Seismic Response of Well Foundation with Dynamic Soil Structure Interaction

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SUMMARY:
For both steel and RCC Bridges passing rivers or creeks, common practice in many countries is to provide concrete wells to support the bridge girders. For many bridges that are strategically important in terms of defense or trade, it is essential that they remain functional even after a strong earthquake hits the structure. The present state of the art for design of well foundation is still marred with a number of uncertainties where a simplistic pseudo static analysis of its response only prevails, though it is a well-known fact that load from super structure, character of soil and its stiffness plays an important role in defining its dynamic characteristics. The present paper is thus an attempt to present a dynamic analysis model trying to cater to a number of such deficiencies as cited above and also provide a practical model (amenable to design office application) that can be used to estimate the pier, well and soil's dynamic interaction

Keywords: Well foundation, Bridge Pier, Modal analysis, Puzrevsky functions, DSSI.

1. INTRODUCTION

Well or caissons are very popular type of foundations deployed to support bridge girders and cross country pipelines over river crossings and ravines in India and many other countries. In many places these bridges are built in zones susceptible to moderate/strong earthquakes. Considering bridges are an important connection between two areas, it becomes essential that many of them remain under operations even after a strong earthquake, enabling relief team to quickly access the areas of maximum damage. Thus understanding the behavior of well foundations and pier under such seismic forces are of paramount importance to ensure operational safety of these bridges.

Shown in Fig. 1.1 is a well foundation with bridge deck as installed in a river bed. Standard procedure for seismic analysis is the total lateral force coming from the bridge deck $\alpha_h.W$ is transferred to the top of the well as shear $V=\alpha_h.W$ and Moment as $M=\alpha_h.W.H$. It is assumed that this force is resisted by the mobilized passive force of the soil on well steining such that a Factor of Safety (*FOS*) of 2-2.5 is ensured as a minimum, thus well foundation is usually assumed to be a rigid block subjected to nominal stress only. Major digression from reality in the above concept is that dynamic effect of soil and well foundation is completely ignored. It should be realized that seismic waves propagate through soil and excites the well foundation first. How much will this in turn excite the pier and top deck will depend upon relative stiffness of the well and its surrounding soil and interaction among them. Realistically, this is a classic structure-foundation system where Dynamic Soil Structure Interaction (DSSI) plays a crucial role in the overall response and should be taken into cognizance to arrive at overall response of the system. In the present paper a mathematical model for a well pier foundation is proposed considering the effect of DSSI that is amenable to design office practice.

$$FOS = \frac{\alpha_h.W.H}{K_p.\gamma_s.D.L^3/6} \geq 2\text{to}2.5 \tag{1.1}$$

Here K_p passive pressure of soil, γ_s weight density of soil in kN/m^3, L height and D is diameter of well.

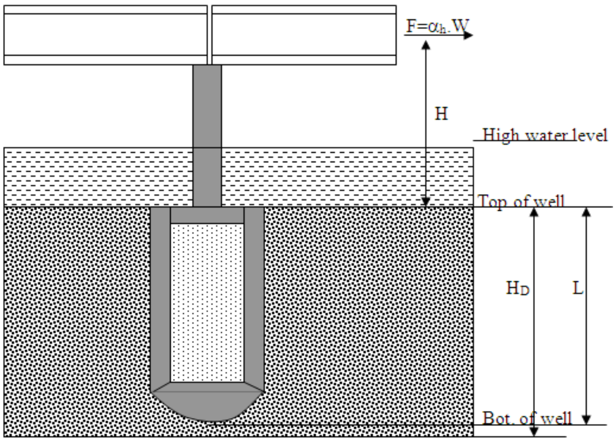


Figure 1.1. A Well foundation and pier supporting bridge deck

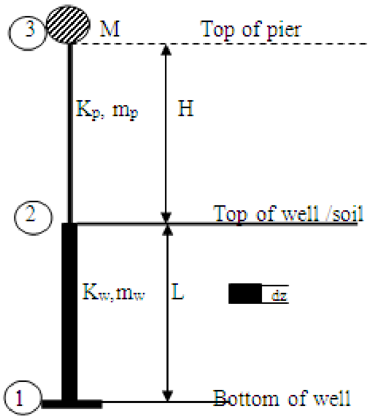


Figure 2.1. Mathematical model of well foundation with pier

2. PROPOSED METHOD

Shown in Fig. 2.1 is the mathematical model proposed for the pier –well system. In this model M is mass transmitted to the top bridge pier from the bridge deck, m_p mass of the pier, m_w mass of well foundation including sand fill if any, K_p stiffness of bridge pier, K_w stiffness of well foundation. Applying unit load successively at node 1, 2 and 3 we have flexibility matrices as given in Eqn. 2.1.

