Equivalent 1-DOF Model for Approximate Multi-Modal Analysis of Soil-Structure Interaction

A. Lanzi
University of California San Diego, La Jolla, California, USA
Sapienza Università di Roma, Rome, Italy

J. E. Luco
University of California San Diego, La Jolla, California, USA

SUMMARY:
This paper presents a new equivalent model, consisting of a single degree of freedom oscillator supported on an elastic soil, selected to approximate the response of a multi-story soil-structure system in the vicinity of each system mode. The model is based on the modal properties of the structure on a fixed-base and on the foundation impedance functions. It is shown that the effects of the lower fixed-base modes can be accounted for by increasing the stiffness and damping of the foundation while the effects of the higher fixed-base modes can be approximated by increasing the mass of the foundation. The new effective model provides considerable insight into the effects of SSI on the dynamic response of structures and can be used to obtain approximate values of the system natural frequencies, mode shapes, damping ratios and participation factors of an *N*-story structure by solving *N* reduced eigen-value problems of dimension 3.

Keywords: Soil-structure interaction, Modal analysis, Dynamics

1. INTRODUCTION

In his pioneering study of the dynamic building-foundation interaction effects, Parmelee (1967) considered the response of the now classical model of a one-story structure resting on a rigid foundation supported on an elastic soil. The model had three degrees of freedom corresponding to horizontal translation of the top mass, horizontal translation of the base mass and rocking of the base. This model was later studied in much detail by Bielak (1971), Jennings and Bielak (1973) and Veletsos and Meek (1974). To extend the applicability of the 3-DOF model to the study of the response of multi-story structures (such as that shown in Fig. 1.1), Parmelee (1967) proposed to focus on the response in the vicinity of each fixed-base mode by replacing the top mass of the 3-DOF model by an appropriate modal mass, and by selecting an appropriate height and foundation mass for the model which would account for the inertia of the remaining modes. In Parmelee's work, the stiffness of the super-structure was selected to correspond to the modal stiffness, and the stiffness and damping characteristics of the soil were left unchanged by the effects of the remaining modes.

In this paper, we revisit the problem of deriving equivalent 3-DOF models consisting of a single degree of freedom oscillator resting on a rigid foundation supported on an elastic soil to approximately represent the response in the vicinity of each mode of a multi-story structure when the effects of soil-structure interaction are included. Conceptually, the process is illustrated in Figs. 1.2a and 1.2b. As first shown by Tajimi (1967), the multi-story model illustrated in Fig. 1.1 can be replaced by the equivalent model shown in Fig. 1.2a, which consists of *N* oscillators, attached to the same foundation, each oscillator being characterized by the corresponding modal quantities of the structure on a fixed-base: natural frequency ω_i, damping ratio ξ_i, effective modal mass M_i and height H_i. The model in Fig. 1.2b consists of several separate single-degree-of-freedom oscillators, each one supported by a foundation on an elastic soil. The mass, stiffness and damping of the foundation are modified to account for the effects of the remaining modes. The model considers explicitly only one fixed-base mode at a time to describe the structural response, while the remaining fixed-base modes

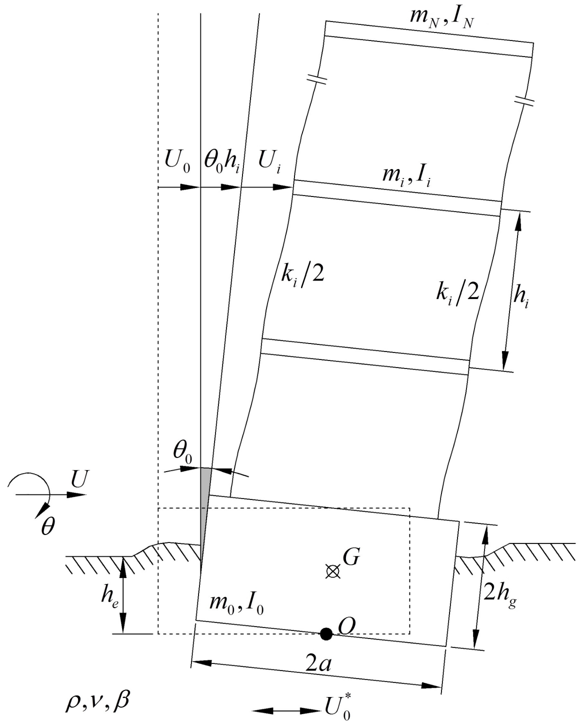

Figure 1.1 Shear-type model for in-plane coupled horizontal-rocking vibration

are accounted for by modifying the properties of the foundation. It is shown that the effects of the lower fixed-base modes can be approximated by increasing the stiffness and damping of the foundation while the effects of the higher fixed-base modes can be approximated by increasing the mass of the foundation. The equivalent model can be used to obtain approximate values of the system modal quantities of an *N*-story structure on an elastic soil by solving *N* reduced eigenvalue problems of dimension 3, instead of one eigenvalue problem of the complete interacting system. Although the

