

Damage Identification of a Continuous Beam using Sequential Nonlinear LSE Approach

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SUMMARY: (10 pt)

The ability to detect damages real-time on-line, based on vibration data measured from sensors, will ensure the reliability and safety of structures. Innovative data analysis techniques for the on-line damage detection of structures have received considerable attentions recently. However, most of the techniques have been studied based on shear beam type of structures with small degree of freedoms. A challenging problem in structural damage detection is to minimize the number of sensors and reduce the numerical difficulty in obtaining reasonably accurate results when the system is complex. In this connection, a newly proposed data analysis method for structural damage identifications, referred to as the sequential nonlinear LSE (SNLSE) approach, will be studied in this paper using a continuous beam structure. Experimental study will be carried out with limited number of measured response data. The capability of the SNLSE approach in identifying the structural damage will be verified.

Keywords: Structural health monitoring, Sequential nonlinear LSE, Damage identification of structures

1. INTRODUCTION

Installation of structure health monitoring (SHM) system is necessary and plays an important role in the assessment of structure conditions to ensure the reliability and safety of structures. Early detection of structural damage is an important goal of SHM systems and critical for the decision making of repair and replacement maintenance. Analysis methodologies based on vibration data measured from sensors has received increasing attention recently in the field of civil engineering for damage identification of structures [e.g., Bernal and Beck (2004)]. Most of the methodologies available in the literature [e.g., Bernal and Beck (2004)] are capable of identifying the constant system parameters, such as the stiffness, and require both the referenced data (data for the structure without damage) and the data after structural damage. Then, the damage is obtained by a comparison of the constant structural parameters prior to and after damages. In practice, however, the referenced data may not be available or difficult to obtain, and after a severe event, such as a strong earthquake, it may not be feasible to conduct vibration tests to obtain meaningful data for damage identifications. Hence, it is desirable for an analysis method to be capable of detecting the structural damage based solely on the vibration data measured during a severe event without a prior knowledge of the undamaged structure. In this connection, several on-line damage identification methodologies have been developed recently, including the least square estimation (LSE) [e.g., Lin et al (2001), Smyth, et al (2003), Yang and Lin (2004, 2005)], the extended Kalman filter (EKF) [e.g., Hoshiya and Saito (1984), Sato et al (2001), Yang et al (2006a)], the sequential nonlinear least square estimation (SNLSE) [Yang et al (2006b)], the quadratic sum-squares error (QSSE) [Yang et al (2009)], and others.

For the application of LSE approach, all the responses including acceleration, velocity and displacement have to be available. However, in practice, acceleration responses are usually measured on-line, whereas the velocity responses can be obtained through a single numerical integration. For the displacement response, however, a double numerical integration from the acceleration response results in a significant numerical drift that is also magnified seriously when damages occur [Yang and Lin

(2005)]. Nevertheless, such a numerical drift can be removed using special approaches [see Yang and Lin (2005)].

With only the measurements of acceleration responses, the EKF, SNLSE and QSSE approaches can be used for the damage identification. However, the EKF approach requires that the estimates of the initial values of the unknown parameters should not be far away from their theoretical values in order to obtain convergent solutions, as compared with the SNLSE and QSSE approaches. Further, the dimension of the extended state vector \mathbf{Z} in EKF is quite large, especially for large and complex structures. Hence, the computational efforts required for estimating \mathbf{Z} is quite involved.

Among the approaches described above, the SNLSE and QSSE approaches are more suitable for on-line damage identification of structures, in terms of accuracy, convergence and efficiency, and they have been proved to be effective for damages detection of building structures that can be represented as shear-beam type of structures with spring-mass systems [Yang and Lin (2006b, 2009)]. In this paper, the recently proposed SNLSE approach will be examined for identifying damages of relatively more complex structures such as bridge structures. Recursive solutions will be derived based on the finite element model of the bridge structures. Due to space limitation, only experimental verification of a three span reinforced concrete continuous beam will be demonstrated to show the accuracy and effectiveness of the proposed SNLSE approach.