$$f_{11} = 0,\ f_{22} = 1/k_w,\ f_{33} = (1/k_w) + (1/k_p) \tag{2.1}$$

By Dunkerley's (1894) principle the equivalent frequency of the system can then be expressed as Eqn. 2.2. Eqn. 2.2 on simplification gives Eqn. 2.3, which can be further expressed as Eqn. 2.4.

$$1/\omega_e^2 = (0) + [m_w(1/K_w)] + (M + m_p)[(1/K_w) + (1/K_p)] \tag{2.2}$$

$$1/\omega_e^2 = (M + m_p)/K_p + (M + m_p + m_w)/K_w \tag{2.3}$$

$$1/\omega_e^2 = (1/\omega_p^2) + (1/\omega_w^2) \tag{2.4}$$

It may be observed that Eqn. 2.4 is similar to what has been proposed by Wolf (1988) for coupled frequency of soil-structure system for SDOF. The equivalent frequency of the pier-well system vides Eqn. 2.3 shows that the model can be broken down into individual systems as shown in Fig. 2.2. It is thus observed that if we can find out the fixed base natural frequency/time period of the individual system as shown in Fig. 2.2 and couple them based on Eqn. 2.4 we arrive at the equivalent natural frequency of the pier–well system. Eqn. 2.4 can be further expressed as Eqn. 2.5 and multiplying each term of Eqn. 2.5 by acceleration due to gravity g we get Eqn. 2.6.

$$(M + m_p + m_w)/K_e = (M + m_p)/K_p + (M + m_p + m_w)/K_w \tag{2.5}$$

$$\delta_e = \delta_p + \delta_w \tag{2.6}$$

Here, δ_p and δ_w are fixed base displacement of the pier and well respectively. Eqn. 2.6 in terms of modal response analysis can be expressed as Eqn. 2.7 and on solving we get Eqn. 2.8.

$$\delta_e = (S_{ap}/\omega_p^2) + (S_{aw}/\omega_w^2) \tag{2.7}$$

$$\delta_e = (S_{ap}/\omega_p^2)[1 + (S_{aw}/S_{ap})(\omega_p^2/\omega_w^2)] \tag{2.8}$$

$$\delta_e = \kappa_i.CF.(S_{ap}/\omega_p^2)[1 + (S_{aw}/S_{ap})(\omega_p^2/\omega_w^2)] \tag{2.9}$$

In terms modal definition Eqn. 2.8 can be expressed as Eqn. 2.9. Here κ_i modal mass participation factor, *CF* code factor expressed as *ZI/2R* where *Z* zone factor, *I* importance factor and *R* response reduction factor. Eqn. 2.9 actually represents the fixed base amplitude of the bridge pier amplified by the DSSI effect represented by term within parenthesis. Thus

$$AF_p = \left[1 + \left(S_{aw}/S_{ap}\right)\left(\omega_p^{\,2}/\omega_w^{\,2}\right)\right] \tag{2.10}$$

$$\left\lfloor AF_p \right\rfloor_i = [I] + \left\lfloor S_{awi}/S_{api} \right\rfloor + \left\lfloor \lambda_{pi}/\lambda_{wi} \right\rfloor \tag{2.11}$$

Here AF_p is an amplification factor due to DSSI of the pier and is a dimensionless term. For multi degree freedom system Eqn. 2.10 can be expressed (Chowdhury and Dasgupta (2011)) by Eqn. 2.11. Here *I* identity matrix having diagonal terms as 1, λ with subscript represents the Eigenvalues of pier and well, *i* represents the number of modes considered in the analysis. It is thus observed that if we are in a position to determine correctly the stiffness and mass matrix of the pier and well vis-à-vis it's Eigenvalues we can easily find out the overall response based on DSSI from Eqn. 2.11.

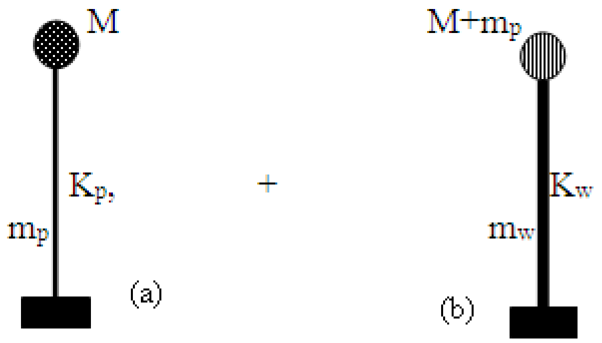


Figure 2.2. Equivalent mathematical model for the DSSI of the well pier system

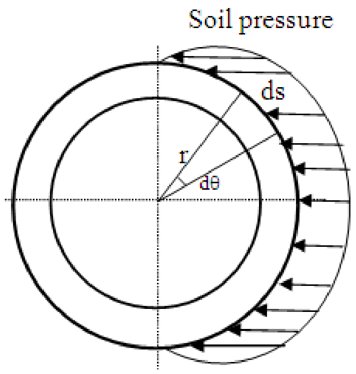


Figure 2.3. Soil pressure on well shaft viewed in plan

2.1 Stiffness and Mass Matrix of Bridge Pier

Considering the bridge pier to be fixed at top of well head, the system acts as cantilever beam with a mass M lumped at its head (Fig. 2.2a). It can be shown (Sadeghpour & Chowdhury (2008)) that stiffness matrix can be expressed as

