic forces. In the current study, the excitation of the system is supposed to be harmonic and applied in the horizontal direction. As a result, the response of the system is also harmonic and may be written as $\mathbf{r}(t) = \mathbf{r}\exp(\mathrm{i}\omega t)$. On the other hand, the damping is assumed as a hysteretic type, i.e., $\mathbf{C} = (2\beta/\omega)\mathbf{K}$; where β is the hysteretic damping factor. Accordingly, Eqn. (2.1), would yield:

$$\left(-\omega^2\mathbf{M} + (1 + 2\mathrm{i}\beta)\mathbf{K}\right)\mathbf{r} = -\mathbf{M}\mathbf{J}\mathbf{a}_\mathrm{g}^\mathrm{h} + \mathbf{B}^\mathrm{T}\mathbf{P} \tag{2.2}$$

It should be mentioned that the superscript h on the acceleration vector refers to the horizontal type of excitation. That is: $\mathbf{a}_\mathrm{g}^\mathrm{h} = \begin{pmatrix} a_\mathrm{g}^x & 0 \end{pmatrix}^\mathrm{T}$.

For the fluid domain, both velocity potential and hydrodynamic pressure may be used as the independent variable to establish the governing equation. The latter is more common in engineering applications and has been utilized to describe the reservoir's state in this paper. Considering the water to be an inviscid and compressible fluid, with small irrotational movements, the hydrodynamic pressures inside the reservoir are governed by the scalar wave equation (Fig. 1):

$$\nabla^2 p(t) - \frac{1}{c^2}\ddot{p}(t) = 0 \quad \text{in } \Omega \text{ and } D \tag{2.3}$$

where c is the sonic velocity in water and p denotes the hydrodynamic pressures. Since the analysis is going to be carried in the frequency domain, the hydrodynamic pressure is supposed to have a harmonic form, i.e., $p(t) = p\,\exp(\mathrm{i}\omega t)$. Hence, Eqn. (2.3) and its boundary conditions would find the following form:

$$\nabla^2 p + \frac{\omega^2}{c^2} p = 0 \quad \text{in } \Omega \text{ and } D \tag{2.4}$$

$$p = 0, \quad \text{on the water surface} \tag{2.5}$$

$$\partial_n p = -\rho a_\mathrm{g}^n - \mathrm{i}\omega q p, \quad \text{at the reservoir's bottom} \tag{2.6}$$

ρ is the mass density of water and q is the absorption coefficient of reservoir's bottom. Although, Eqn. (2.4) does not contain any dispersion term, its boundary conditions are arranged such that, dispersion occurs in all of the reservoir's modes of vibration.

In order to solve Eqn. (2.4) by finite element method, one can employ the weighted residual approach and apply the boundary conditions of the problem to obtain:

$$-\omega^2\mathbf{G}\,\mathbf{P} + \mathrm{i}\omega\, q\,\mathbf{L}_\mathrm{II}\,\mathbf{P} + \mathbf{H}\,\mathbf{P} = \mathbf{R}_\mathrm{I} + \omega^2\mathbf{B}\,\mathbf{r} - \mathbf{B}\,\mathbf{J}\,\mathbf{a}_\mathrm{g}^\mathrm{h} \tag{2.7}$$

where, $\mathbf{G}$, $\mathbf{L}_\mathrm{II}$ and $\mathbf{H}$ are characteristic matrices of the fluid domain and may be found elsewhere (Samii & Lotfi (2012)). While $\mathbf{B}$ and $\mathbf{J}$ have already been defined, $\mathbf{R}_\mathrm{I}$ is obtained by assembling the element matrices with the following definition:

$$\mathbf{R}_\mathrm{I}^e = \frac{1}{\rho}\oint_{\Gamma_\mathrm{I}^e} \mathbf{N}(\partial_n p)\,\mathrm{d}\Gamma_\mathrm{I}^e \tag{2.8}$$

In order to calculate the above integral, we should know $\partial_n p$ on the truncation boundary. This is achieved by formulating the absorbing boundary condition.

