Mechanical models for the vulnerability assessment of existing reinforced concrete buildings

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SUMMARY:
Vulnerability assessment at large scale requires referring to reliable models which are able to establish a correlation between hazard and structural damage. Among the different approaches proposed in literature, the attention is focused on mechanical models based on the displacement-based approach, which describe the inelastic response of buildings by capacity curves able to provide essential information in terms of stiffness, overall strength and ultimate displacement capacity. In the paper an extensive sensitivity analysis is carried out by considering the various expressions proposed in literature for these entities (in order to define the more reliable ones) and by evaluating how each parameter (e.g. strength and ductility of materials, structural element dimensions, interstory heigth, ...), which mechanical models may be founded on, affect the structual response (in terms of main parameters which define the capacity curve). Particular attention is paid to the model adopted in Lagomarsino *et al.* (2010) by proposing some improvements as pointed out from the sensitivity analyses results.

Keywords: mechanical models, reinforced concrete buildings, vulnerability assessment

1. INTRODUCTION

Earthquake loss estimation procedures require data capture on the vulnerability of both building classes and exposed elements, in term of number and size of buildings and number of occupiers. Losses are usually related – by proper correlation laws – to structural damages classified in various levels (damage states) as a function of the heaviness and the spread. The damage assessment needs referring to reliable vulnerability models, able to establish a correlation between hazard and structural damage. Several methods for the vulnerability assessment of reinforced concrete buildings have been developed and proposed in recent years. They are based on various approaches which may basically classified according to two classes: the observational (macroseismic) models and the mechanical ones. The first approach is derived and calibrated from damage assessment data, collected after earthquakes in areas that suffered from different intensities; it is therefore only based on qualitative data. The second one allows to take expressly into account the influence of a limited number of mechanical and geometrical parameters on the seismic response.
This latter approach seems particularly effective for R.C. structures built using an "engineering approach" based on principles and rules of design unlike other building types like masonry structures mainly designed following empirical rules of art. Mechanical models are formulated following two basic approaches: a force-based seismic assessment and a displacement-based one. This paper will deal with the latter approach, in line with performance based assessment which all research trends and codes usually refer to.
In particular, the structure response is described by mean of a force – displacement curve which aims to describe the overall inelastic response of the structure, providing essential information to idealize its actual behaviour in terms of stiffness, overall strength and ultimate displacement capacity. The use of mechanical models, depending expressly on geometrical and mechanical parameters, is particularly attractive for different types of vulnerability assessment at territorial scale, as:

- assessment on building stocks characterized by homogeneous behaviour for a seismic loss estimation. In this case, results assume a statistical meaning in order to perform risk analysis,

that is to evaluate the probability by having certain consequences on the examined area (country, region, town …);
- assessment on single buildings for planning preventive interventions for the seismic risk mitigation. In this case, the main goal is to evaluate the relative seismic capacity of a building within a group to identify the more vulnerable structures and thus to define a list of priority. Starting from this list, then it is possible to select buildings where more detailed analyses may be focused on.

In the paper, an overview of models, based on a displacement approach and a wide sensitivity analysis are carried out, in order to evaluate how each parameter (e.g. strength and ductility of materials, structural element dimensions, interstory heigth, ...), which mechanical models are founded on, may affect the structural response. Results of sensitivity analyses may be also useful to select parameters where efforts on the knowledge phase has to be more effectively focused in order to increase the reliability of assessment. Particular attention is paid to the mechanical model adopted in Lagomarsino *et al.* (2010), named in the following as DBV-*concrete* (Displacement Based Vulnerability) method; it is useful to both the above-mentioned aims.

2. MECHANICAL MODELS BASED ON A DISPLACEMENT APPROACH

In the following, a general overview of procedure which mechanical models based on a displacement approach is referred to, is provided.

Traditionally, mechanical models based on a displacement approach refers to the direct displacement - based design method (Calvi, 1999) and does not strictly require for its application the outline of the capacity curve; however, all variables necessary to define it are implicitly introduced. Assuming a bilinear curve without hardening, three quantities basically need to be defined to fully describe the capacity curve. Usually, in models proposed in literature, the entities directly defined on mechanical basis are the period of vibration (T_{LSi}) and the displacement capacities (D_{LSi}), which may be characterized for different limit states (LS_i) and two global failure modes (either beam-sway or column-sway). Starting from these parameters, the ultimate strength of the capacity curve (A_y) is obtained through the intersection of the period and the displacement capacity at the yielding (which corresponds to LS_2). In the following, generally four limit states are introduced (from 1 to 4); for example, according to the definition of Eurocode 8, they could correspond to Damage Limitation (LS_2), Significant Damage (LS_3) and Near Collapse (LS_4) Limit States, respectively. In general, models consider three limit states, starting from LS_2; in order to introduce a further limit state LS_1 associated to the non-structural light damage condition (structure almost in elastic phase), the capacity curve could be modified through appropriate principles (e.g. by defining the elastic period T_{LS1} and relating it to a proper percentage of overall strength, as shown in Fig.2.1 through a dashed line). In the following, the attention is paid only on LS_2 and LS_4, focusing on the DBV-*concrete* model as proposed in Lagomarsino *et al.* (2010), which the sensitivity analyses discussed in §3 has been focused on.