$$[K]_p = \frac{EI}{H^3}\begin{bmatrix} \mu_1^{\,4}\int_0^1 f''(\xi)_1^{\,2}\,d\xi & & \text{Symmetric} \\ \mu_2^{\,2}\mu_1^{\,2}\int_0^1 f''(\xi)_2 f''(\xi)_1\,d\xi & \mu_2^{\,4}\int_0^1 f_2''(\xi)^2\,d\xi & \\ \mu_3^{\,2}\mu_1^{\,2}\int_0^1 f''(\xi)_3 f''(\xi)_1\,d\xi & \mu_3^{\,2}\mu_2^{\,2}\int_0^1 f''(\xi)_3 f''(\xi)_2\,d\xi & \mu_3^{\,4}\int_0^1 f''(\xi)_3^{\,2}\,d\xi \end{bmatrix} \tag{2.12}$$

Here $f''(\xi)_i = [-\sin\mu_i\xi - \sinh\mu_i\xi + \alpha_i(\cos\mu_i\xi + \cosh\mu_i\xi)]$, $\mu_i = 1.8751,\ 4.6941,\ 7.8548,\ 10.966,\ \ldots$ and $\alpha_i = (\sin\mu_i + \sinh\mu_i)/(\cos\mu_i + \cosh\mu_i)$, *i* mode number, *E* elastic modulus , *I* moment of inertia of the pier. The mass matrix of the pier can be similarly be expressed by Eqn. 2.13 where, $f(\xi) = \sin\mu_i\xi - \sinh\mu_i\xi - \alpha_i(\cos\mu_i\xi - \cosh\mu_i\xi)$, ρ Mass density of pier material, *A* Cross sectional area of pier and *H* Height of pier.

$$[\mathrm{M}] = \rho \mathrm{AH} \begin{bmatrix} \int_0^1 \mathrm{f}(\xi)_1^2 d\xi + \frac{\mathrm{M}}{\rho \mathrm{AH}} f_1(1)^2 & & \text{Symmetric} \\ \int_0^1 \mathrm{f}_2(\xi)\mathrm{f}_1(\xi) d\xi + \frac{\mathrm{M}}{\rho \mathrm{AH}} f_2(1) f_1(1) & \int_0^1 \mathrm{f}_2(\xi)^2 d\xi + \frac{\mathrm{M}}{\rho \mathrm{AH}} f_2(1)^2 & \\ \int_0^1 \mathrm{f}_3(\xi)\mathrm{f}_1(\xi) d\xi + \frac{\mathrm{M}}{\rho \mathrm{AH}} f_3(1) f_1(1) & \int_0^1 \mathrm{f}_3(\xi)\mathrm{f}_2(\xi) d\xi + \frac{\mathrm{M}}{\rho \mathrm{AH}} f_3(1) f_2(1) & \int_0^1 \mathrm{f}_3(\xi)^2 d\xi + \frac{\mathrm{M}}{\rho \mathrm{AH}} f_3(1)^2 \end{bmatrix} \quad (2.13)$$

2.2 Stiffness of the Well Shaft

In most of the dynamic analysis the well is assumed to be a rigid cylindrical mass having infinite stiffness (Gazzetas et al (2006), Mandal & Jain (2008), Thakkar et al (2002) etc., where only the soil stiffness contribute to the dynamic response. This is however an oversimplification of the issue as in most of the case the well foundation behaves as a hollow cylindrical shell filled by sand/soil to increase its weight. In reality for wells deeply embedded in soil the well foundation behaves as a finite beam on elastic foundation, whose displacement function can expressed as

$$x = C_0 \cosh pz \cos pz + C_1 \cosh pz \sin pz + C_2 \sinh pz \sin pz + C_3 \sinh pz \cos pz \quad (2.14)$$

Where $p = \sqrt[4]{k_s D / 4E_w I_w}$, k_s subgrade modulus of soil, $E_w I_w$ flexural stiffness of well. Expressing in terms of Puzrevsky's function (Karnovsky and Lebed 2001), Eqn. 2.14 can be expressed as

$$x = C_0 V_0(pz) + C_1 V_1(pz) + C_2 V_2(pz) + C_3 V_4(pz) \quad (2.15)$$

Where $V_0(pz) = \cosh(pz)\cos(pz)$ (2.16)

$V_1(pz) = (1/\sqrt{2})(\cosh(pz)\sin(pz) + \sinh(pz)\cos(pz))$ (2.17)

$V_2(pz) = \sinh(pz)\sin(pz)$ (2.18)