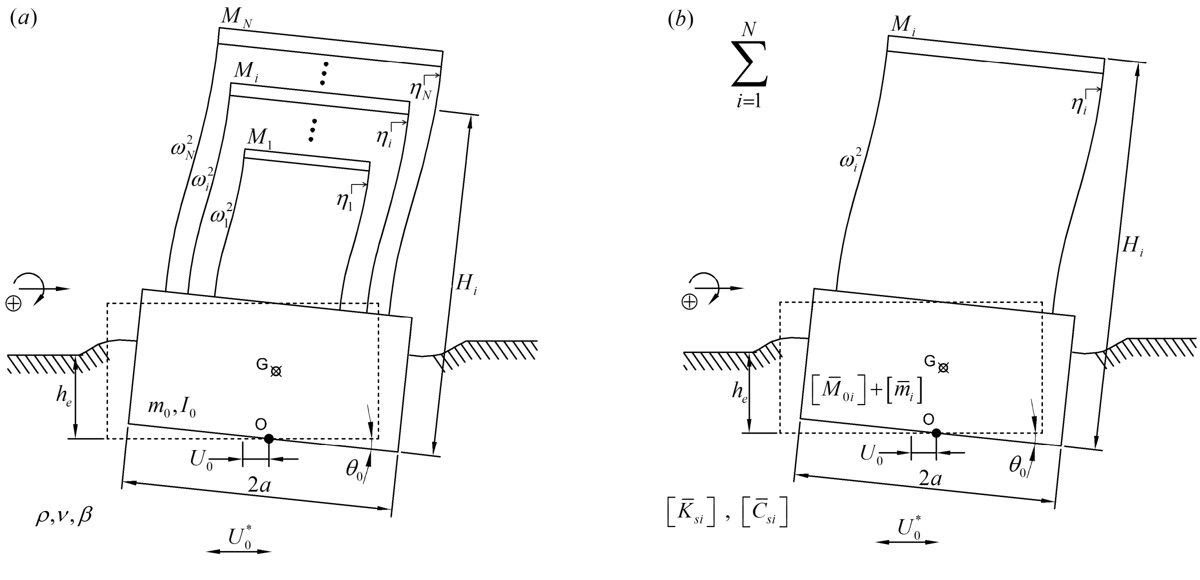

Figure 1.2. Equivalent models of the interacting system: (a) obtained by expressing the relative displacements of the superstructure in terms of fixed-base modes, and (b) obtained by using the approximate 1-DOF systems.

proposed model provides an alternative way to obtain the modal quantities that appear in the approximate classical normal mode approach for soil-structure interaction problems [Roesset et al. (1973), Tsai (1974), Novak (1974), Rainer (1975), Bielak (1975, 1976), Clough and Mojtahedi (1976), Beredugo (1976), Warburton and Soni (1977), Vaidya et al (1986)], its main characteristic is that it offers considerable physical insight into the nature of the soil-structure interaction effects.

2. STATEMENT OF THE PROBLEM AND BASIC INTERACTION EQUATIONS

Consider the problem of in-plane coupled horizontal-rocking vibrations of a linear elastic structure on a rigid foundation supported on a viscoelastic soil, excited by elastic waves propagating into the soil medium. The superstructure is discretized in a set of *N* nodes interconnected by elastic members; the soil is represented by a continuous, three-dimensional half-space. Referring to the model shown in Fig. 1.1, in which a shear-type behavior of the superstructure is assumed, the deformed configuration of the system can be described in terms of the $N \times 1$ vector $\{U\}$ of relative displacements of the nodes with respect to a frame of reference attached to the moving rigid foundation, and of the 2×1 vector $\{U_s\} = (U_s, \theta_s)^T$ expressing the relative motion of the foundation with respect to the input motion. With these definitions and for harmonic excitation, the motion of the superstructure and of the foundation is governed by the following system of equations:

$$\left[\begin{array}{c|c} M_b & M_b\alpha \\ \hline \alpha^T M_b & M_o + \alpha^T M_b \alpha \end{array}\right]\left\{\begin{array}{c} \ddot{U} \\ \hline \ddot{U}_s \end{array}\right\} + \left[\begin{array}{c|c} C_b & 0 \\ \hline 0 & C_s \end{array}\right]\left\{\begin{array}{c} \dot{U} \\ \hline \dot{U}_s \end{array}\right\} + \left[\begin{array}{c|c} K_b & 0 \\ \hline 0 & K_s \end{array}\right]\left\{\begin{array}{c} U \\ \hline U_s \end{array}\right\} = -\left[\begin{array}{c} M_b\alpha \\ \hline M_o + \alpha^T M_b \alpha \end{array}\right]\{\ddot{U}_o^*\} \qquad (2.1)$$

where $[M_b],[C_b],[K_b]$ are the mass, damping and stiffness matrix for the superstructure on a fixed-base, $[M_o]$ is the mass matrix of the foundation and $[K_s(\omega)] + i\omega[C_s(\omega)]$ represents the foundation impedance matrix. In Eq. (2.1), the harmonic time dependent term $e^{i\omega t}$ is omitted for clearness, and the displacement vectors $\{U\}$ and $\{U_s\}$ are intended to be frequency-dependent. The total displacements of the superstructure $\{U_t\}$ and of the foundation $\{U_0\}$ are given by:

$$\{U_t\} = [\alpha]\{U_0\} + \{U\}, \qquad \{U_0\} = \{U_0^*\} + \{U_s\} \qquad (2.2a, b)$$

where $[\alpha] = [\{1\},\{h\}]$ is the $N \times 2$ rigid-displacement influence matrix in which $\{1\}$ is a column of ones and $\{h\}$ represents the nodal heights with respect to the bottom of the foundation. The vector $\{U_0^*\}$ is the foundation input motion, assumed known from the solution of the scattering problem, [e.g., Luco (1980), Luco and Wong (1987)]. Equation (2.1) represents a generalization of an equation first formulated by Parmelee et al. (1969) for the case of a surface foundation and no scattering.