2. SEQUENTIAL NONLINEAR LSE

The equation of motion of a m-DOF nonlinear structure can be expressed as

$$\mathbf{M} \ddot{\mathbf{x}}(t) + \mathbf{F}_c [\dot{\mathbf{x}}(t), \boldsymbol{\theta}] + \mathbf{F}_s [\mathbf{x}(t), \boldsymbol{\theta}] = \boldsymbol{\eta} \mathbf{f}(t) \quad (2.1)$$

in which $\mathbf{x}(t) = [x_1, x_2, \dots, x_m]^T$ = m-displacement vector; \mathbf{M} = (m×m) mass matrix; $\mathbf{F}_c [\dot{\mathbf{x}}(t), \boldsymbol{\theta}]$ = m-damping force vector; $\mathbf{F}_s [\mathbf{x}(t), \boldsymbol{\theta}]$ = m-stiffness force vector; $\mathbf{f}(t) = [f_1(t), f_2(t), \dots, f_s(t)]^T$ = s-excitation vector; and $\boldsymbol{\eta}$ = (m × s) excitation influence matrix associated with $\mathbf{f}(t)$. In Eq.(1), $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_n]^T$ is an n-unknown parametric vector with θ_i ($i = 1, 2, \dots, n$) being the i th unknown parameter of the structure, including damping, stiffness, nonlinear and hysteretic parameters. We shall assume for the time being that the unknown parametric vector $\boldsymbol{\theta}$ is constant, i.e., $\boldsymbol{\theta} = \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \dots = \boldsymbol{\theta}_{k+1}$, where $\boldsymbol{\theta}_i = \boldsymbol{\theta}$ ($t = i\Delta t$) for $i = 1, 2, \dots, k+1$. In what follows, the bold face letter represents either a vector or a matrix.

The observation equation associated with the equation of motion can be written as

$$\boldsymbol{\varphi}[\mathbf{X}; t]\boldsymbol{\theta} + \boldsymbol{\varepsilon}(t) = \mathbf{y}(t) \quad (2.2)$$

in which $\mathbf{y}(t) = \boldsymbol{\eta}\mathbf{f}(t) - \mathbf{M}\ddot{\mathbf{x}}(t)$ is known and $\boldsymbol{\varepsilon}(t)$ is the model noises. Eq.(2.2) can be discretized at $t = t_k = k\Delta t$ as

$$\boldsymbol{\varphi}_k(\mathbf{X}_k)\boldsymbol{\theta}_k + \boldsymbol{\varepsilon}_k = \mathbf{y}_k \quad (2.3)$$

in which $\boldsymbol{\varphi}_k(\mathbf{X}_k) = \boldsymbol{\varphi}[\mathbf{X}(t_k); t_k]$, $\boldsymbol{\theta}_k = \boldsymbol{\theta}(t_k)$, $\boldsymbol{\varepsilon}_k = \boldsymbol{\varepsilon}(t_k)$ and $\mathbf{y}_k = \mathbf{y}(t_k)$. Instead of solving \mathbf{X}_k and $\boldsymbol{\theta}_k$ simultaneously by forming an extended state vector as in the EKF approach, we shall solve \mathbf{X}_k and $\boldsymbol{\theta}_k$ in two steps. The first step is to determine $\boldsymbol{\theta}_k$ by assuming (or under the condition) that

\mathbf{X}_k is given using the LSE solution, and the second step is to determine \mathbf{X}_k through a nonlinear LSE approach, referred to as the SNLSE, as follows.