3. APPLYING H-W NRBC ON THE UPSTREAM BOUNDARY OF RESERVOIR

H-W boundary condition was proposed as a modification of Higdon's boundary conditions by Hagstrom & Warburton (2004), to enhance its efficiency. This NRBC leads to a set of balanced symmetrizable systems of equations on Γ_{I}, and its reflection coefficient is proved to be much less than Higdon boundary condition. H-W radiation condition of order J may be written as a recursive sequence of auxiliary variables, as below:

$$\partial_t \phi_1 = (a_0 \partial_t - c\partial_x) p \tag{3.1}$$
$$(a_j \partial_t + c\partial_x)\phi_{j+1} = (a_j \partial_t - c\partial_x)\phi_j, \quad j \in \{1, \cdots, J\} \tag{3.2}$$
$$\phi_{J+1} = 0. \tag{3.3}$$

It may be shown that by combining the above relations along with Eqn. (2.3), one would obtain the following boundary relations on Γ_{I}. The derivation process is explained by Samii & Lotfi (2012):

$$\partial_x p = \frac{\mathrm{i}\omega}{c}(a_0 p - \phi_1) \tag{3.4}$$
$$\begin{aligned} &- \omega^2 a_j (a_{j-1}^2 - 1)\phi_{j-1} + a_j c^2 \partial_y^2 \phi_{j-1} + \omega^2 (1 + a_j a_{j-1})(a_j + a_{j-1})\phi_j \\ &+ c^2 (a_j + a_{j-1})\partial_y^2 \phi_j - \omega^2 a_{j-1}(a_j^2 - 1)\phi_{j+1} + a_{j-1} c^2 \partial_y^2 \phi_{j+1} = 0, \quad j \in \{2, \cdots, J\} \end{aligned} \tag{3.5}$$
$$\begin{aligned} &-2\omega^2 a_1 (a_0^2 - 1) p + 2a_1 c^2 \partial_y^2 p + \omega^2 (2a_0 a_1 + a_1^2 + 1)\phi_1 \\ &\qquad + c^2 \partial_y^2 \phi_1 - \omega^2 (a_1^2 - 1)\phi_2 + c^2 \partial_y^2 \phi_2 = 0 \end{aligned} \tag{3.6}$$
$$\phi_{J+1} = 0 \tag{3.7}$$

3.1. Utilizing the absorbing boundary condition in dam-reservoir equations

In order to employ this boundary condition in reservoir's equation, $\partial_n p$ in Eqn. (2.8) may be substituted with $-\partial_x p$, which is equal to $-\mathrm{i}\omega(a_0 p - \phi_1)/c$ from Eqn. (3.4). As a result, the contribution related to Γ_{I}^e in Eqn. (2.7) finds the following form:

$$\mathbf{R}_{\mathrm{I}}^e = -\frac{\mathrm{i}\omega}{\rho c}\int_{\Gamma_{\mathrm{I}}^e} \mathbf{N}(a_0 p - \phi_1)\,\mathrm{d}\Gamma_{\mathrm{I}}^e \tag{3.8}$$

For H-W boundary condition of order zero, one has $\phi_1 = 0$, which results in Sommerfeld boundary condition by choosing a_0 equal to 1. For higher orders, ϕ_1 may be interpolated similar to p; hence, one would obtain:

$$\mathbf{R}_{\mathrm{I}}^e = -\frac{\mathrm{i}\omega}{c}\mathbf{L}_{\mathrm{I}}^e\left(a_0 \mathbf{P}^e - \mathbf{\Phi}_1^e\right) \tag{3.9}$$

Now $\mathbf{R}_{\mathrm{I}}^e$ may be assembled into (2.7) and combining the resulting equation with the governing relation (2.2) of the solid domain would yield:

$$\begin{bmatrix} \mathbf{S}_{\mathrm{dam}} & -\mathbf{B}^{\mathrm{T}} & \mathbf{0} \\ -\omega^2 \mathbf{B} & (\mathbf{S}_{\mathrm{res}} + \frac{\mathrm{i}\omega a_0}{c}\mathbf{L}_{\mathrm{I}}) & -\frac{\mathrm{i}\omega}{c}\mathbf{L}_{\mathrm{I}} \end{bmatrix} \begin{Bmatrix} \mathbf{r} \\ \mathbf{P} \\ \mathbf{\Phi}_1 \end{Bmatrix} = \begin{Bmatrix} -\mathbf{M}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} \\ -\mathbf{B}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}}^{\mathrm{h}} \end{Bmatrix} \tag{3.10}$$