Some specific aspects related to the evaluation of T_{LS2} and D_{Lsi} are discussed and summarized in Table 2.1 (in which for example some references are introduced).

Concerning the evalution of T_{LS2}, formula proposed in the literature may be ascribed to two main classes:
- a first one, basically related to the building height and some coefficient (C,β) defined as a function of different structural system (if bare, infilled frame or dual system, depending on systems designed or not to design capacity) ;
- a second one, aimed to also include the dependence on additional mechanical parameters which may influence the structural response.

The proposal of the expression of T_{LS2} in DBV-*concrete* belongs to the second group. In this case, an additional coefficient (ψ) is proposed in order to take into account the dependence of the period on some additional geometric and mechanical parameters, like: height section of column and beam (h_s

and h_{st}, respectively), inter-storey height (h_i) and the compressive strength of concrete (f_c). This coefficient derives from simple considerations on the parametric dependence of the period with these factors, as derived from modal analyses. This proposal basically refers to consider the values obtained from the application of formula of the first group as representative of the mean behaviour of a whole building stock; thus, starting from these values, they are modified if the structural parameters vary from the mean ones defined for the corresponding stock. When these latter parameters coincide with the mean ones ($\bar{h}_s, \bar{h}_{sT}, \bar{h}_i, \bar{f}_c$), the ψ coefficient is equal to 1. The introduction of ψ coefficient seems particularly useful to vulnerability assessment addressed to define a list of priority since is more capable to take into account – also for T_{LS2} evaluation - the specific characteristic of a building within a group.

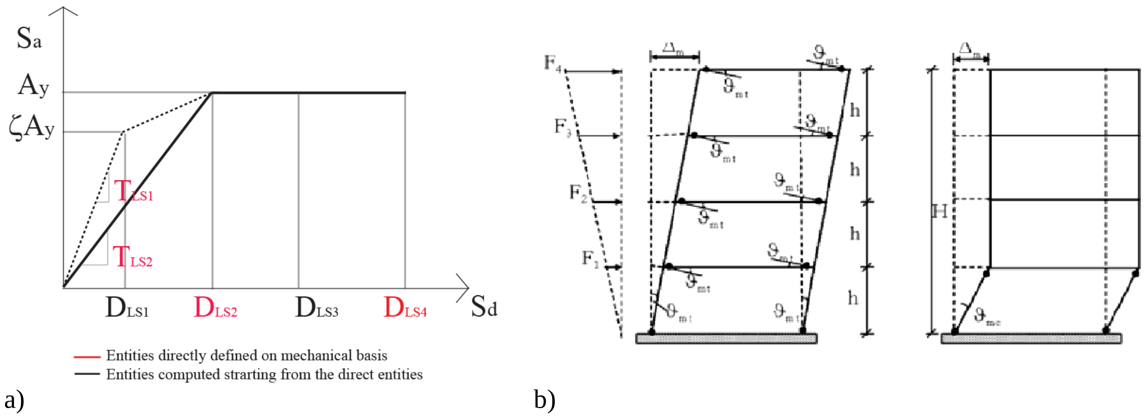

Figure 2.1. a) Capacity curve assumed for the reinforced concrete building class. b) Beam-sway (left) and column-sway (right) mechanism (from Paulay and Priestley 1992)

The evaluation of displacement capacities at the limit state 2 and 4 (D_{LS2} and D_{LS4}) is basically related to the chord rotation (θ) of the main structural element, column or beam, depending on the global failure mode; to compute θ at first, for D_{LS2}, only the yielding contribution is considered, then, progressing the response in non linear range, the plastic one is also added for D_{LS4}. The formulations of chord rotation are based on two main approaches: analytical and empirical. These approaches are discussed in §3.1, where some expressions proposed in literature have been examined.

The comparison between the “capacity” and the “demand” (the seismic input in terms of response spectrum), aimed to evaluate the seismic performance, requires the conversion of the “force-displacement curve” - representative of the actual multi degree of freedom structure - in the “capacity curve”, aimed to idealize the response of an equivalent single degree of freedom.
To this aim, mechanical models based on displacement approach mainly refer to the introduction of a κ_1 coefficient (like defined in Table 2.1). With respect to the original proposal of Priestley (1997), in DBV-*concrete* a slight modification to the expressions proposed for the definition of κ_1, particularly relevant in case of buildings characterized by few stories and with masses prevailing concentrated at floor level, has been introduced. In particular, it is proposed to multiply κ_1 for a factor $\kappa'(= (2N+1)/N)$ aimed to take into account the mismatch noticed in the position of the centre of seismic mass in the abovementioned cases. For example, in case of the inverted triangle load pattern, the centre of seismic forces is located at $0.67H_T$ (that is 2/3 of H_T, as proposed in Priestley 1997) only in case of a continuum system; whereas, in case of a building characterized by a single level, by concentrating all the seismic force at the top, the centre of seismic mass is located at H_T. .

In conclusion, the target displacement D_{PP}, or performance point, is evaluated by comparing the seismic demand, represented by elastic spectra properly reduced (by either overdamped or inelastic approach), to the capacity curve of the equivalent *SDOF*. Once evaluated the performance point D_{PP} and defined proper damage states on the capacity curve $D_{SL,k}$, it is possible proceed to the vulnerability assessment.