3. EQUIVALENT SINGLE DEGREE OF FREEDOM MODEL

3.1. Analytical formulation

To start the derivation of the equivalent models, it is assumed that the superstructure, when attached to a fixed base, possesses classical normal modes. Next, the relative displacement vector of the superstructure $\{U\}$ is expressed in terms of the modal coordinates $\{\eta\}$ through the standard transformation

$$\{U\} = [\Phi]\{\eta\} = \sum_{j=1}^{N+2} \{\phi_j\}\eta_j \qquad (3.1)$$

where $[\Phi]$ is the fixed-base modal matrix. The fixed-base eigenvectors satisfy the eigen-value problem

$$\omega_j^2 M_b \{\phi_j\} = [K_b]\{\phi_j\} \tag{3.2}$$

in which ω_j are the fixed-base natural frequencies of the superstructure. Introducing the transformation (3.1) into Eq. (2.1) leads to

$$\left[\begin{array}{c|c} M & \beta^T \\ \hline \beta & M_o + \alpha^T M_b \alpha \end{array}\right]\left\{\begin{array}{c} \ddot{\eta} \\ \hline \ddot{U}_s \end{array}\right\} + \left[\begin{array}{c|c} 2D\Omega M & 0 \\ \hline 0 & C_s \end{array}\right]\left\{\begin{array}{c} \dot{\eta} \\ \hline \dot{U}_s \end{array}\right\} + \left[\begin{array}{c|c} \Omega^2 M & 0 \\ \hline 0 & K_s \end{array}\right]\left\{\begin{array}{c} \eta \\ \hline U_s \end{array}\right\} = -\left[\begin{array}{c} \beta^T \\ \hline M_o + \alpha^T M_b \alpha \end{array}\right]\{\ddot{U}_o^*\} \tag{3.3}$$

in which $[M] = [M_r]$, $[D] = [\xi_r]$, and $[\Omega] = [\omega_r]$ are diagonal matrices of fixed-base modal masses, damping ratios, and natural frequencies of the superstructure, respectively, and $[\beta] = [\alpha]^T [M_b][\Phi]$ is the matrix of participation factors of the superstructure on a fixed-base.

For harmonic time-dependence, the upper part of Eq. (3.3) leads to

$$\eta_i = (B_i - 1) M_i^{-1} \{\beta_i\}^T \{U_0\} \tag{3.4}$$

where $B_i = \left[1 + 2i\xi_i(\omega/\omega_i)\right] \Big/ \left[1 - (\omega/\omega_i)^2 + 2i\xi_i(\omega/\omega_i)\right]$ is a dynamic amplification coefficient and $\{\beta_i\}^T = \{\phi_i\}^T [M_b](\{1\},\{h\})$.

The lower portion of Eq. (3.3) corresponds to the equations of motion of the foundation, under the action of the external excitation and of the forces $\{F\}_{sf}$ exerted by the superstructure:

$$-\omega^2 [M_0]\{U_0\} + i\omega[C_s]\{U_s\} + [K_s]\{U_s\} = -[\alpha]^T [M_b]\left(\{\ddot{U}\} + [\alpha]\{\ddot{U}_0\}\right) \tag{3.5}$$

where the right-hand-side of Eq. (3.5) corresponds to the force $\{F\}_{sf}$. Making use of Eqs. (3.1) and (3.4), and of the relation $\sum_{r=1}^{N} M_r^{-1}\{\beta_r\}\{\beta_r\}^T = [\alpha]^T [M_b][\alpha]$, the force $\{F\}_{sf}$ can be expressed, in the case of harmonic time-dependence, as:

$$\{F\}_{sf} = \omega^2 \sum_{i=1}^{N} B_i [\bar{m}_i]\{U_0\} \tag{3.6}$$

where the matrix $[\bar{m}_i] = M_i^{-1}\{\beta_i\}\{\beta_i\}^T$ can be written in the form

$$[\bar{m}_i] = \hat{M}_i \begin{bmatrix} 1 & H_i \\ H_i & H_i^2 \end{bmatrix} \tag{3.7}$$

where $\hat{M}_i = M_i^{-1}\left(\{\phi_i\}^T [M_b]\{1\}\right)^2$ and $H_i = \{\phi_i\}^T [M_b]\{h\} / \{\phi_i\}^T [M_b]\{1\}$ are effective modal masses and modal heights $(i = 1, N)$, respectively.