Step I: Suppose the state vector \mathbf{X}_{k+1} is known and the parametric vector $\boldsymbol{\theta}_{k+1}$ is constant. The classical LSE recursive solution $\hat{\boldsymbol{\theta}}_{k+1}$ that is the estimate of $\boldsymbol{\theta}_{k+1}$ is obtained as follows

$$\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_k + \mathbf{K}_{k+1}(\mathbf{X}_{k+1})[\mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\hat{\boldsymbol{\theta}}_k] \quad (2.4)$$

$$\mathbf{K}_{k+1}(\mathbf{X}_{k+1}) = \mathbf{P}_k \boldsymbol{\varphi}_{k+1}^T(\mathbf{X}_{k+1})[\mathbf{I} + \boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\mathbf{P}_k \boldsymbol{\varphi}_{k+1}^T(\mathbf{X}_{k+1})]^{-1} \quad (2.5)$$

$$\mathbf{P}_k = \mathbf{P}_{k-1} - \mathbf{K}_k(\mathbf{X}_k)\boldsymbol{\varphi}_k(\mathbf{X}_k)\mathbf{P}_{k-1} \quad (2.6)$$

in which $\mathbf{K}_{k+1}(\mathbf{X}_{k+1})$, $\boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})$ and \mathbf{P}_k are defined similarly in the LSE approach, except that the former two matrices are functions of \mathbf{X}_{k+1} .

Step II: As observed from Eqs. (2.4) – (2.5), $\hat{\boldsymbol{\theta}}_{k+1}$ is a function of the unknown state vector \mathbf{X}_{k+1} , i.e. $\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_{k+1}(\mathbf{X}_{k+1})$. The estimate $\hat{\mathbf{X}}_{k+1|k+1}$ of \mathbf{X}_{k+1} was obtained by the following recursive solution [Yang et al (2006b)],

$$\hat{\mathbf{X}}_{k+1|k+1} = \mathbf{X}_{k+1|k} + \bar{\mathbf{K}}_{k+1}[\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1}(\hat{\mathbf{X}}_{k+1|k})] \quad (2.7)$$

where $\hat{\mathbf{y}}_{k+1}(\hat{\mathbf{X}}_{k+1|k}) = \boldsymbol{\varphi}_{k+1}(\hat{\mathbf{X}}_{k+1|k})\hat{\boldsymbol{\theta}}_{k+1}$, and

$$\hat{\mathbf{X}}_{k+1|k} = \boldsymbol{\Phi}_{k+1,k}\hat{\mathbf{X}}_{k|k} + \mathbf{B}_1\ddot{\mathbf{x}}_k + \mathbf{B}_2\ddot{\mathbf{x}}_{k+1} \quad (2.8)$$

In Eqs. (2.7) and (2.8), $\boldsymbol{\Phi}_{k+1,k}$ is the transition matrix for the state vector from k to $k+1$, and \mathbf{B}_1 , \mathbf{B}_2 and $\bar{\mathbf{K}}_{k+1}$ are appropriate matrices [see Yang et al (2006b)].

This approach is referred to as the sequential nonlinear LSE (SNLSE), and it will be verified by experimental studies later.

3. EXPERIMENTAL STUDIES

Experimental studies have been carried out using a three span reinforced concrete continuous beam model. The parameters of the structure were identified using the SNLSE approach with data measured from the vibration tests and compared with the results from the finite element model, to verify the effectiveness and accuracy of SNLSE parametric identification and damage detection of bridge structures.

3.1 Experiment setup



Figure 1. Three span reinforced concrete continuous beam model

The experimental model of three-span reinforced concrete continuous beam was setup as shown in Fig.1. The concrete was reinforced by the galvanized wires which were distributed uniformly in both tension and compression zones of the cross section. The density of the beam was $\rho = 2500\text{Kg} / \text{m}^3$, and the elastic modulus was $E = 3.2 \times 10^{10}\text{Pa}$. The beam was of size $3800\text{mm} \times 200\text{mm} \times 30\text{mm}$ with span arrangement of $1000+1700+1000\text{mm}$ and 50mm overhang on each end. The boundary conditions of the beam were as shown in Fig.1c. Five accelerometers were installed with three on the mid-span and one on each side-span of the beam. The vibration test data was transmitted to the computer through a data acquisition device with a conditioning amplifier. The detailed configuration of the experimental setup is illustrated in Fig.2.