Table 2.1. The procedure of mechanical models based on a displacement approach (note: for the meaning of coefficients and parameters introduced see Table 2.2)

<table>
<tr>
<td rowspan="5">Definition of the capacity curve parameters</td>
<td rowspan="4">Entities directly defined on mechanical basis</td>
<td rowspan="2">T_{LS2}</td>
<td>f(H, structural system)</td>
<td>Linear or exponential functions like:
$$T_{LS2} = CH_T^{\beta}$$
References: Goel and Chopra (1997), Crowley and Pinho (2010)</td>
</tr>
<tr>
<td>f (H, structural system, mechanical or geometrical parameters)</td>
<td>Exponential functions like:
$$T_{LS2} = C' H_T^{\beta'}$$
Where: $\psi = \left(\frac{\overline{h_s}}{h_{si}}\right)^{0.5} \left(\frac{\overline{h_{sT}}}{h_{sTi}}\right)^{\beta_3} \left(\frac{h_i}{\overline{\overline{h_i}}}\right)^{0.75} \left(\frac{\overline{f_c}}{f_{ci}}\right)^{0.125}$
As proposed in Lagomarsino et al. 2010
Or
$$T_{LS2} = \alpha H_T^{\beta} S$$
As proposed in Verderame et al. (2010), where S is the plan area of building</td>
</tr>
<tr>
<td rowspan="2">D_{LSi}
i=2,4</td>
<td rowspan="2">f(H, failure mode, θ_{LSi})

Failure modes: beam or column sway.
θ_{Dsi} are computed on basis of analytical or empirical approches (for some specific references see Table 3.1)</td>
<td>LS_2
$$D_{LS2} = \kappa_1 \theta_{LS2} H_T$$</td>
</tr>
<tr>
<td>LS_4
Beam sway
$$D_{LS4} = \kappa_1 \theta_{LS2} H_T + (\theta_{LS4} - \theta_{LS2}) \kappa_1 H_T$$
Column sway
$$D_{LS4} = \kappa_1 \theta_{LS2} H_T + (\theta_{LS4} - \theta_{LS2}) h_1$$
References: Glaister and Pinho 2003, Crowley et al. (2004)</td>
</tr>
<tr>
<td>Derived entities</td>
<td>A_y</td>
<td colspan="2">$$A_y = D_{LS2} \left(\frac{2\pi}{T_{LS2}}\right)^2$$</td>
</tr>
<tr>
<td>Conversion in SDOF</td>
<td colspan="4">Through an effective height coefficient (κ_1) defined as the ratio between the height to the centre mass of the SDOF substitute structure and the total height of the original structure (H_T):
$\kappa_1 = f$ (failure mode, N, DS_i)
Moreover for $LS_{3,4}$ κ_1 also depends on the steel strain or the ductility. For example, in case of the column sway mode, as proposed in Priestley 1997 it results:
$$\kappa_1 = \begin{cases} 0.67 & LS_i, i = 1,2 \\ 0.67 - 0.17 \frac{\mu_{LSi} - 1}{\mu_{LSi}} & LS_i, i = 3,4 \end{cases}$$
Additional references: Glaister and Pinho 2003; Lagomarsino et al. 2010</td>
</tr>
</table>

Table 2.2 summarizes all the parameters that need to be defined in order to apply the mechanical model proposed in DBV-*concrete*. It is worth noting that, in addition, an a priori choice has to be made on the collapse mode hypothesized (i.e. either beam-sway or column sway type) and the structural type. The different colours (red and grey) assigned to the entities in Table 2.2, highlight the parameters necessary for the evaluation of chord rotation, as well as grey and red, respectively, are for

the empirical and analytical approaches.

Table 2.2. Building parameters for the mechanical model definition

Geometrical features of the member	N (storey number); H_T (total height); h_i (inter-storey height); h_1 (inter-story height at ground floor); L_t (beam length)
Geometrical features of the section	h_s (height section of the main structural element ruling the global response, that is the r.c. beam or the r.c. column); d_b (longitudinal bar diameter); A_s (column longitudinal reinforcement), A_{st} (beam tension longitudinal reinforcement), A'_{st} (beam compression longitudinal reinforcement), A_{sw} (transversal reinforcement), p (stirrup spacing), b_c and d_c (width and depth of the confined core of the section)
Mechanical parameters and loads	ε_{cu} (ultimate concrete strain); ε_y (yielding steel strain); ε_{su} (ultimate steel strain); f_y (yielding steel strength); f_c (concrete resistance) ; L_V (shear span); f_{yw} (yielding transverse steel strength; ν (axial load ratio).
Corrective factors	β_3 (equal to 0.25 in case of column sway mechanism and 0.5 in case of the beam sway one); C' and β'(are defined as function of the structural type, for example in case of moment resistant frames designed only for vertical load without significant seismic details stands for 0.089 and 1 as proposed in Crowley and Pinho 2006); γ_{el} (is equal to 1.5 for primary structural elements).
Note: For the evaluation of coefficient ψ, the reference mean values of $\overline{h}_s, \overline{h}_{sT}, \overline{h}_i, \overline{f}_c$ should be specified.	