$V_3(pz) = (1/\sqrt{2})(\cosh(pz)\sin(pz) - \sinh(pz)\cos(pz))$ (2.19)

Puzrevsky's functions, defined in Eqns. 2.16 to 2.19, have some unique functional properties (Chowdhury and Dasgupta, 2008), which will be used for subsequent analysis for derivation of the stiffness, damping and mass of the piles. For a solution of the well shaft one may use the model shown in Fig. 2.2b. For analysis, the well shaft may be assumed as fixed at base and can undergo unit deflection at the head. Considering base of shaft at z = 0, shown in Fig. 2.2b, one may write at $z = 0, x = 0 \Rightarrow C_0 = 0$ and at $z = 0, x' = 0 \Rightarrow C_1 = 0$

This gives, $x = C_2 V_2(pz) + C_3 V_3(pz)$ (2.20)

At the well head, i.e. at $z = L$, $x = 1$ yielding, gives $C_2 V_2(pz) + C_3 V_3(pz) = 1$ (2.21)

Again at $z = L,\ x' = 1/L$ which gives, $C_2 V_2'(pz) + C_3 V_3'(pz) = 1$ (2.22)

Using properties of Puzrevsky's function, one may write Eqn. 2.22 as Eqn. 2.23. Eqn. 2.23 may be expressed in matrix form as given in Eqn. 2.24.

$$C_2 V_1(pL) + C_3 V_2(pL) = 1/pL\sqrt{2} \quad (2.23)$$

$$[V]\{C\} = \{P\} \text{ which gives } \{C\} = [V]^{-1}\{P\} \quad (2.24)$$

Performing the above operation gives

$$\begin{Bmatrix} C_2 \\ C_3 \end{Bmatrix} = \frac{1}{\Delta} \begin{bmatrix} V_2(pL) & -V_3(pL) \\ -V_1(pL) & V_2(pL) \end{bmatrix} \begin{Bmatrix} 1 \\ 1/pL\sqrt{2} \end{Bmatrix} \quad (2.25)$$

Where $\Delta = V_2^2(pL) - V_1(pL)V_3(pL)$ which implies

$$C_2 = \left(V_2(pL) - \left(V_3(pL)/pL\sqrt{2}\right)\right) \text{ and } C_3 = \left[\left(V_2(pL)/pL\sqrt{2}\right) - V_1(pL)\right] \tag{2.26}$$

Thus, the displacement for the given boundary condition is then expressed as given in Eqn. 2.27. Based on Eqn. 2.27, the generic shape function in dimensionless form considering $\beta = pL$ is given by Eqn. 2.28. Where $A = C_2/\Delta$ and $B = C_3/\Delta$ and $\Delta = V_2^2(\beta) - V_1(\beta)V_3(\beta)$.

$$x = (x_0/\Delta)[C_2V_2(pz) + C_3V_3(pz)] \tag{2.27}$$

$$\phi(z) = AV_2(\beta z/L) + BV_3(\beta z/L) \tag{2.28}$$

$$\phi''(z) = \left(2\beta^2/L^2\right)[AV_0(\beta z/L) + BV_1(\beta z/L)] \tag{2.29}$$

Differentiating twice, Eqn. 2.28 leads to Eqn. 2.29 and using functional properties mentioned earlier one could have Potential energy $d\Pi$ of an element of depth dz as shown in Fig. 2.2b is then given (Shames and Dym 1995) by Eqn. 2.30. Where K_h lateral dynamic stiffness of soil in kN/m and the displacement u may be written as $u = \phi(z)q(t)$.

$$d\Pi = (E_wI_w/2)\left[d^2u/dz^2\right]^2 + \left(K_hu^2/2\right) \tag{2.30}$$

$$K_x = [8G_sr_0/(2-\nu)][1 + (h/r_0)] \tag{2.31}$$

For a rigid circular disc embedded in soil of depth h the stiffness under earthquake force can be expressed as (Wolf-1988) by Eqn. 2.31. Where K_x static foundation stiffness in horizontal direction in kN/m, G_s dynamic shear modulus of soil, r_0 radius of foundation, h depth of embedment of the foundation and ν Poisson's ratio. Ignoring the first term within bracket in Eqn. 2.31 which contributes to base resistance and substituting the same in Eqn. 2.30, for a cylindrical element of depth dz embedded in soil the potential energy Π for a well of length L may be expressed as Eqn. 2.32.

$$\Pi = (E_wI_w/2)\int_0^L \left[d^2u/dz^2\right]^2 dz + [8G_s/2(2-\nu)]\int_0^L u^2 dz \tag{2.32}$$

Considering $u(z,t) = \phi(z)q(t)$ it can be shown (Hurty & Rubenstein 1967) that

$$K_{ij} = E_wI_w\int_0^L \phi_i''(z)\phi_j''(z)dz + [8G_s/(2-\nu)]\int_0^L \phi_i(z)\phi_j(z)dz \tag{2.33}$$