Now, we focus on frequencies in the vicinity of the i*th* fixed-base natural frequency. For frequencies ω very different from ω_i, the coefficient B_i can be approximated by:

$$B_i \approx \begin{cases} 1\ , & \omega/\omega_i \ll 1 \\ -(\omega_i/\omega)^2\left[1+2i\xi_i(\omega/\omega_i)\right]\ , & \omega/\omega_i \gg 1 \end{cases} \tag{3.8}$$

Separating in Eq. (3.6) the contribution from the i*th* mode and introducing the approximation given by Eq. (3.8) leads to

$$\{F\}_{sf} = \left(\omega^2\sum_{j=i}^{N}\left[\overline{m}_j\right] - \sum_{j=1}^{i-1}\omega_j^2\left(1+2i\xi_j\frac{\omega}{\omega_j}\right)\left[\overline{m}_j\right]\right)\{U_0\} + \omega^2\{\beta_i\}\eta_i \tag{3.9}$$

At this point, it is convenient to introduce the normalization of the fixed-base modes given by

$$M_i = \{\phi_i\}^T[M_b]\{\phi_i\} = \{\phi_i\}^T[M_b]\{1\} \tag{3.10}$$

which leads to the result $\hat{M}_i = M_i$. Now, using only the equation related to η_i on the upper part of Eq. (3.3) and combining Eqs. (3.5) and (3.9) for the bottom part, leads to the equations of motion for the equivalent model involving the three degrees of freedom η_i, U_s and θ_s:

$$\left(-\omega^2\left[\begin{array}{c|c} \hat{M}_i & \{\beta_i\}^T \\ \hline \{\beta_i\} & [\overline{M}_{oi}]+[\overline{m}_i] \end{array}\right] + i\omega\left[\begin{array}{c|c} 2\xi_i\omega_i\hat{M}_i & \{0\} \\ \hline \{0\} & [\overline{C}_{si}] \end{array}\right] + \left[\begin{array}{c|c} \omega_i^2\hat{M}_i & \{0\} \\ \hline \{0\} & [\overline{K}_{si}] \end{array}\right]\right)\left\{\begin{array}{c} \eta_i \\ \{U_s\} \end{array}\right\} = \left[\begin{array}{c} \omega^2\{\beta_i\}^T \\ \hline [\overline{F}_i] \end{array}\right]\{U_0^*\} \tag{3.11}$$

in which the equivalent mass, damping and stiffness matrices of the foundation are:

$$\left[\overline{M}_{0i}\right] = [M_0] + \sum_{j=i+1}^{N}\left[\overline{m}_j\right],\quad \left[\overline{C}_{si}\right] = [C_s] + \sum_{j=1}^{i-1}2\xi_j\omega_j\left[\overline{m}_j\right],\quad \left[\overline{K}_{si}\right] = [K_s] + \sum_{j=1}^{i-1}\omega_j^2\left[\overline{m}_j\right] \tag{3.12}$$

The equivalent external forces on the foundation are obtained by pre-multiplying the foundation input motion by:

$$\left[\overline{F}_i\right] = \omega^2\left([M_0] + \sum_{j=i}^{N}\left[\overline{m}_j\right]\right) - i\omega\sum_{j=1}^{i-1}2\xi_j\omega_j\left[\overline{m}_j\right] - \sum_{j=1}^{i-1}\omega_j^2\left[\overline{m}_j\right] \tag{3.13}$$

The solution of Eq. (3.11) gives the response of the system in proximity of the i*th* natural frequency. A number of different reduced systems, equal to the number of structural degrees of freedom, would need to be solved, in order to obtain the complete solution over the full frequency range.

3.2. Physical interpretation

As already mentioned, the significance of the equivalent model presented here relates more to the physical insight that it provides, than to any significant numerical advantages. Analysis of Eq. (3.11) reveals that the equivalent model can indeed be interpreted as a single-degree of freedom oscillator, supported on an elastic soil. The first degree of freedom η_i describes the relative motion of the superstructure, in terms of the amplitude of the i*th* fixed-base mode shape. The mass $\left(\hat{M}_i\right)$, stiffness $\left(\omega_i^2\hat{M}_i\right)$, height $\left(H_i\right)$, and damping ratio $\left(\xi_i\right)$ of the superstructure correspond to the respective modal quantities for the i*th* fixed-base mode. The last two degrees of freedom correspond to the translation and rocking of an equivalent rigid foundation. The mass, damping and stiffness matrices of the equivalent foundation are modified to account for the effects of the remaining fixed-base modes

$(j \neq 1)$ of the superstructure. The fixed-base modes with frequencies $\omega_j > \omega_i$ (higher modes) are accounted for by increasing the mass (Eq. 3.12a) of the foundation. The fixed-base modes with frequencies $\omega_j < \omega_i$ (lower modes) are accounted for by increasing the stiffness (Eq. 3.12c) and the damping (Eq. 3.12b) of the spring and dashpot representing the soil impedances.