With the following definitions:

$$\mathbf{S}_{\mathrm{dam}} = -\omega^2 \mathbf{M} + (1 + 2\beta \mathrm{i})\mathbf{K} \tag{3.11}$$
$$\mathbf{S}_{\mathrm{res}} = -\omega^2 \mathbf{G} + \mathrm{i}\omega q \mathbf{L}_{\mathrm{II}} + \mathbf{H} \tag{3.12}$$

Obviously, the above system of equations is not complete. Therefore, an extra set of equations are required in terms of $p, \phi_1, \cdots, \phi_J$ which will be produced by discretization of Eqn. (3.5) and Eqn.

(3.6) on Γ_{I}.

3.2. Discretization of the non-reflecting boundary

In order to utilize the finite element method for solving Eqn. (3.5) and Eqn. (3.6), one can apply weighted residual method on these equations. While the derivation procedure of these equations may be found elsewhere (Samii & Lotfi (2012)), their final discretized form are as follows

$$\omega^2 \mathbf{L}_{\mathrm{I}} \left(-\gamma_j \mathbf{\Phi}_{j-1} + \delta_j \mathbf{\Phi}_j - \bar{\gamma}_j \mathbf{\Phi}_{j+1}\right) \\ - c^2 \bar{\mathbf{D}}_{\mathrm{I}} \left(a_j \mathbf{\Phi}_{j-1} + \eta_j \mathbf{\Phi}_j + a_{j-1} \mathbf{\Phi}_{j+1}\right) = 0, \quad j \in \{2, \cdots, J\} \tag{3.13}$$

$$\omega^2 \mathbf{L}_{\mathrm{I}} \left(-2\gamma_1 \mathbf{P} + \bar{\delta}_0 \mathbf{\Phi}_1 - \frac{\gamma_2}{a_2} \mathbf{\Phi}_2\right) - c^2 \bar{\mathbf{D}}_{\mathrm{I}} \left(2a_1 \mathbf{P} + \mathbf{\Phi}_1 + \mathbf{\Phi}_2\right) = 0 \tag{3.14}$$

Where,

$$\mathbf{Q}_{\mathrm{I}} = \frac{1}{\rho} \tilde{\mathbf{I}}_{\mathrm{I}} \tilde{\mathbf{I}}_{\mathrm{I}}^{\mathrm{T}} \tag{3.15}$$

$$\bar{\mathbf{D}}_{\mathrm{I}} = \mathbf{D}_{\mathrm{I}} + \mathrm{i}\omega q \mathbf{Q}_{\mathrm{I}} \tag{3.16}$$

$$\begin{cases} \gamma_j = a_j (a_{j-1}^2 - 1); & \delta_j = (1 + a_j a_{j-1})(a_j + a_{j-1}); \\ \bar{\gamma}_j = a_{j-1} ({a_j}^2 - 1); & \eta_j = (a_j + a_{j-1}); \end{cases} \quad \text{for } j \in \{1, \cdots, J\}$$

$$\bar{\delta}_0 = \left(2a_0 a_1 + a_1^2 + 1\right); \tag{3.17}$$

In these relations, $\tilde{\mathbf{I}}_{\mathrm{I}}$ is a vector, whose elements are all equal to zero, except the one that corresponds to the bottom node of Γ_{I}; $\mathbf{D}_{\mathrm{I}}$ may also be obtained by assembling the corresponding element matrices, which are defined as below:

$$\mathbf{D}_{\mathrm{I}}^e = \frac{1}{\rho} \int_{\Gamma_{\mathrm{I}}^e} \mathbf{N}_y \mathbf{N}_y^{\mathrm{T}} \mathrm{d}\Gamma_{\mathrm{I}}^e \tag{3.18}$$

Now one should establish a system of equations based on (3.10), (3.13) and (3.14), and solve it to find the response of the relevant fluid-structure system.