Here the shape function $\phi(z)$ is expressed by Eqn.2.28. For the fundamental mode stiffness of the pile is given by Eqn. 2.34. Expansion of Eqn. 2.34 finally gives Eqn. 2.35.

$$K_{ij} = E_wI_w\int_0^L \phi''(z)^2 dz + [8G_s/(2-\nu)]\int_0^L \phi(z)^2 dz \tag{2.34}$$

$$K_w = \left(4\beta^4E_wI_w/L^4\right)\int_0^L [AV_0(\beta z/L) + BV_1(\beta z/L)]^2 dz + [8G_s/(2-\nu)]\int_0^L (AV_2(\beta z/L) + BV_3(\beta z/L))^2 dz \tag{2.35}$$

Now considering $\xi = z/L$, $L.d\xi = dz$ and as $z \to 0$, $\xi \to 0$ and as $z \to L$, $\xi \to 1$, when Eqn. 2.35 can be expressed in natural co-ordinates as

$$K_w = \left(4\beta^4 E_w I_w / L^3\right)\int_0^1 \left[AV_0(\beta\xi) + BV_1(\beta\xi)\right]^2 d\xi + \left[8G_s L/(2-\nu)\right]\int_0^1 \left(AV_2(\beta\xi) + BV_3(\beta\xi)\right)^2 d\xi \quad (2.36)$$

or
$$K_w = \left(4\beta^4 E_w I_w / L^3\right) I_1 + \left[8G_s L/(2-\nu)\right] I_2 \quad (2.37)$$

Where
$$I_1 = \int_0^1 \left[AV_0(\beta\xi) + BV_1(\beta\xi)\right]^2 d\xi \text{ and } I_2 = \int_0^1 \left(AV_2(\beta\xi) + BV_3(\beta\xi)\right)^2 d\xi \quad (2.38)$$

I_1 and I_2 are integral functions that need to be determined numerically. However, prior to that relationship between dynamic subgrade modulus k_s and Wolf's parameter as shown in Eqn. 2.36 needs to be established. Let us assume a circular shaft of outer diameter $D=2r$ and wall thickness t as shown in Fig. 2.3. Observing Eqn. 2.37 it is seen that the first term represents the structural stiffness of well and the second term expresses the contributing soil stiffness. Thus in terms of k_s the stiffness of the soil over a segmental area $ds \times L$ of the well shaft can be expressed as

$$k_{soil} = k_s.ds.L.I_2 \quad (2.39)$$

Here $ds = r.d\theta$. This gives total soil stiffness about the shaft by Eqn. 2.40. Equating Eqn. 2.40 to second term of 2.37, we get β given by Eqn. 2.41. Based on β as mentioned in Eqn. 2.41 and dynamic modulus of soil G_s Eqn. 2.37 can be finally expressed by Eqn. 2.42.

$$k_{soil} = k_s \int_0^\pi r.d\theta.L.I_2 \quad \rightarrow k_{soil} = \pi.k_s.D.L.I_2/2 \quad (2.40)$$

$$k_s = 16G_s / \left[\pi(2-\nu)D\right] \qquad \rightarrow \beta = pL = \sqrt[4]{4G_s L^4 / [(2-\nu)\pi.E_w.I_w]} \quad (2.41)$$

$$K_w = \left(8G_s L/(2-\nu)\right)\chi_{12} \quad (2.42)$$

Table 2.1. Stiffness and Integral coefficient for mass and damping of well foundation

β	2	2.25	2.5	2.75	3	3.25	3.5
χ_{12}	0.351	0.387	0.409	0.407	0.372	0.290	0.149
I_2	0.184	0.242	0.304	0.366	0.422	0.464	0.478

Here $\chi_{12} = I_1 + I_2$ is well shaft stiffness coefficient. For a well foundation which is usually massive L/r is normally less than 20 and E/G_s varying from 1000 to 10,000 (the usual range when wells will be deployed), the value of β usually varies from 2.2 to 3.5. Thus considering β varying from 2.0 to 3.5, the values of χ_{12}, I_2 are furnished in Table 2.1 for ready reference.