3. SENSITIVITY ANALYSES

Among the different mechanical models proposed in literature, the DBV-*concrete* method adopted in Lagomarsino *et al.* (2010) has been assumed as reference for the sensitivity analyses discussed in the following. This model basically starts from the one originally proposed by Crowley *et al.* (2004 and 2008), with some modifications mainly related to the definition of the yielding period (by the introduction of ψ coefficient) and the SDOF (by the introduction of κ' coefficient).

The validation of the changes proposed in DBV-*concrete* has been carried out by the numerical simulation of the damage scenario occurred in L'Aquila (with particular reference to the data relating the Pettino village and its surrounding area). The simulation has been conducted for different classes (as a function of the age, storey number, structural type) characterized by homogeneous behaviour to which associate a proper mechanical model. Figure 3.1 shows the comparison between the simulated and real damage scenario as a function of the ages and storeys number of examined classes. Despite the need of some improvements of mechanical models adopted, the proposed methodology seems to provide a quite good and realistic assessment of the damage scenario occurred. In fact, from the application a percentage of not safe buildings equal to 27% against surveyed scenario equal to 35% have been obtained (for further detail see Lagomarsino *et al.* 2010).

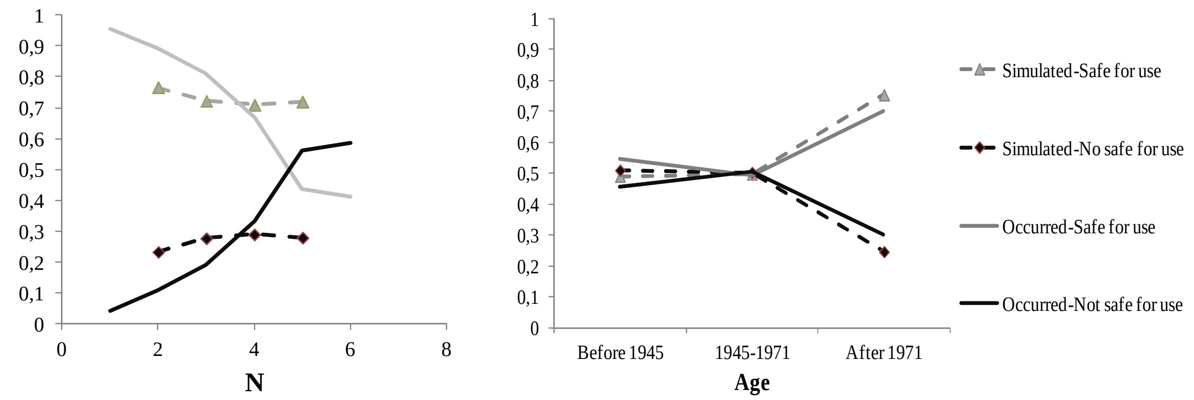

Figure 3.1. Comparison between trends of simulated and surveyed scenario varying both ages and N

In the following, the results of sensitivity analyses performed are discussed. In particular, first of all the sensitivity to the different expressions proposed in literature for the chord rotation is examined. Then, a more extensive sensitivity analysis to all the parameters which concur to the definition of A_y and displacement capacities for the different limit states is presented.

3.1. Sensivity to different expressions proposed in literature for the chord rotation

The evolution of deformation capacity of RC members increased interest in recent years, due to the widespread of displacement-based concepts for seismic design of new structure and seismic assessment of old ones. In the abovementioned mechanical models, this deformation capacity affect D_{LSi} (i=2,3,4) and A_y (since it is computed from the intersection of D_{LS2} and T_{LS2}).
The deformation capacity of beams, columns and walls is defined in terms of chord rotation (θ_{LS2} in case of D_{LS2} and θ_{LS4} in case of D_{LS4}, respectively). The formulations of chord rotation are based on two main approaches: analytical and empirical. The analytical approach presents the following main advantages: a) it represents a mechanical and physical model, and b) curvature (ϕ_y and ϕ_u) can be quantified in terms of section parameters and material properties on the basis of the plane-section hypothesis. On the contrary, the empirical approach is based on statistical analysis conducted on a specific sample of experimental data (which may differ for samples number, structural type, detail of elements – plain or deformed bars -, mechanical parameters etc…); as a consequence, the reliability of these expressions depends strongly on the sample, upon which were calibrated (50 or 1000 beams); so in some cases the generalization of these expressions could appear conventional. In any case, the empirical approach presents the advantage to take into account the functional dependence of some parameters, not present in the analytical approach. Many different formulations of the chord rotation are proposed in literature; Table 3.1 summarized some of the most noteworthy ones.

In case of θ_{LS2} the empirical approach provides a simplified formula based on: the curvature at yield (ϕ_y) as a function of the yield strain of steel (ε_y); the beam section height or column depth, for the case of beam and a column-sway mechanism, respectively, and the empirical coefficients that aim at introducing the effects of flexure and shear flexibility of joints and framing members. For the analytical approach, the formula of θ_{LS2} considers more parameters, that may be summarized in three main factors: the flexural contribution ($\theta_{LS2,flex}$), the shear deformation contribution ($\theta_{LS2,shear}$) and the anchorage slip of bars ($\theta_{LS2,slip}$).