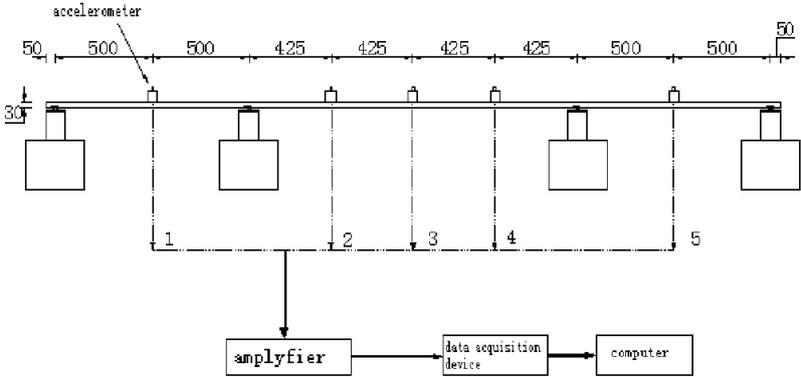


Figure 2. Arrangement of sensors and data acquisition devices in the model test

3.2 Finite element modelling

The finite element model of the three-span reinforced concrete continuous beam was established using 8 beam elements and 9 nodes. The layout of elements and nodes and their numbers is illustrated in Fig.3.

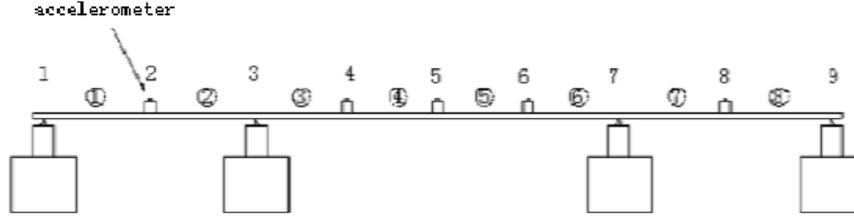


Figure 3. Finite element model of the three-span reinforced concrete continuous beam

Let \mathbf{M}_i and \mathbf{K}_i be the local mass matrix and the local stiffness matrix, respectively, of the i th element (member) with a uniform cross-section in the local coordinate system, one has

$$\mathbf{M}_i = \frac{\bar{m}L_i}{420} \begin{bmatrix} 156 & 22L_i & 54 & -13L_i \\ 22L_i & 4L_i^2 & 13L_i & -3L_i^2 \\ 54 & 13L_i & 156 & -22L_i \\ -13L_i & -3L_i^2 & -22L_i & 4L_i^2 \end{bmatrix}; \mathbf{K}_i = \frac{E_i I_i}{L_i} \begin{bmatrix} 12/L_i^2 & 6/L_i & -12/L_i^2 & 6/L_i \\ 6/L_i & 4 & -6/L_i & 2 \\ -12/L_i^2 & -6/L_i & 12/L_i^2 & -6/L_i \\ 6/L_i & 2 & -6/L_i & 4 \end{bmatrix} \quad (3.1)$$

in which L_i and \bar{m}_i are the length and the mass per unit length of the i th element (or member) of the sub-structure, respectively, and $k_i = E_i I_i / L_i$ is the equivalent stiffness parameter, where E_i and I_i are the Young's modulus and moment of inertial of the i th element (or member), respectively. The local element mass and element stiffness matrices \mathbf{M}_i and \mathbf{K}_i are transformed into $\bar{\mathbf{M}}_i$ and $\bar{\mathbf{K}}_i$, which are the element matrices in the global coordinate system of the structure, using the transformation matrix \mathbf{T} , i.e.,