2.3 Calculation of Mass and Damping

The well foundation's mass consists of two parts, i) the self-weight and ii) the lumped mass as its head as shown in Fig. 2.2b. The contribution of this can be expressed as (Meirovitch, 2001) Eqn. 2.43, where m_x is mass per unit length. For the present case Eqn. 2.43 can be expressed as Eqn. 2.44. Here γ_w and A_w unit weight of and cross sectional area of well. Eqn. 2.44 in natural co-ordinates can be expressed by Eqn. 2.45.

$$M_w = m_x \int \phi_i(z)\phi_j(z)dz + (M + m_p)\phi_i(L)\phi_j(L) \quad (2.43)$$

$$M_w = (\gamma_w A_w / g)\int_0^L \phi(z)^2 dz + (M + m_p)\phi(L)^2 \quad (2.44)$$

$$M_w = (\gamma_w A_w / g) I_2 + (M + m_p)\phi(1)^2 \quad (2.45)$$

Here I_2 is the integral function explained in Eqn. 2.38. It may be noted that though here the derivation of stiffness and mass of the well foundation is for the fundamental mode but would still have a multi-degree characteristics as that of the pier as the second term in Eqn. 2.45 will keep varying with each mode of the pier. The value of $(M+m_p)$ is actually represented by Eqn. 2.13 where though the stiffness matrix is diagonal (considering they are derived from Eigenvector basis when $\phi_i.\phi_j=0$), the mass matrix is non-diagonal but symmetric. To extract the uncoupled modal contribution of this mass on well we as first step find out the fixed base Eigenvalues from the expression $[K_p][\phi]=\lambda[M][\phi]$. It is apparent that the λ matrix is diagonal. Considering the $[K_p]$ matrix is also diagonal in this case a diagonal mass matrix contributing to each mode can be obtained as

$$[M_x]_{i,i} = \left[\left(\mu_i^4 \int_0^1 (f_i(\xi)'')^2 \, d\xi\right) \Big/ [\lambda]_{i,i}\right] \tag{2.46}$$

Eqn. 2.45 based on above can now be expressed for each mode as $M_{wi} = (\gamma_w A_w L/g) I_2 + M_{xi}\phi(1)^2$, here i = 1,2,3 the mode numbers. Damping of well foundation embedded in soil medium will consist of two parts: material and radiation damping. Material damping of soil is also a part of the vibrating system, however, it has been found that for translational motion this effect is insignificant and may be ignored. As a first step for calculating the total damping one may ignore material damping for the time being. For a rigid circular disc embedded in soil for a depth h Wolf (1988) has shown that radiation damping may be expressed as given in Eqn. 2.47.

$$c_x = (r_0 K_x / V_s)\left[0.68 + 0.57\sqrt{h/r_0}\right] \tag{2.47}$$

Here, K_x lateral stiffness of the embedded disc; V_s shear wave velocity of the soil. Thus for an infinitesimally thin circular disc of thickness dz Eqn. 2.47 can be expressed as Eqn. 2.48 and considering Eqn. 2.48 it has been shown by Chowdhury & Dasgupta (2010) that the damping ratio for the well shaft can be finally expressed as by Eqn. 2.49. In Eqn. 2.49 ω_n is the natural frequency of the well shaft ($\sqrt{K_w/(M_{xi})}$) and I_2 are the integral functions furnished in Table 2.1. To Eqn. 2.49 now, a suitable material damping ratio for the well material (ς_m), may be added to arrive at total damping ratio of the system.

$$c_x = (r_0 K_x / V_s)\left[0.68 + 0.57\sqrt{dz/r_0}\right] \tag{2.48}$$

$$\varsigma_x = (0.43 L \omega_n / V_s) I_2 \tag{2.49}$$

2.4 Moment and Shear of Pier and Well Considering DSSI

For the pier based on the operation $[K_p][\phi]=\lambda[M][\phi]$ based on Eqns. 2.12 & 2.13, let the Eigenvalues and vectors be represented by $\lfloor \lambda_p \rfloor_{i,i} = \lfloor \lambda_{i,i} \rfloor$ and $\lfloor \varphi_p \rfloor_{i,i} = \lfloor \phi_{i,j} \rfloor$ where $i = j$ = 1, 2, 3 and $\lambda_p=\omega_p^2$, the square of natural frequency of the pier. Similarly for the well let the fixed base Eigenvalues be $\lambda_{wi} = K_w / M_{wi}$, where $\omega_{wi} = \sqrt{K_w / M_{wi}}$. For each of these natural frequencies (ω_p and ω_w) we can find out the corresponding values S_{ap}/g and S_{aw}/g from code and putting these expressions in Eqn. 2.11 we can find out the amplification factor AF_{pi} due to DSSI for each mode of the pier due to the well foundation. Based on modal analysis the amplitude of vibration for the pier can then be expressed as

$$\delta_{pi} = \kappa_i CF \left(S_{api} / \omega_{pi}^2\right) AF_{pi} \{\phi_{ij}\}^T \{f_i(\xi)\} \tag{2.50}$$

where $$\kappa_i = \frac{\rho AH \int_0^1 \{\phi_{ij}\}^T f_i(\xi) d\xi + M\{\phi_{ij}\}^T \{f_i(1)\}}{\rho AH \int_0^1 \left[\{\phi_{ij}\}^T f_i(\xi)\right]^2 d\xi + M\left[\{\phi_{ij}\}^T \{f_i(1)\}\right]^2} \tag{2.51}$$