It is interesting to point out that in the equivalent model of Parmelee (1967) for the vicinity of a particular mode (say, the i*th* mode), all remaining modes (higher and lower) contribute to the additional foundation mass ($\Delta[m] = \sum_{\substack{j=1 \\ j \neq i}}^{N} [\bar{m}_j]$), while in the present equivalent model only the higher modes contribute to the additional foundation mass ($\sum_{j=i+1}^{N} [\bar{m}_j]$). The two results are equivalent for the fundamental mode but they are significantly different for the higher modes. Also, Parmelee (1967) did not consider any additional soil stiffness or damping associated with the lower modes of the superstructure. The results obtained here for the additional damping are consistent with the numerical results of Bielak (1975, 1976) which show that the structural contribution to the system damping ratio for a particular mode stems primarily from the lower fixed-base modes.

A schematic representation of the equivalent 1-DOF model for the simpler case of vertical vibrations is shown in Fig. 3.1. The single story superstructure is represented by the modal mass, stiffness and damping ratio for the i*th* fixed-base mode; the effects of the higher fixed-base modes are represented as an additional mass, rigidly connected to the foundation, and the effects of the lower fixed-base modes are represented as an additional spring and dashpot connected to the foundation.

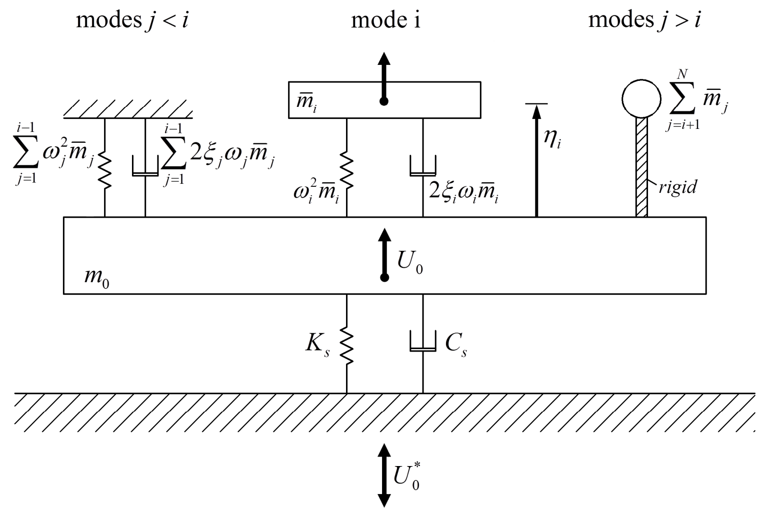

Figure 3.1 Schematic representation of the equivalent single degree of freedom model for vertical vibrations

4. NUMERICAL RESULTS

4.1. Description of the test model

To provide some numerical results, a test structure corresponding to a nine-story building, supported on a rigid foundation, partially embedded in a homogeneous, linear elastic half-space is considered. Table 1 lists the data for the coordinates, masses, stiffnesses of the different floors and the natural frequencies of the structure on a fixed-base. The superstructure is supported on a foundation which is represented by an equivalent rigid, circular base mat, with radius $a = 11m$ and height $h_0 = 5.5m$; the base of the foundation is $5.5m$ below the ground level, thus the embedment ratio is $h_e/a = 0.5$. The foundation soil has a unit mass ρ of $1850\,Kg/m^3$ and Poisson's ratio $\nu = 1/3$. Different values for the shear wave velocity $\beta(m/sec)$ of the soil are considered. To account for the effects of soil-structure interaction, the foundation impedance coefficients provided in Apsel and Luco (1987) are used. Structural damping is accounted for by assuming a fixed-base modal damping ratio of 1% for all modes. A more detailed description of the test structure is provided in Lanzi (2011).

Table 4.1. Properties of the 9-story structure and fixed-base natural frequencies

Floor #	Mass	Moment of inertia	Stiffness	Height	Mode #	F.B. frequency
i	m_i [10^6 Kg]	I_i [10^6 Kgm^2]	k_i [MN/m]	Z_i [m]	j	f_j [hz]
9	1.20	67.70	8103	45.70	1	2.16
8	1.20	67.70	8103	41.40	2	6.42
7	1.20	67.70	8103	37.10	3	10.51
6	1.20	67.70	8103	32.80	4	14.31
5	1.20	67.70	8103	28.50	5	17.72
4	1.20	67.70	8103	24.20	6	20.64
3	1.20	67.70	8103	19.90	7	23.00
2	1.20	67.70	8103	15.60	8	24.74
1	1.20	67.70	8103	11.30	9	25.80
G	1.40	90.20	---	2.75	---	---

4.2. Contribution of the foundation added mass, added stiffness and added damping

To provide quantitative results as to the added foundation mass, stiffness and damping for different modes, the modified matrices for the foundations have been computed for the test nine-story structure. Fig. 4.1 shows the added mass and stiffness terms of the matrices $\overline{M}_{0i}$ and $\overline{K}_{si}$ for the nine reduced eigenvalue problems. The terms relative to the mass are normalized to the fixed-base total mass, total moment of masses and total moment of inertia of the superstructure:

$$\Delta m/M_b = M_b^{-1}\sum_{j=i+1}^{N}\hat{M}_i,\qquad \Delta S/S_b = S_b^{-1}\sum_{j=i+1}^{N}\hat{M}_i H_i,\qquad \Delta I/I_b = I_b^{-1}\sum_{j=i+1}^{N}\hat{M}_i H_i^2 \tag{4.1}$$

The terms relative to the stiffness are normalized to the translational, rotational and coupling stiffness of a superstructure fixed at all of the upper nodes (fixed-structure mode).

$$\Delta K_{HH}/k_1,\qquad \Delta K_{HM}/k_1 h_1,\qquad \Delta K_{MM}\Big/\left[k_1 h_1^2 + \sum_{r\neq 1} k_r\left(h_r - h_{r-1}\right)^2\right] \tag{4.2}$$

The reason for this normalization of the stiffness is that the motion at high frequencies involves small total displacements of the superstructure, which tends to respond like a fixed-superstructure.