4. NUMERICAL EXPERIMENTS

The introduced method is employed to analyze a typical dam-reservoir system. Since the rigorous solution of our problem has been calculated in the frequency domain, we will also present the analysis results in the frequency domain; however, the whole analysis procedure may be carried out in the time domain. The mentioned rigorous solution, which is proposed by Hall and Chopra (1982), is based on the hyper-element method. This method treats the infinite dimension of the reservoir analytically and uses the finite element discretization for the cross section of the reservoir.

The general setup of the considered dam-reservoir system is illustrated in Fig. 2. The height of the dam and reservoir is taken as 200 meters in all of the analysis cases. Dam and reservoir are assumed to be placed on a rigid foundation, and the system is excited with horizontal ground motion of frequency ω. The material properties of the dam's concrete and reservoir's water are listed in Table 1.

Table 1. Material properties of the model

Concrete modulus of elasticity	27.5 GPa
Concrete Poisson's ratio	0.2
Unit weight of concrete	24 kN/m^3
Pressure wave velocity in water	1440 m/s
Unit weight of water	9.81 kN/m^3

4.1. Range of excitation frequency and FE mesh

One of the important aspects of the analysis procedure is its reliability in the frequency range, which is applied to the real structure. In this study, we will calculate the response of the system for excitation frequencies below 12 Hz.

The employed finite element mesh for dam and reservoir consists of 2D quadratic isoparametric elements. The size of the elements should be able to simulate the shape of waves, which are propagating inside dam and reservoir. Besides, it should also be noticed that, in this study, the accuracy of higher order NRBCs are going to be compared with the semi-analytical exact solutions, with the same mesh and material properties. Therefore, satisfying the minimums for the mesh size should make this study perfectly reliable. As a result, the size of the fluid and solid elements are taken smaller than 40 m. This will result in a mesh with 5 layers of elements along the height of the dam; the number of elements in x-direction is variable with the length of the reservoir. Nevertheless, In order to evaluate the sensitivity of the response to the mesh size, one of the experiments has been carried out for a model with 20 m mesh size.

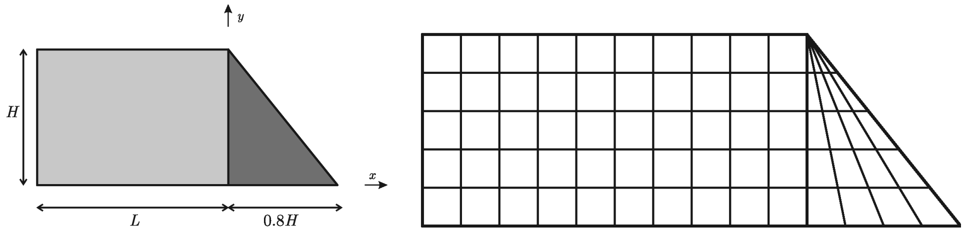

Figure 2. (a) Geometry of the model, used in numerical analyses, and (b) the corresponding FE mesh for $L/H = 2.0$.