In case of θ_{LS4} the expressions are based on both analytical and empirical approaches. In the analytical approach, the value of the total chord rotation capacity (elastic plus inelastic part) is based on a purely flexural behaviour through the concepts of plastic hinge and plastic hinge length, in which the entire inelasticity of the shear span is considered to be lumped and uniformly distributed. This approach depends on: the chord rotation at yield (θ_{LS2}), the ultimate curvature at the end section (ϕ_u); the yield curvature at the end section (ϕ_y) and the plastic hinge length (L_{pl}). The effects of shear, bond-slip, tension stiffening, etc., should be dealt with through L_{pl}. The empirical expressions are based on the same parameters of the analytical approach, with the addition of the following terms: the axial load ratio (ν); the mechanical ratio of the tension and compression longitudinal reinforcements (respectively ω and ω'); the yield strength of transverse steel (f_{yw}); the ratio of transverse steel parallel to the direction of loading (ρ_{sx}); the steel ratio of diagonal reinforcement (ρ_d); factors aimed to taken into account the effectiveness of confinement and constructive details (like as anchorage, slip and type of bars - α_l and δ_l respectively).

Once introduced the expressions of the chord rotation at yielding and ultimate, an extensive sensitivity analysis is carried out in order to define the more reliable ones to be adopted in mechanical models. Table 3.2 shows the values which have been assumed for the parameters required to define the chord rotation at yielding and the chord rotation capacity at the ultimate. The geometrical and mechanical data have been taken from the literatures and codes. It is worth pointing out that the corrective factors have been applied only for the sensitivity analyses discussed in the §3.2. Figure 3.2 shows the comparison between some expressions proposed in literature (as summarized in Table 3.1).

Table 3.1. Classification of some expressions proposed in literature for chord rotation