It can be observed that: (i) the equivalent added foundation mass for the first and second modes are less than 15% and 6% of the total mass of the superstructure, respectively. The corresponding added moment of mass and mass moment of inertia are extremely small. The added foundation mass tends to zero for the higher modes; (ii) the lower fixed-base modes contribute to the added foundation stiffness and damping. The added translational stiffness increases gradually with the order of the mode considered. The added rotational stiffness increases mainly from the first to the second mode and then remains almost constant.

The magnitude of the added stiffness of the foundation ΔK_{HH}, ΔK_{HM}, and ΔK_{MM} due to the lower fixed-base modes are shown in Fig. 4.2a versus the normalized shear wave velocity of the soil. The added stiffnesses are normalized by the corresponding values of the foundation impedances, and only the curves relative to mode 2 (added stiffness is due to the fundamental fixed-base mode only) and mode 9 (added stiffness is due to all the lower fixed-base modes) are plotted. It can be observed that the relative importance of the added foundation stiffness due to the superstructure monotonically decreases with increasing shear wave velocity. The added translational and rotational stiffness can be, for soft soils, of the same order of the foundation impedances. The added coupling stiffness can be seven times larger than the actual foundation coupling impedance, thus increasing significantly the effect of coupling in the stiffness matrix of the foundation.

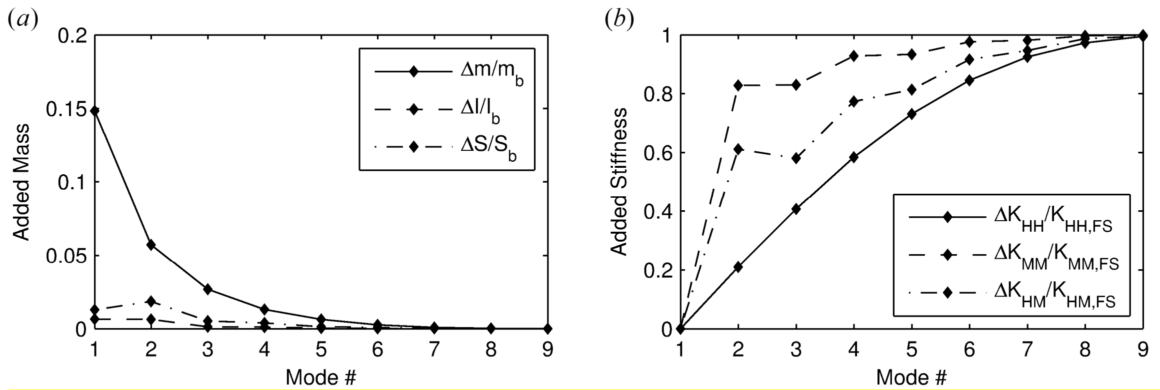

Figure 4.1 (a) Added mass and (b) added stiffness terms for the equivalent 1-DOF model of the test structure

The added foundation damping terms normalized by the corresponding foundation damping constants (imaginary parts of the foundation impedance functions) are shown in Figs. 4.2b. It can be observed that the added foundation damping is small as a consequence of the small amount of modal damping ratios assumed for the superstructure (1%) compared to the damping due to wave radiation into the soil. The increase in the damping term C_{HH} is almost zero, as a result of the large amount of radiation damping for foundation translation. The most significant (but still small) increments correspond to the rocking damping. Most of the added damping is associated with the effect of the fundamental mode and can be seen in the second and higher modes.

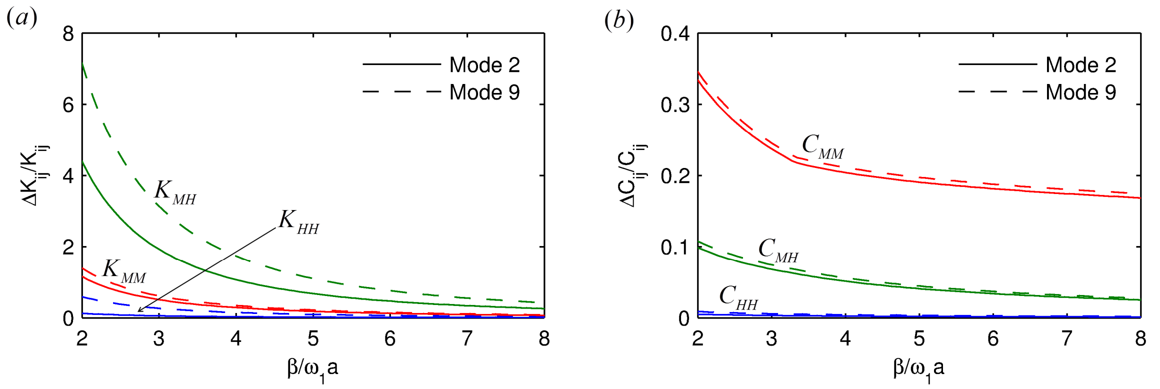

Figure 4.2 (a) Added stiffness and (b) added damping for modes 2 and 9 versus normalized shear wave velocity