4.2. Analysis results

In this section, several analysis results are presented to investigate the performance of H-W boundary condition. The first set of results are produced by using the Sommerfeld boundary condition, which is known to converge to the exact solution of the problem, when the truncation boundary is located at an infinitely large distance from the wave source. In dam-reservoir interaction problem, this distance is often characterized by L/H, which is the length to height ratio of the reservoir. As shown in Fig. 3, for this case, the results are converging to the exact solution; however, even for $L/H = 3.0$, Sommerfeld BC exhibits some major instabilities, in our desired frequency range. The horizontal axis of Fig. 3, shows the excitation frequency of the system, which is normalized with respect to the first natural frequency of the dam with an empty reservoir. The reflection coefficient (α) for all the analysis cases has been taken equal to 1.0. This reflection coefficient corresponds to a rigid reservoir's bed, which results in $q = 0$ in Eqn. (2.6).
Example 1: Now, as the first experiment on H-W boundary condition, we consider a model with $L/H = 1$ and the reflection coefficient at the reservoir's bed is taken as $\alpha = 1$. By applying horizontal excitation on the system, the transfer function of the dam crest acceleration will be plotted for different orders of boundary condition. In this case, all of the a_js in Eqn. (3.2) are taken equal to 1.0. The results are shown in Fig. 4, along with the exact solution of the problem. By increasing the order of the boundary condition, some oscillations can be observed before the first peak of the response. In order to show the behavior of the system at this range, a blow up of the response is also plotted beside each full range graph. This peak corresponds to a frequency, where the whole system is resonated. In our current problem, this frequency is very close to the cutoff frequency of the reservoir. At this frequency, which is in fact equal to reservoir's first natural frequency, the vibrational behavior of reservoir's water alternates from evanescent waves, at which the phase velocity is zero, to propagating waves, where the phase velocity is variable with excitation frequency. It is observed that the higher order NRBCs are very effective in absorption of propagating waves; however, they do not particularly enhance the performance of the method below the cutoff frequency. This issue has been recognized by Hagstrom et al. (2008), and in later improvements of the H-W NRBC, they have included the effect of

evanescent modes in the analysis of the boundary condition.

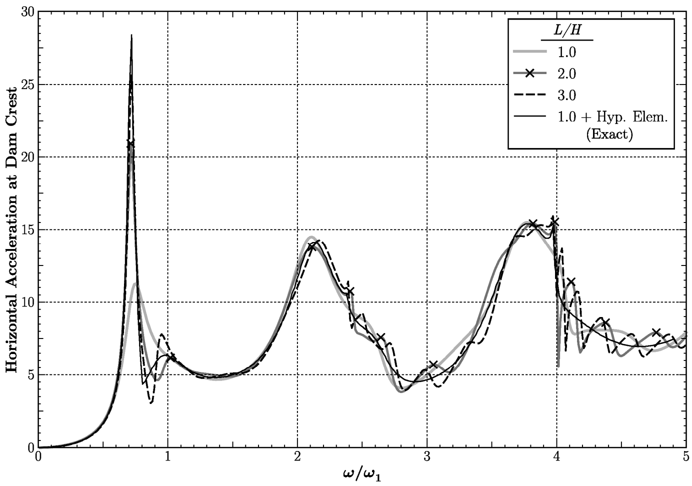

Figure 3. Horizontal acceleration at the dam crest due to horizontal excitation for models with different ratios of L/H and applying Sommerfeld BC on Γ_{I}.

Example 2: As mentioned before, the mesh size of the current model may seem to be a matter of concern. Therefore, we have examined one of the above cases, for a model, in which, each element is divided into four smaller elements. Hence, the element size in this refined mesh is less than 20 m. Fig. 5 compares the results of regular and refined meshes for order 5 boundary condition. We have also included the exact results for both refined and regular meshes. As can be observed, the results of high order boundary condition and hyper-element method follow similar trends for both regular and refined meshes. Especially, it should be mentioned that the mesh size cannot be accounted for the oscillations around the cutoff frequency. It is also noted that, mesh refinement has affected the response more noticeably at higher frequencies even for the hyper-element method (i.e., our exact solution).

Example 3: In example 1, the effect of dam vibration was included in the response of the system. In order to find a more detailed scope on the performance of the boundary condition, we consider the reservoir to be placed next to a rigid wall. Again, α is taken equal to 1.0, which results in no absorption at the reservoir's bottom. By applying horizontal excitation on this setup, the total hydrodynamic force (F_{water}) over the wall is calculated. The results have been calculated for the same orders and same a_js as the previous example. The corresponding results are shown in Fig. 6; the exact solution of the problem is also plotted alongside, for comparison purposes. The exact solution of hydrodynamic pressure distribution along the height of the dam may be calculated by means of the following relation:

$$p = \sum_{j=1}^{\infty} B_j \cos(\lambda_j y) \exp(k_j x) \tag{4.1}$$

where,

$$\lambda_j = \frac{2j-1}{2h}\pi; \quad k_j = \sqrt{\lambda_j^2 - \frac{\omega^2}{c^2}}; \quad B_j = \frac{2\rho a_g^x}{h}\left(\frac{(-1)^{j+1}}{k_j \lambda_j}\right) \tag{4.2}$$

Again, it is worth noticing that, for the current boundary conditions, none of λ_js in the above relations is zero; therefore dispersion occurs in all of the modes of the reservoir.