$$\bar{\mathbf{M}}_i = \mathbf{T}^T \mathbf{M}_i \mathbf{T}; \bar{\mathbf{K}}_i = \mathbf{T}^T \mathbf{K}_i \mathbf{T} \quad (3.2)$$

in which \mathbf{T} is a (4×4) matrix with its (i, j) element, \mathbf{T}_{ij} , as: $\mathbf{T}_{11} = \mathbf{T}_{22} = \mathbf{T}_{33} = \mathbf{T}_{44} = \cos \phi$, $\mathbf{T}_{12} = \mathbf{T}_{34} = \sin \phi$, $\mathbf{T}_{21} = \mathbf{T}_{43} = -\sin \phi$, and $\mathbf{T}_{ij} = 0$ for other i and j , where ϕ is the angle between the local and global coordinates. Finally, the element mass and stiffness matrices $\bar{\mathbf{M}}_i$ and $\bar{\mathbf{K}}_i$ are expanded to $(m \times m)$ matrices denoted by $\tilde{\mathbf{M}}_i$ and $\tilde{\mathbf{K}}_i$, and the global mass and stiffness matrices \mathbf{M} and \mathbf{K} of the structure are obtained by summing up $\tilde{\mathbf{M}}_i$ and $\tilde{\mathbf{K}}_i$ for all the elements, i.e.

$$\mathbf{M} = \sum_{i=1}^p \tilde{\mathbf{M}}_i; \mathbf{K} = \sum_{i=1}^p \tilde{\mathbf{K}}_i = \sum_{i=1}^p k_i \mathbf{S}_i \quad (3.3)$$

in which for simplicity of presentation $\tilde{\mathbf{K}}_i$ is expressed in terms of $k_i \mathbf{S}_i$, where $k_i = E_i I_i / L_i$ is the equivalent stiffness parameter and \mathbf{S}_i is a $(m \times m)$ matrix of the i th element. In Eq.(3.3), p is the total number of elements (members).

For the three-span reinforced concrete continuous beam, the structural parameters are as follows: (1) mass per unit length is $\bar{m} = \rho A = 15 \text{Kg} / \text{m}$, (2) elastic modulus is $E_i = 3.2 \times 10^{10} \text{Pa}$ ($i = 1 : 9$), (3) length of the elements are $L_1 = L_2 = L_7 = L_8 = 0.5 \text{m}$, $L_{2-5} = 0.425 \text{m}$, and (4) first 4 natural frequencies are $f_1 = 24.2 \text{Hz}$, $f_2 = 53.3 \text{Hz}$, $f_3 = 62.9 \text{Hz}$, $f_4 = 91.0 \text{Hz}$.

An impulse excitation was applied at the location of 3/8 of the main span by a hammer. Vertical accelerations at nodes 2, 4, 5, 6, 8, denoted by a2, a4, a5, a6, a8 respectively, were acquired by the accelerometers. Structural parameters were identified using the data of the first 3s starting from the 10th sample so that the vibration was considered as free vibration. Two different cases were studied where the beam was first considered to be undamaged and then damaged.

3.3 Case 1: undamaged

3.3.1 Data processing

The time history of measured acceleration are shown in Fig.4, with sampling rate of 0.0031s. The first 4 modal frequencies can be obtained by frequency spectrum analysis of the acceleration response at node 2, which are $f_{a1} = 25.9$ Hz, $f_{a2} = 57.6$ Hz, $f_{a3} = 64.4$ Hz, and $f_{a4} = 85.7$ Hz respectively. The maximum difference between the theoretical frequencies obtained from the finite element modeling and the measured frequencies is about 8% as given in Table 1, which shows that the finite element model reflects the dynamic characteristics of the actual structure very well and thus, it can be used for identification of structural parameters.