<table>
<tr><td rowspan="3">Damage State 2</td><td>Empirical</td><td colspan="12">MORE SIMPLIFIED APPROACHES
$$\theta_{LS2} = \alpha\varepsilon_Y \frac{H}{h}$$
Where α, H and h are function of the collapse mode (if beam-sway or column-sway). As proposed in Glaister e Pinho (2003): in case of beam-sway: α=0.5; H=L_T; h=h_{st}; in case of column-sway: α =0.43; H=h_1; h=h_s.</td></tr>
<tr><td rowspan="2">Analitycal</td><td colspan="12">APPROACHES ACCOUNT FOR FLEXURAL, SHEAR AND SLIP CONTRIBUTIONS
$$\theta_{LS2} = \theta_{LS2,FLEXURAL} + \theta_{LS2,SHEAR} + \theta_{LS2,SLIP}$$
Where:</td></tr>
<tr><td colspan="6">As proposed in NTC 2008 and EC8 (2005)
$$\begin{cases} \theta_{LS2,FLEXURAL} = \Phi_y \frac{L_v}{3} \\ \theta_{LS2,SHEAR} = 0.0013\left(1+1.5\frac{h_{s(t)}}{L_v}\right) \\ \theta_{LS2,SLIP} = 0.13\Phi_y \frac{d_b f_y}{\sqrt{f_c}} \end{cases}$$</td><td colspan="6">As proposed in Panagiotakos e Fardis (2001).
$$\begin{cases} \theta_{LS2,FLEXURAL} = \Phi_y \frac{L_v}{3} \\ \theta_{LS2,SHEAR} = 0.0025 \\ \theta_{LS2,SLIP} = a_{sl} \frac{0.25\varepsilon_y d_b f_y}{(d-d')\sqrt{f_c}} \end{cases}$$</td></tr>
<tr><td rowspan="10">Damage State 4</td><td rowspan="5">Analitycal</td><td colspan="12">$$\theta_{LS4} = \frac{1}{\gamma_{el}}\left(\theta_{LS2} + \theta_{pl} + \theta_{slip}\right)$$
Where:</td></tr>
<tr><td colspan="4">NTC 2008, EC 8 (2005),
Fib Bulletin N° 25 (2003)</td><td colspan="4">Fardis (2007)</td><td colspan="4">Glaister e Pinho (2003)</td></tr>
<tr><td colspan="4">$$\begin{cases} \theta_{pl} = (\phi_u - \phi_y)L_{pl}\left(1-\frac{0.5L_{pl}}{L_v}\right) \\ \theta_{slip} = 0 \end{cases}$$</td><td colspan="4">$$\begin{cases} \theta_{pl} = (\phi_u - \phi_y)L_{pl}\left(1-\frac{0.5L_{pl}}{L_v}\right) \\ \theta_{slip} = (\theta_{u,slip} - \theta_{y,slip}) \end{cases}$$</td><td colspan="4">$$\begin{cases} \theta_{pl} = (\phi_u - \phi_y)L_{pl} \\ \theta_{slip} = 0 \end{cases}$$</td></tr>
<tr><td colspan="12">Where the plastic hinge length, the ultimate and yielding curvature at the end section for the beam sway mechanism and column sway mechanism stand respectively for:</td></tr>
<tr><td colspan="3">$L_{pl} = 0.1L_v + 0{,}17h + 0.24\frac{d_{bl} f_y}{\sqrt{f_c}}$</td><td colspan="3">$\Phi_u = \frac{\varepsilon_{cu} - \varepsilon_{su}}{h_{s(t)}}$</td><td colspan="3">$\Phi_y = 1.7\frac{\varepsilon_y}{h_{st}}$</td><td colspan="3">$\Phi_y = 2.4\frac{\varepsilon_y}{h_s}$</td></tr>
<tr><td rowspan="5">Empirical</td><td colspan="12">$$\theta_{LS4} = f(empir.coefficient;\nu;w,w';f_c;L_v;h;\alpha;\rho_{sx};f_{yw};\rho_d)$$</td></tr>
<tr><td colspan="9">$\theta_{LS4} = \frac{1}{\gamma_{el}}0.016\left(0.3^{\nu}\right)\left[\frac{\max(0.01;w')}{\max(0.01;w)}f_c\right]^{0.225}\left(\frac{L_v}{h}\right)^{0.35}25^{\left(\alpha\rho_{sx}\frac{f_{yw}}{f_c}\right)}\left(1.25^{100\rho_d}\right)$</td><td colspan="3">NTC 2008
EC8 (2005)</td></tr>
<tr><td colspan="9">$\theta_{LS4} = \alpha_{st}\left(1-0.38\alpha_{cyc}\right)\left(1+\frac{\alpha_{sl}}{1.7}\right)\left(10.375\alpha_{wall}\right)\left(0.3^{\nu}\right)\left[\frac{\max(0.01;w')}{\max(0.01;w)}f_c\right]^{0.2}\left(\frac{L_v}{h}\right)^{0.425}25^{\left(\alpha\rho_{sx}\frac{f_{yw}}{f_c}\right)}\left(1.45^{100\rho_d}\right)$</td><td colspan="3">Fib Bulletin N° 25 (2003)</td></tr>
<tr><td colspan="9">$\theta_{LS4} = \alpha_{st}\alpha_{cyc}\left(1+\frac{\alpha_{sl}}{2.3}\right)\left(1-\frac{\alpha_{wall}}{3}\right)\left(0.2^{\nu}\right)\left[\frac{\max(0.01;w')}{\max(0.01;w)}f_c\right]^{0.275}\left(\frac{L_v}{h}\right)^{0.45}1.1^{\left(\alpha\rho_{sx}\frac{f_{yw}}{f_c}\right)}\left(1.3^{100\rho_d}\right)$</td><td colspan="3">Panagiotakos and Fardis (2001)</td></tr>
<tr><td colspan="9">$\theta_{LS4} = \alpha_{st}\left(1-0.43\alpha_{cyc}\right)\left(1+\frac{\alpha_{sl}}{2}\right)\left(1-\frac{3\alpha_{wall}}{8}\right)\left(0.3^{\nu}\right)\left[\frac{\max(0.01;w')}{\max(0.01;w)}f_c\right]^{0.225}\left(\frac{L_v}{h}\right)^{0.35}25^{\left(\alpha\rho_{sx}\frac{f_{yw}}{f_c}\right)}\left(1.275^{100\rho_d}\right)$</td><td colspan="3">Fardis (2007)</td></tr>
<tr><td colspan="14">It is worth noting that more expressions have been proposed in literature, like that present in OPCM 3274 and Fib Bulletin N° 24. These formula have been also adopted in the sensitivity analysis.</td></tr>
</table>

From Figure 3.2, it may be stated as follows. The equations for the computation of θ_{LS4} show in general a greater scatter than the equations for θ_{LS2}; in the case of θ_{LS4}, the relationships of analytical approach are more susceptible to variations both of f_y and h_{st}, than the empirical approach.The empirical approach takes into account more parameter, like ν, $A_{s(t)}$, A_{sw},… than the analytical, but except ν and $A^{(')}_{s(t)}$ the variation of these features do not affect significantly the chord rotation at ultimate; for example, the figure shows the chart of ν, where there are not analytical expressions since values are constant. By normalizing the values, obtained by the different equations, at the value of EC8, both analytical and empirical approaches bring out the fact that, on average, scatters are: 70% in the case of ν, 35% for θ_{LS2}, 44% (for the analytical one) and 60% (for the empirical one) for θ_{LS4}, in the case of F_y, 42% for θ_{LS2}, 23% (for the analytical one) and 65% (for the empirical one) for θ_{LS4} , in the case of H_{st}, 35% for θ_{LS2} in the case of F_c. In case of θ_{LS4}, Borzi's expression provides higher estimations than other ones.