4.3. Validation of the proposed approach by comparison with the steady-state response

To validate the equivalent 3-DOF models, the frequency response of the nine-story test building has been calculated using the exact solution [Eq. (2.1)] and the approximate solution calculated by use of Eqs. (3.11), (2.2b), (3.4) and (3.1). Figures (4.3a, b) show the amplitude of the frequency response for the total displacement at the top of the building for an excitation corresponding to the (a) translational and (b) rocking (normalized by the height of the building) components of the foundation input motion for soils with shear wave velocities of 180, 360 and 720 m/sec. The results for a particular mode are shown in a frequency range which extends until agreement is found with the results for the next mode. The results obtained show that the 3-DOF models accurately account for the response in the vicinity of each system mode, and fully validate the equivalent models for soils ranging from very stiff to relatively soft. It should be mentioned that the calculations reflected in Fig. 4.3 utilize frequency-dependent impedance functions.

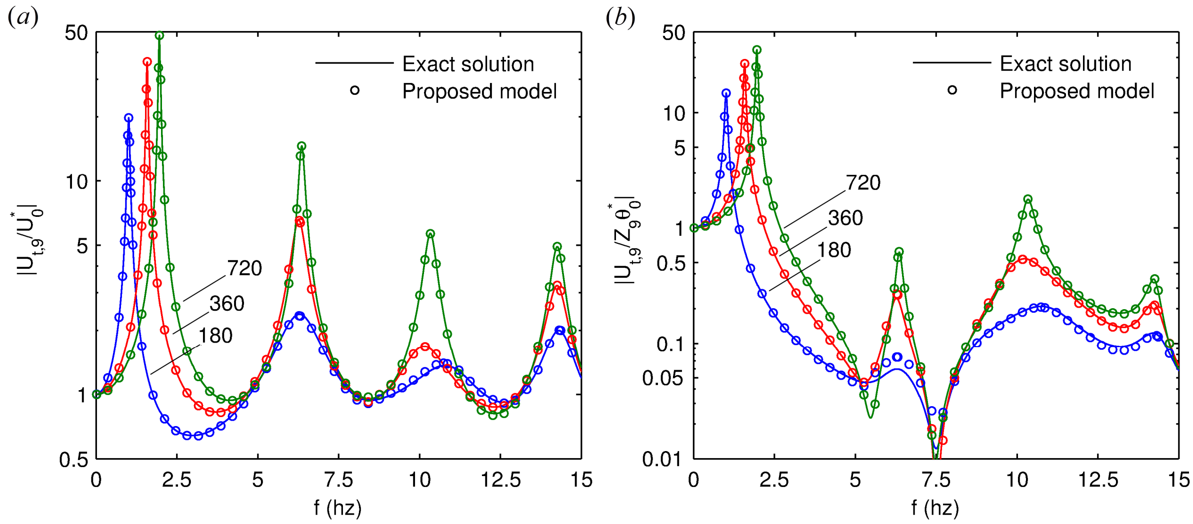

Figure 4.3 Transfer function of the normalized absolute displacement at the top of the nine-story structure for unit translation (a) and unit rotation (b) input motion and three values of soil shear wave velocity β(m/s)

5. CONCLUSIONS

A new equivalent model, consisting of a single degree of freedom oscillator supported on an elastic soil, selected to approximate the response of a soil-structure system in the vicinity of each system mode has been presented. The model is based on the modal properties of the structure on a fixed-base and on the foundation impedance functions. It is shown that the effects of the lower fixed-base modes can be accounted for by increasing the stiffness and damping of the foundation while the effects of the higher fixed-base modes can be approximated by increasing the mass of the foundation. The new equivalent model provides considerable insight into the effects of SSI on the dynamic response of structures.

The magnitudes of the additional foundation masses associated with the effect of the higher fixed-base modes have been quantified for a nine-story test structure. The added foundation mass decreases rapidly with mode number from a maximum, for the first mode, of about 15% of the total mass of the superstructure. The added moment of mass and moment of inertia are extremely small. The added mass proposed here coincides with that proposed by Parmelee (1967) for the fundamental mode, but is significantly smaller than the earlier added mass for the higher modes.

The lower fixed-base modes contribute to the stiffness and the damping of the foundation. The added translational stiffness gradually increases with mode order, while the added rotational stiffness increases mainly from the first to the second mode. The added translational and rotational stiffness of the foundation can be, for soft soils, of the same order of the foundation impedances. The added coupling stiffness can be several times larger than the corresponding foundation impedance, thus increasing significantly the effect of the translation-rocking coupling in the effective stiffness matrix of the foundation. The effects of the lower fixed-base modes in increasing the damping matrix of the foundation are less pronounced than those on the stiffness. This is a result of the typically small amount of structural modal damping compared to the larger radiation damping in the soil. The analytical results obtained here for the additional damping are consistent with the numerical results of Bielak (1975, 1976) which show that the structural contribution to the system damping ratio for a particular mode stems primarily from the lower fixed-base modes. Finally, the new 3-DOF model has been validated by comparison of the local steady-state response with that obtained by exact solution of the equation of motion for the complete interacting system.