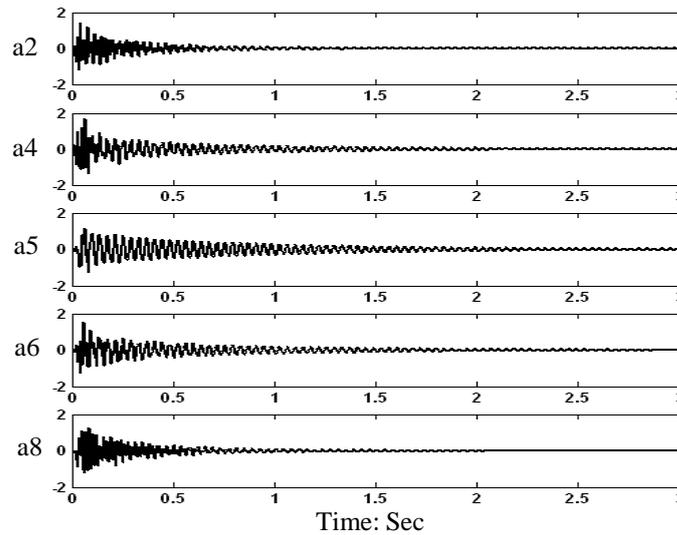


Figure 4. Acceleration time histories of measured nodes

Table 1. Comparison between theoretical and measured undamaged frequencies

	1st	2nd	3rd	4th
Theoretical Value (Hz)	24.2	53.3	62.9	91.0
Measured Value (Hz)	25.9	57.6	64.4	85.7
Relative Error (%)	7.02	8.07	2.38	5.82

3.3.2 Angular acceleration

There are 2 degrees of freedom at each node of the beam element, namely, the vertical and angular responses. In order to compute the structural parameters using SNLSE, the angular response is also needed in addition to the vertical response of the structure. However, in practice it is difficult to measure the angular response directly, and therefore, it will be estimated from the vertical response using the following approach.

Let γ denote the nodal rotational angle, according to the relationship between deformations, one has

$$\dot{\gamma} = \frac{dy}{dx} \quad (3.4)$$

in which y denotes the vertical displacement and x denotes the transverse coordinate. The angular velocity and acceleration can be obtained as

$$\dot{\gamma} = \frac{d^2y / dx}{dt} \quad (3.5)$$

$$\ddot{\gamma} = \frac{d^3y / dx}{dt^2} \quad (3.6)$$

Under vertical loads, the nodal transverse coordinate x does not vary with time for small deformation, and the differential orders can be changed as

$$\dot{\gamma} = \frac{d(dy / dt)}{dx} = \frac{dv}{dx} \quad (3.7)$$

$$\ddot{\gamma} = \frac{d(d^2y / dt^2)}{dx} = \frac{da}{dx} \quad (3.8)$$

where v is nodal vertical velocity, a is nodal vertical acceleration. In summary, the nodal angular responses equal to the derivation of the corresponding vertical responses with respect to the transverse coordinates. In numerical computation, the differentiation process requires adequate density of measuring nodes, whereas the number of accelerometers installed in the experiment is limited. According to the mechanics of material, the deformation of a continuous beam at any time instance is accordance with a spline curve which in mathematics is a sufficiently smooth piecewise-polynomial function. Therefore, the vertical accelerations at unmeasured nodes were first estimated by spline interpolation from the measured vertical accelerations, and then the angular acceleration were computed using Eq.(3.8). Selected angular accelerations at the measured nodes are shown in Fig.5.