Table 3.2. The values assigned to the parameters in the sensitivity analysis

Geometrical features of the member	N (1-15; mean value 4); h_i (2 – 5 m, mean value is 3 m); h_1(2 – 5 m, mean value is 3.40 m); L_t (3 – 10 m, mean value is 4.20 m).
Geometrical features of the section	h_s (0,15 – 1 m, mean value for the column is 0.33 m; mean values for the beam are different between the column sway and the beam sway, respectively stands for 0.6 m and 0.3 m); d_b (6 – 30 mm, mean value 16 mm); $A_{s(t)}$ (4 – 55 cm^2, mean value 10 cm^2), $A'_{s(t)}$ (3 – 55 cm^2, mean value 10 cm^2), A_{sw} (0.6 – 5 cm^2, mean value 1 cm^2), p (5 – 30 cm, mean value 20 cm)
Mechanical parameters and loads	ε_{cu} (0.005 – 0.01, mean value is 0.075); ε_{su} (0.02 – 0.05, mean value is 0.025); f_y (150 – 600 MPa, mean value is 235 MPa); f_c (5 – 45 MPa, mean value is 11 MPa); f_{yw} (150 – 600 MPa, mean value is 235 MPa); ν (0 - 1).
Corrective factors	β_3 (equal to 0.25 in case of column sway mechanism and 0.5 in case of the beam sway one); C' and β'(0.089 and 0.9 in case of frames designed "post 1971" and 0.089 and 1 in case of frames designed "before 1971"); γ_{el} (is equal to 1.5).
Note:	Beam-sway mechanism $\overline{h}_s = 0.33m, \overline{h}_{sT} = 0.3m, h_i = 3m, \overline{f}_c = 20MPa$ Column-sway mechanism $\overline{h}_s = 0.33m, \overline{h}_{sT} = 0.6m, \overline{h_i} = 3m, \overline{f}_c = 20MPa$

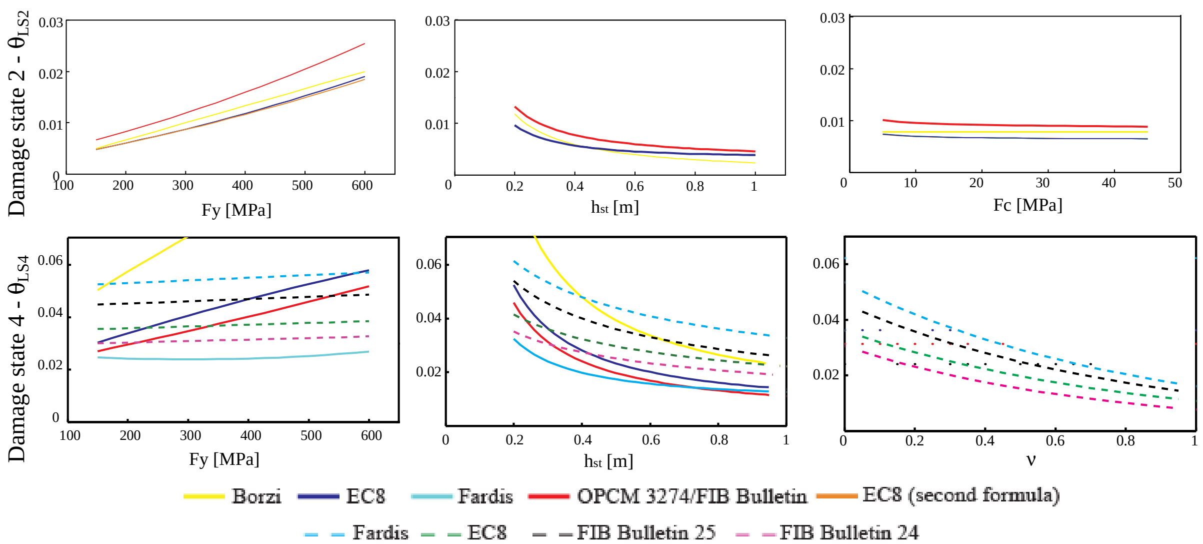

Figure 3.2. Variation of chord rotation at yielding and ultimate for some parameters

3.2. Sensitivity to the geometrical and mechanical parameters which models are based on

Once defined the effective period of vibration and the corresponding displacement capacities at different limit state, the ultimate strength of the capacity curve is obtained with the formula proposed in literature for the chord rotation (Table 3.1). In the following, the sensitivity of seismic response to the parameters that describe the mechanical model presented in §2 is discussed. Since three factors, defining the capacity curve (the initial period, the ultimate strength and the ultimate displacement capacity), affect also the seismic verification, it seems useful to discuss the results through a synthetic parameter, that may describe the combined effects, on the seismic response, of every data which models are based on. To this end, the sensitivity analysis has been conducted in terms of peak ground acceleration (PGA) consistent either with the limit state 2 (PGA_{maxLS2}) or 4 (PGA_{maxLS4}); these values have been obtained by adopting inelastic spectra (according to the N2 method proposed in Fajfar 2000 and adopted in Eurocode 8) and by imposing the target displacement (D_{PP}) as equal to D_{LS2} and D_{LS4}, respectively. It is worth noting that an elastic spectrum consistent with type 1 – ground type A as proposed in Eurocode 8 (corner period T_c equal to 0.4) was adopted. Figure 3.3 shows the results obtained relative, for example, to N e F_y and h_{st}. Results are represented in terms of box plot normalized at the values of EC8 equation (analytical approach); the dashed line represents the mean value provided by this equation. In particular, it may be stressed as follows. The PGA_{maxLS2} shows a greater scatter due to the variations of F_y and less for N while results obtained by different expressions compared to the values of EC8 appear acceptable.Regarding Limit State 4, the empirical approach tends to provide conservative results. In addition, in most cases the evaluations achieved by the analytical approach are less scatter than the empirical relation. This is due to the influence of the different sample on which these formula have been calibrated. In the case of N, the values of PGA_{maxLS4} obtained by the empirical approach take into account the influence of ν, so it explains the extents of the box plot.