AKCNOWLEDGEMENT
The work described here was completed while A. Lanzi was at the University of California, San Diego with a Fellowship from the US-Italy Fulbright Commission.

REFERENCES

Apsel, R. J., & Luco, J. E. (1987). Impedance Functions for Foundations Embedded in a Layered Medium: an Integral Equation Approach. *Earthquake Engineering and Structural Dynamics,* **15:2**, 213-231.

Beredugo, Y. O. (1976). Modal analysis of coupled motion of horizontally excited embedded footings. E*arthquake Engineering and Structural Dynamics*, **4:4**, 403-410.

Bielak, J. (1971). *Earthquake Response of Building-Foundation Systems.* Report EERL 71-04, Earthquake Engineering Research Laboratory. Pasadena, CA: California Institute of Technology.

Bielak, J. (1975). Modal Analysis for Building-Soil Interaction. Report E17, Instituto de Ingenieria. Mexico, D.F.: Universidad Autonoma de Mexico.

Bielak, J. (1976). Modal Analysis for Building-Soil Interaction. (ASCE, Ed.) *Journal of the Engineering Mechanics Division,* **102:EM5**, 771-786.

Clough, R. W., & Mojtahedi, S. (1976). Earthquake Response Analysis considering Non-proportional Damping. *Earthquake Engineering and Structural Dynamics,* **4:5**, 489-496.

Jennings, P. C., & Bielak, J. (1973). Dynamics of Building-Soil Interaction. *Bulletin of the Seismological Society of America,* **63:1**, 9-48.

Lanzi, A. (2011). Approximate Analysis of Dynamic Soil-Structure Interaction, M.S. thesis, University of California San Diego, La Jolla, California.

Luco, J. E. (1980). *Linear Soil-Structure Interaction.* Report UCRL - 15272, PSA No. 7249808. Livermore, California: Lawrence Livermore Laboratory.

Luco, J. E., & Wong, H. L. (1987). Seismic Response of Foundations Embedded in a Layered Half-Space. *Earthquake Engineering and Structural Dynamics,* **15:2**, 233-247.

Novak, M. (1974). Effect of Soil on Structural Response to Wind and Earthquake. *Earthquake Engineering and Structural Dynamics,* **3:3**, 79-96.

Novak, M. (1974). Additional Note on the Effect of Soil on Structural Response. *Earthquake Engineering and Structural Dynamics,* **3:3**, 312-315.

Parmelee, R. A. (1967). Building-Foundation Interaction Effects. (ASCE, Ed.) *Journal of the Engineering Mechanics Division,* **93:2**, 131-155.

Parmelee, R. A., Perelman, D. S., & Lee, S. L. (1969). Seismic Response of Multiple Story Structure on Flexible Foundations. *Bulletin of the Seismological Society of America,* **59:3**, 1061-1070.

Rainer, J. H. (1971). Structure-Ground Interaction in Earthquakes. (ASCE, Ed.) *Journal of the Engineering Mechanics Division,* **97:EM2**, 1431-1450.

Rainer, J. H. (1975). Damping in Dynamic Structure-Foundation Interaction. *Canadian Geotechnical Journal,* **12:1**, 13-22.

Roesset, J. M., Whitman, R. V., & Dobry, R. (1973). Modal Analysis for Structures with Foundation Interaction. (ASCE, Ed.) *Journal of the Structural Division,* **99:ST3**, 399-416.

Sarrazin, M., Roesset, J. M., & Whitman, R. V. (1972). Dynamic Soil-Structure Interaction. (ASCE, Ed.) *Journal of the Structural Division,* **98:ST7**, 1525-1544.

Tajimi, H. (1967). Discussion: building-foundation interaction effects. (ASCE, Ed.) *Journal of Engineering Mechanics Division,* **93:EM6**, 294-298.

Tsai, N. C. (1974). Modal Damping for Soil-Structure Interaction. (ASCE, Ed.) *Journal of the Engineering Mechanics Division*, **100:EM2**, 323-341.

Tsai, N. C., Niehoff, D., & Swatta, M. (1974). The use of Frequency-independent Soil-Structure Interaction Parameters. *Nuclear Engineering and Design*, **31:2**, 168-183.

Vaidya, N. R., Bazan-Zurita, E., & Rizzo, P. C. (1986). On the Validity of Modal Superposition Technique in Soil-Structure Interaction Analysis. *Proc. 3rd U.S. National Conference on Earthquake Engineering*, (pp. 683-692). Charleston, SC.

Velestos, A. S., & Meek, J. W. (1974). Dynamic Behavior of Building-Foundation Systems. *Earthquake Engineering and Structural Dynamics,* **3:2**, 121-138.

Warburton, G. B., & Soni, S. R. (1977). Errors in Response Calculations for Non-classically Damped Structures. *Earthquake Engineering and Structural Dynamics,* **5:4**, 365-376.