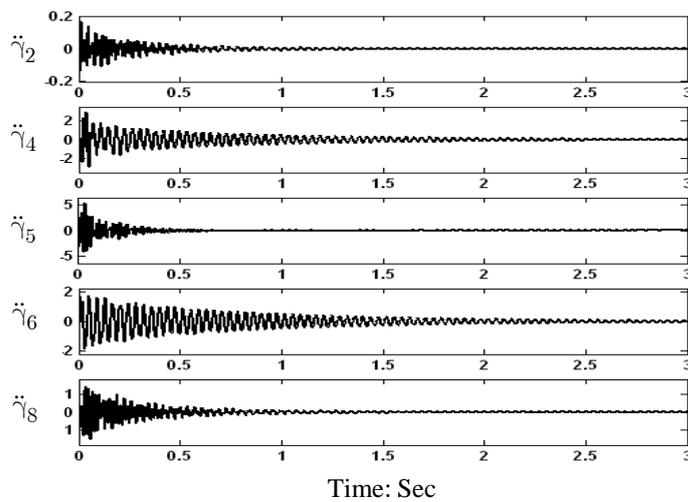


Figure 5. Angular acceleration time histories at the measured nodes

3.3.3 Parametric estimation

With the measured accelerations at nodes 2, 4, 5, 6, 8 and the estimated accelerations at nodes 1, 3, 7, 9 as well as the corresponding angular accelerations computed, structural parameters k_i are identified based on the SNLSE approach. The computed theoretical and estimated values of structural parameters are listed in Table 2. It can be seen from both Table 2 that the proposed SNLSE approach is capable of identifying structural parameters accurately.

Table 2. Comparison between the theoretical and estimated structural parameters

Structural Parameters	Theoretical Values (10^4 N.m)	Estimation (10^4 N.m)	Error (%)
k1	2.52	2.58	2.21
k2	2.52	2.61	3.42
k3	2.96	3.11	4.79
k4	2.96	3.12	5.32
k5	2.96	2.90	2.11
k6	2.96	2.67	9.78
k7	2.52	2.34	7.21
k8	2.52	2.61	3.54

3.4 Case 2: damaged

The damage is simulated by symmetrically reducing the cross sectional area of the beam from 1150mm to 1350mm along the longitudinal direction, as illustrated in Fig.6. In the damage region, two blocks were cut off and each is of the dimension 200mm \times 50mm \times 30mm. Similar to the undamaged case, an impact load was applied at the location of 3/8 of the main span and only the free vibration period was considered for signal processing and parametric identification.

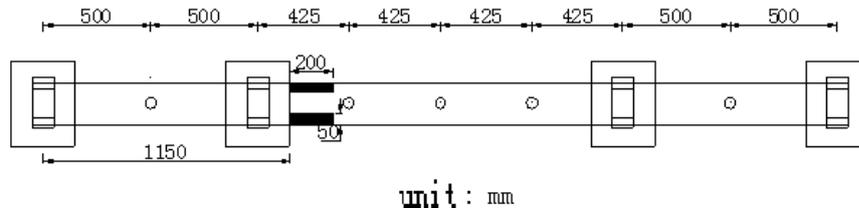


Figure 6. Damage pattern of the beam

3.4.1 Data processing

The time history of the first 2 seconds of the measured acceleration are shown in Fig.7, with sampling rate of 0.0031s. The first 4 modal frequencies were obtained by frequency spectrum analysis of the acceleration response at node 2, which are $f_{a1} = 26.4$ Hz, $f_{a2} = 53.3$ Hz, $f_{a3} = 63.5$ Hz, and $f_{a4} = 90.6$ Hz, respectively. The maximum difference between the theoretical frequencies obtained from the finite element modeling and the measured frequencies is about 9% as given in Table 3. It can be seen by comparing Table 1 and 3 that the measured damaged frequencies are closer to the corresponding theoretical values than the measured undamaged frequencies. This shows that using frequency alone is not able to draw conclusions on structural damage.