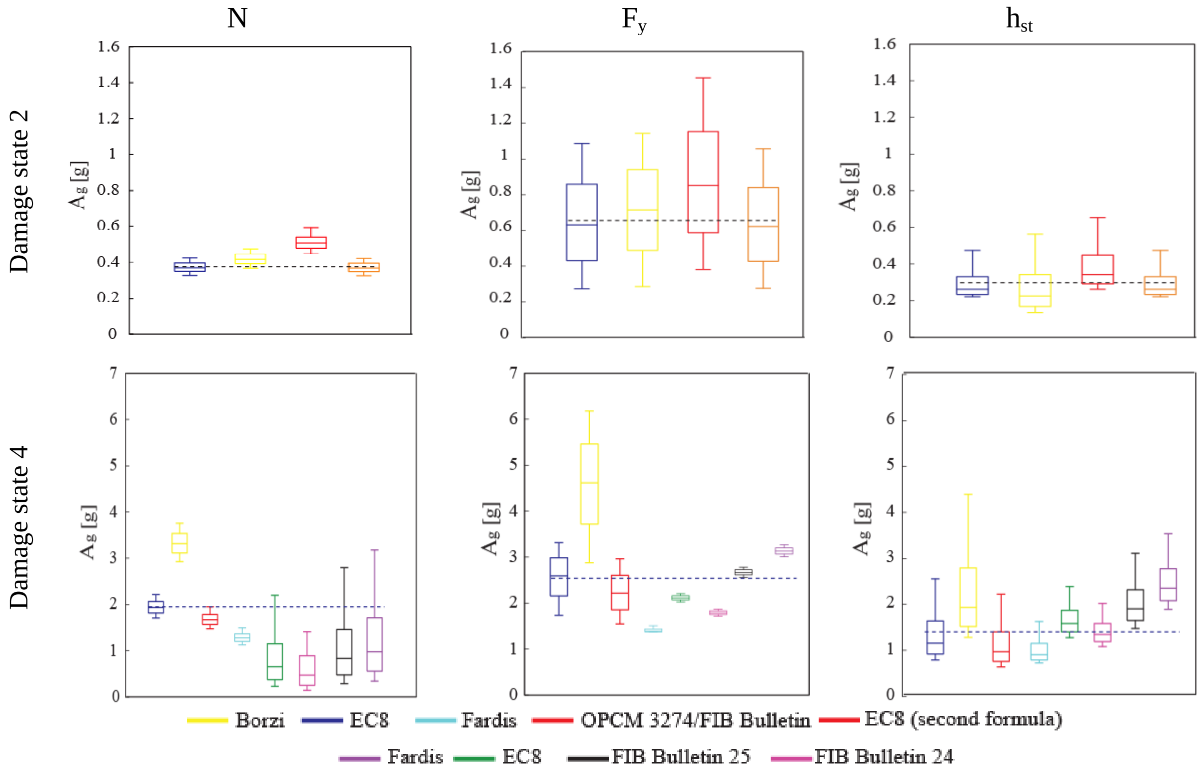

Figure 3.3. Variation of PGA_{maxDS2} and PGA_{maxDS4} to N, F_y and h_{st}

4. FINAL REMARKS

The sensitivity analysis performed and discussed in §3 allowed: on the one hand, to evaluate the scatter of the results obtained using the different formula proposed in literature (for chord rotation), on the other hand, to identify the more powerful parameters, which the mechanical model is founded on, may affect the structural response. These results may orient to a more reliable and precautionary assessment. In particular, the analyses have been developed with reference to the DBV-*concrete* model (proposed in Lagomarsino *et al*. 2010): indeed, it provided a quite good and realistic assessment of the damage scenario occurred in L'Aquila (in particular, Pettino village and its surrounding area).

According to the results of sensitivity analysis some slight improvements could be applied to the model, with regard to the following topics: the choice of the approach (analytical and empirical) to adopt for the evaluation of chord rotation and the estimation of the period at yielding.

Concerning the first issue, the empirical approach seems to be favored, since it leads to precautionary results and takes into account the influence of some parameters, that appear to be important for the vulnerability assessment, as the axial load acting on columns and the quantity of longitudinal and transverse reinforcement. At the same time, the empirical approach requires more parameters and consequently, in principle, more diagnostic techniques and survey, than the analytical one. However, despite this, from the results of the sensitivity analysis, the seismic response seems to be not much affected by the variation of some parameters – like as the amount of reinforcement- so it is possible reduce this encreasing effort on the knowledge phase. Concerning the evaluation of period at yielding, some additional parameters could be inserted in the ψ coefficient. As stressed, it is particularly useful to take into account the dependence of the period on some mechanical parameters which may influence the structural response. In particular, it seems reasonable to introduce other factors, like the shear span of the beams (that now it is included only in the chord rotation capacity evaluation).

Finally, a general issue on mechanical models based on the displacement-based approach concerns the possibility to include the effect also of brittle shear failures; this could be particularly significant in case of vulnerability assessment on existing building.

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