In this example, the frequency axis is normalized with respect to the first cutoff frequency of the reservoir (i.e., ω_1^r). Since, the results are calculated in the frequency domain, real and imaginary parts of F_{water} are plotted separately. Below the cutoff frequency, F_{water} is a completely real variable (refer to Eqn. (4.1) and Fig. 6); this kind of response corresponds to evanescent waves in the reservoir which are decaying as we move farther from the dam body. As it can be observed in Fig. 6, by increasing the order of NRBC, some oscillatory behaviour occur in this frequency range. However, in higher frequencies, where propagating waves are the dominant part of the solution, the response converges to the exact solution by increasing the order of NRBC.

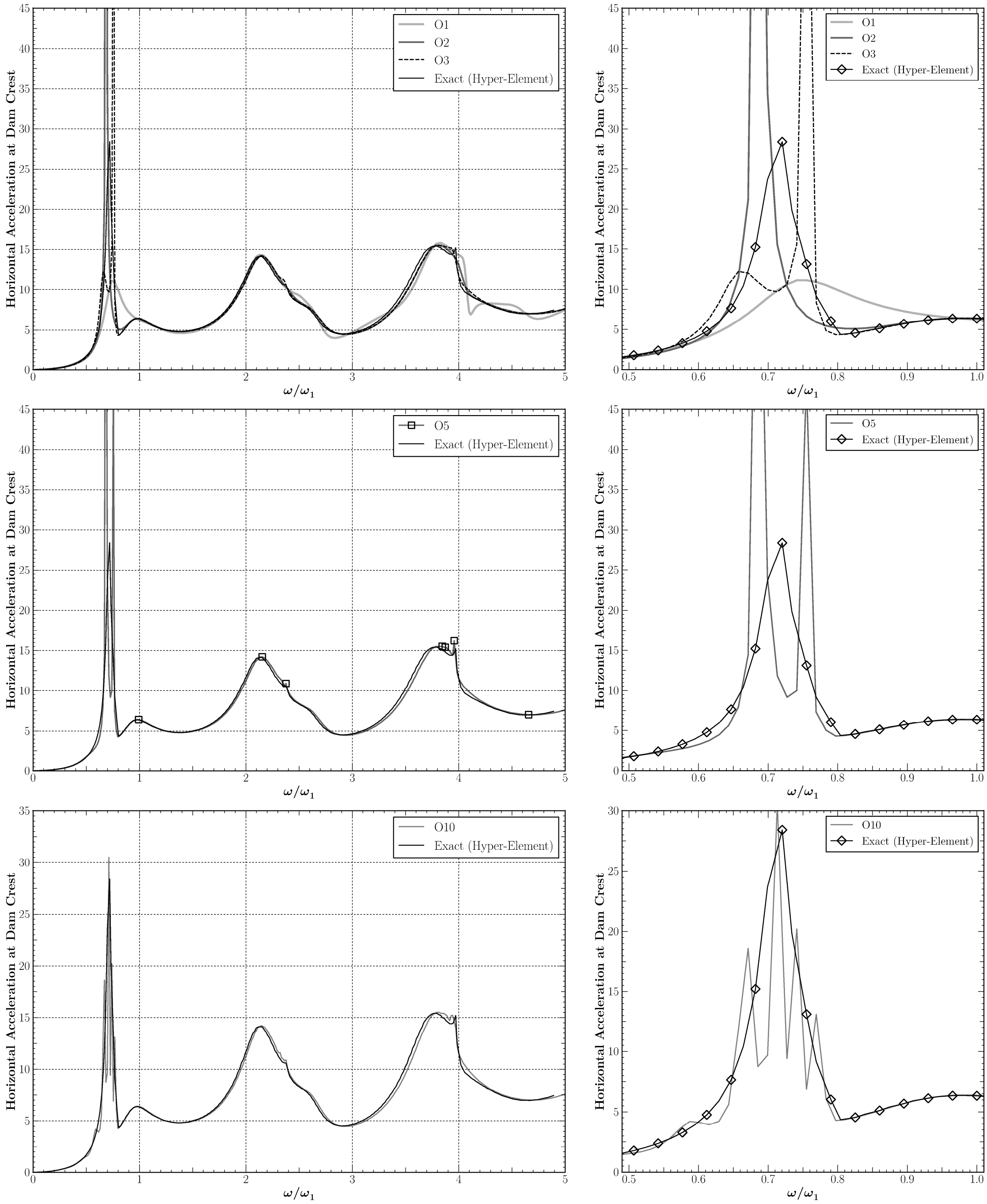

Figure 4. Horizontal acceleration at the dam crest due to horizontal excitation for different orders of H-W boundary condition ($L/H = 1.0$ and $\alpha = 1$).

5. CONCLUSIONS

In this paper, the H-W boundary condition is applied to the dam-reservoir interaction problem. This boundary condition is already developed for scalar wave equation and its performance in the mentioned problem has been the main point of interest in this paper. The following conclusions may be drawn from this study:

- The Sommerfeld boundary condition, which is usually utilized in the dam-reservoir interaction problem, can not effectively simulate the radiation damping lied in the omitted part of the reservoir. Even for large amounts of L/H ratio, this boundary condition result in oscillating responses above the first or second natural frequencies of the reservoir.

- Below the fundamental cutoff frequency of the reservoir, there are no travelling waves included in the response of the fluid domain. Hence, no outgoing energy flux is present and the non-reflecting boundary condition would not cause a noticeable improvement in the solution of the problem. Therefore, as the order of NRBC increases, some oscillatory behaviour may be observed in the response.
- For frequencies higher than the fundamental natural frequency of the reservoir, the non-reflecting boundary condition behaves quite well. By increasing its order, the results are converging to the exact solution and no instability is observed at this range. However, increasing the order of the NRBC would have adverse effects on the response below the cutoff frequency. It should be mentioned that, the improvements which have been proposed by Hagstrom el al (2008) seem to be quite promising to solve this kind of issues.

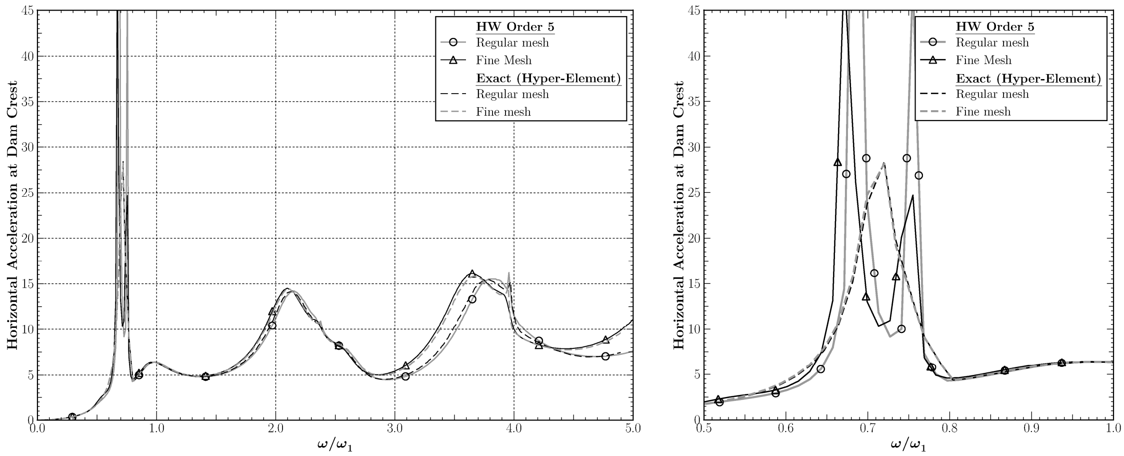

Figure 5. Horizontal acceleration at the dam crest due to horizontal excitation for order 5 boundary condition and hyper-element method, by regular and refined meshes ($L/H = 1.0$ and $\alpha = 1$).