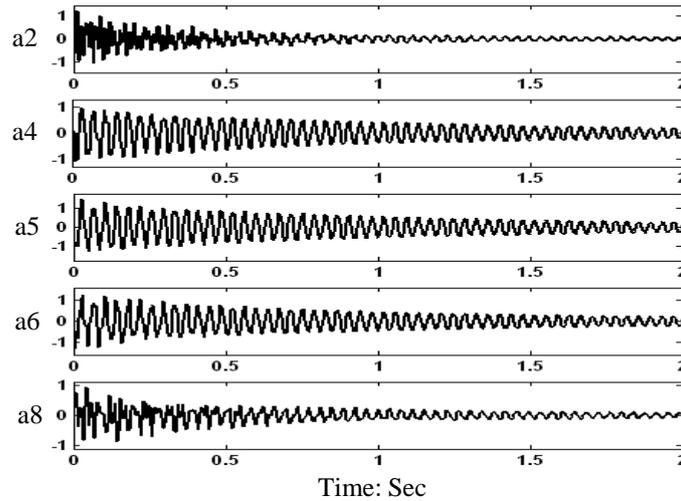


Figure 7. Acceleration time histories of measured nodes

Table 3. Comparison between theoretical and measured damaged frequencies

	1st	2nd	3rd	4th
Theoretical Value (Hz)	24.2	53.3	62.9	91.0
Measured Value (Hz)	26.4	53.3	63.5	90.6
Relative Error (%)	4.58	0	0.95	0.44

3.4.2 Angular acceleration

The accelerations of unmeasured nodes as well as the angular accelerations are computed using the same approach as in the undamaged case. Fig.8 plots the angular accelerations at the measured nodes for representation.

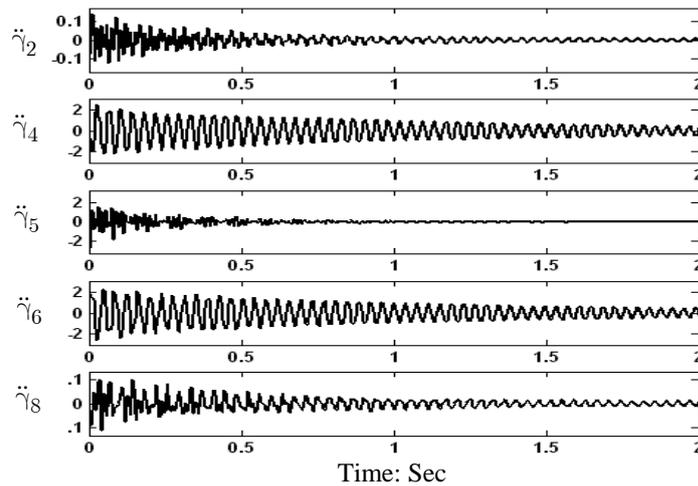


Figure 8. Angular acceleration time histories at the measured nodes

3.4.3 Parametric estimation

Based on the SNLSE approach, the structural parameters k_i were identified. The computed theoretical and estimated values of structural parameters are listed in Table 4. It can be seen from Table 4 that the proposed SNLSE approach is capable of identifying structural parameters as well as damages with reasonable accuracy.

Table 4. Comparison between the theoretical and estimated structural parameters

Structural Parameters	Theoretical Values (10^4 N.m)	Estimation (10^4 N.m)	Error (%)
k1	2.52	2.65	5.33
k2	2.52	2.65	5.14
k3	2.37	2.56	7.99
k4	2.96	3.42	15.41
k5	2.96	2.61	12.01
k6	2.96	2.82	4.81
k7	2.52	2.29	9.11
k8	2.52	2.82	11.73

5. CONCLUSION

In this paper, the recently proposed sequential nonlinear least square estimation (SNLSE) has been used to identify structural damages of a continuous beam structure with limited number of measurements. Experimental studies were carried out using a three span reinforced concrete continuous beam and the proposed SNLSE approach was applied for identification of structural parameters as well as damages. The results showed that SNLSE approach is capable of identifying the structural damages with reasonable accuracy and effectiveness.

ACKNOWLEDGEMENT

This research is supported by the National Natural Science Foundation of China Grant No. 50808138.

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