Here κ_i modal mass participation factor. Thus displacement, bending moment and shear force are expressed as (Meirovitch, 2001)

$$\delta_{pi} = \kappa_i CF\left(S_{api}/\omega_{pi}^2\right) AF_{pi}\left[\phi_{1,i} f_1(\xi) + \phi_{2,i} f_2(\xi) + \phi_{3,i} f_3(\xi)\right] \tag{2.52}$$

$$M_{pi} = -\kappa_i CF\left(EIS_{api}/H^2\omega_{pi}^2\right) AF_{pi}\left[\phi_{1,i}\mu_1^2 f_1''(\xi) + \phi_{2,i}\mu_2^2 f_2''(\xi) + \phi_{3,i}\mu_3^2 f_3''(\xi)\right] \tag{2.53}$$

$$V_{pi} = -\kappa_i CF\left(EIS_{api}/H^3\omega_{pi}^2\right) AF_{pi}\left[\phi_{1,i}\mu_1^3 f_1'''(\xi) + \phi_{2,i}\mu_2^3 f_2'''(\xi) + \phi_{3,i}\mu_3^2 f_3'''(\xi)\right] \tag{2.54}$$

The SRSS values for first three modes are expressed as

$$M_{ps} = \sqrt{M_{p_1}^2 + M_{p_2}^2 + M_{p_3}^2} \text{ and } V_{ps} = \sqrt{V_{p_1}^2 + V_{p_2}^2 + V_{p_3}^2} \tag{2.55}$$

For the well foundation the displacement as elaborated earlier is expressed as given in Eqn. 2.56. Where C_2 and C_3 are integration constants and $V_2(\beta z/L)$ and $V_3(\beta z/L)$ are Puzrevsky's function as explained earlier. Now imposing the boundary conditions a) At $z=L$ $EId^2x/dz^2=M_{ps}$ and *b*) at $z=L$ $EId^3x/dz^3=V_{ps}$, after some simple algebraic manipulation we get Eqn. 2.57.

$$x = C_2V_2(\beta z/L) + C_3V_3(\beta z/L) \tag{2.56}$$

$$C_2 = -\frac{V_0(\beta)}{\Delta}\left(\frac{M_{ps}L^2}{2\beta^2 EI_w}\right) + \frac{V_1(\beta)}{\Delta}\left(\frac{V_{ps}L^3}{2\sqrt{2}\beta^3 EI_w}\right) \quad \text{and} \quad C_3 = -\frac{V_3(\beta)}{\Delta}\left(\frac{M_{ps}L^2}{2\beta^2 EI_w}\right) + \frac{V_0(\beta)}{\Delta}\left(\frac{V_{ps}L^3}{2\sqrt{2}\beta^3 EI_w}\right) \tag{2.57}$$

Where $\Delta = V_0^2(\beta) + V_1(\beta)V_3(\beta)$

Now using the properties of Puzrevsky's function as defined earlier and double differentiating x we have M_w and V_w given by Eqns. 2.58 and 2.59 respectively. These equations give the bending moment and shear force for the complete well foundation along its depth due to inertial interaction with the superstructure including effect of DSSI.

$$M_w = -EI\left(d^2x/dz^2\right) = -\left(2\beta^2 EI_w/L^2\right)\left[C_2V_0(\beta z/L) + C_3V_1(\beta z/L)\right] \tag{2.58}$$

Similarly, $$V_w = -EI\left(d^3x/dz^3\right) = -\left(2\sqrt{2}\beta^3 EI_w/L^3\right)\left[C_2V_3(\beta z/L) - C_3V_0(\beta z/L)\right] \tag{2.59}$$

2.5 Effect of Scour

In many cases due to flow of water a portion of well top loses its grip due to erosion, when the well become partly embedded. In this case let the depth of well be L and depth of embedment be $L_1=\alpha L$ where $0<\alpha<1$. In this case the stiffness expression vide Eqn. 2.36 gets modified to Eqn. 2.60 where $\beta_e = \sqrt[4]{4G_s\alpha^4 L^4 /[(2-\nu)\pi E_w I_w]}$. Subscript e represents the embedded depth. Rest of the steps remains same as explained earlier.

$$K_w = \left(4\beta_e^4 E_w I_w/L^3\right)\int_0^1 \left[AV_0(\beta_e\xi) + C_1V_1(\beta_e\xi)\right]^2 d\xi + \left(8G_s L/(2-\nu)\right)\int_0^\alpha \left(AV_2(\beta_e\xi) + BV_3(\beta_e\xi)\right)^2 d\xi \tag{2.60}$$

2.6 Kinematical Interaction with Ground

What we have discussed till now is the inertial interaction of the well with superstructure load incorporated in the analysis as $M + m_p$. Other than this, free field motion of the ground will also pull the well along with it. Especially deeply embedded wells will move along with ground motion following the same deformation geometry of the ground. The free field time period of the ground referring to Fig. 1.1, can be expressed as $T_n = 4H_D/((2n-1)V_s)$ where H_D depth of soil to the bedrock, V_s Shear wave velocity of the soil, n the mode number. Let S_{an}/g be spectral acceleration corresponding to time period T_n. The ground displacement u_{gn} can be expressed as Eqn. 2.61, which further can be simplified and expressed as Eqn. 2.62.