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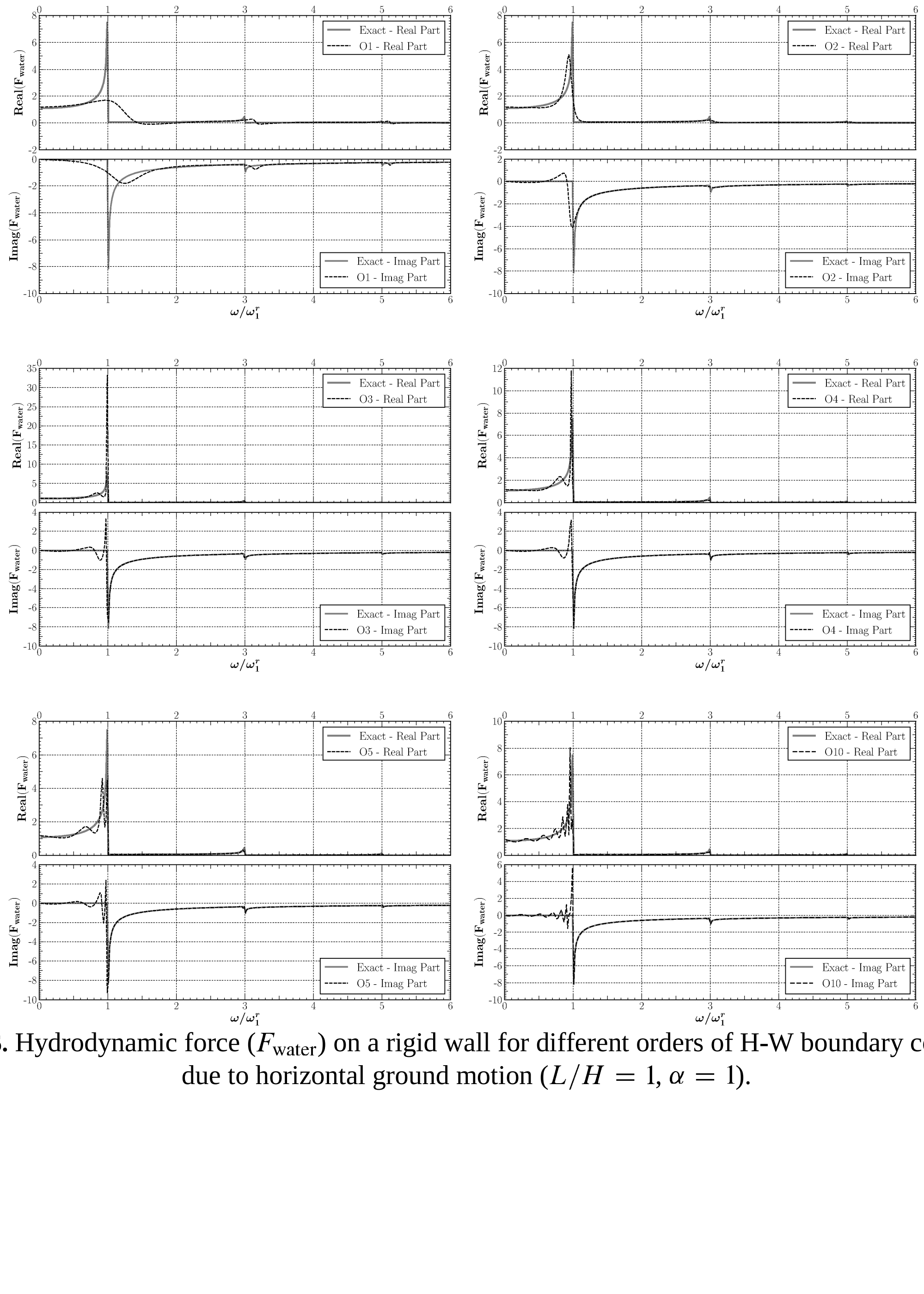

Figure 6. Hydrodynamic force (F_{water}) on a rigid wall for different orders of H-W boundary condition, due to horizontal ground motion ($L/H = 1$, $\alpha = 1$).