$$u_{gn} = \kappa_n C_F \left(S_{an} T_n^2 / 4\pi^2\right) \cos[(2n-1)(\pi z/2H_D)] \tag{2.61}$$

$$u_{gn} = \kappa_n C_F (S_{an}/g)\left[4\gamma_s H_D^2 / \left(\pi^2 (2n-1)^2 G\right)\right] \cos[(2n-1)(\pi z/2H_D)] \tag{2.62}$$

Here γ_s weight density of the soil, G dynamic shear modulus of soil. Considering

$$EI\left(d^2 u_{gn}/dz^2\right) = -M_z \Rightarrow M_z = \kappa_n C_F \gamma_s (S_{an}/g)(E_w I_w/G) \cos[(2n-1)(\pi z/2H_D)] \tag{2.63}$$

and

$$V_z = -\kappa_n C_F \gamma_s (S_{an}/g)(E_w I_w/G)((2n-1)(\pi z/2H_D)) \sin[(2n-1)(\pi z/2H_D)] \tag{2.64}$$

Here z varies from 0 to L(the depth of well) and κ_n is modal participation factor and for first three modes, expressed as $8/(2+\pi)$, $8/(2-3\pi)$ and $8/(2+5\pi)$. SRSS values of moments and shears obtained using Eqns. 2.63 and 2.64 now need to be added to Eqn. 2.55 to get the complete response of the well under seismic load.

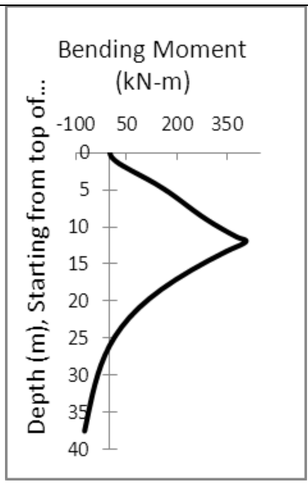
3. RESULTS AND DISCUSSION

Basic Data of Well Foundation: *Bridge Pier* Height (H_p) 12m, Diameter 2m, Mass transmitted to the top bridge from the bridge deck (M) 175500kg, Mod. of Elasticity of Concrete (E_p) 2.55×10^7 kN/m^2 *Well Foundation* Height (L) 25.5m, Diameter (D) 5.5m, Thickness of well 800mm, Thickness of well cap 1.8m, density of fill material 1957 kg/m^3 *Soil Parameters* Dynamic Shear Modulus (G_s) 1.32×10^5 kN/m^2, mass density 1957 kg/m^3, Poisson's ratio (ν) 0.3 *Seismic Condition* According to Indian Standard IS 1893 Part 1, Seismic Zone *V*, Soil Type Soft, Importance factor (I) 1.0, Response reduction factor (R) 3.0

Salient analytical parameters obtained from the analysis including bending moment profile and shear force profile in pier and well, starting from top of pier are as mentioned hereafter.

Modal Parameters and Amplification Factors

		Mode 1	Mode 2	Mode 3
Pier	Time Period	0.47	0.035	0.011
	Damping	5%	5%	5%
	Amplification	1.03	3.5	32.85
	Modal Mass Participation	-0.38	0.44	0.29
Well	Time Period	0.11	0.06	0.06
	Damping	20%	20%	20%



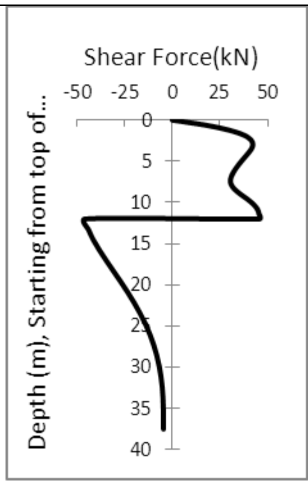


Figure 3.2. Bending moment and Shear force profile along pier and well.

4. CONCLUSION

A comprehensive analytical model is proposed herein that takes into consideration the DSSI effect of the soil and well on the pier supporting the superstructure. It also takes into account the elastic deformation of the well that is usually considered as a rigid cylindrical block in the present state of the art. It is observed that DSSI amplifies the fixed based response of the pier and this is more profound as the soil stiffness reduces. The higher modes though have a higher amplification than the fundamental mode their effect is not high as reduced modal mass participation brings down the amplification effect significantly. It is the fundamental mode and its corresponding amplification, which remains most critical. Considering the procedure is analytical in nature no sophisticated software is required. An Excel or a MathCAD sheet is sufficient to arrive at an accurate result.

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