# A Simplified Pseudo-Dynamic Method of Reinforced Retaining-Wall Subjected to Seismic Loads

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#### SUMMARY:

Reinforced soil wall designed and constructed based on numerous methods for the stability of retaining walls, slops, and embankments have been performed. This paper describes a new simplified pseudo-dynamic method for evaluating the behaviour of reinforced retaining wall under horizontal and vertical seismic accelerations. Stability analyses are conducted to be obtained the required strength of reinforcements, the critical inclination of the failure angle and safety factor of reinforced soil wall. Parametric studies quantify the effect of different factors such as angle of internal friction and magnitude of seismic accelerations on the resistance mobilized in reinforcements and the factor of safety due to axial pullout of reinforcement layers of a reinforced soil wall. Comparison of the present results with previously studies shows a very close agreement.

Keywords: Retaining wall, Pseudo dynamic, Seismic stability, Reinforcement, Safety factor

# **1. INTRODUCTION**

During an earthquake, significant damage can result due to instability of the soil in the area affected by internal seismic waves. In recent years, the research on seismic stability of unreinforced and reinforced soil structures by limit equilibrium method has popularity gained due to their inherent advantage over the conventional retaining walls in performance (Nimbalkar et al., 2006; Narasimha Reddy et al., 2008; Nouri et al., 2008; Basha and Babu, 2009; Choudhury and Ahmad, 2009; Narasimha Reddy et al., 2009; Huang and Lou, 2010). Due to pervasive developments, technical and economical advantage of soil reinforcement, the use of reinforced soil walls and slopes is growing (e.g., Nouri et al., 2008; Shahgholi et al., 2001). Nimbalkar et al. (2006) evaluated the effects of the magnitude of horizontal amplification of pseudo-dynamic approach on reinforced soil slopes and walls. Choudhury et al., (2007) reported the effects of horizontal and vertical seismic accelerations on the external stability of reinforced soil walls using the pseudo-dynamic method.

In this paper a new closed-form approach, fairly simple, of modified limit equilibrium of the sliding soil wedge of the reinforced backfill based on pseudo-dynamic analysis is described. A comprehensive parametric study to investigate the effect of the relative importance of design parameters on the required strength of reinforcement layers and safety factor due to axial pullout of reinforcement layers were conducted.

## 2. METHODOLOGY

Figure 1 shows a reinforced soil wall of height, H, with reinforcement layers of length,  $L_r$ , in a dry cohesionless free-draining backfill with angle of friction,  $\varphi$  and with unit weight,  $\gamma$ . The backfill is reinforced with "n" layers of planar reinforcement. The spacing between the reinforcement layers is  $S_v=H/n$ , expect for the top and bottom layers of reinforcement which have spacing of  $S_v/2$ .



Figure 1. Sketch of the reinforced soil-wall system.

The failure zone is defined by a planar rupture surface AB (dashed line in Figure 2), inclined at an angle of  $\alpha_{cri}$  with the horizontal (Kramer, 1996). The value of  $\alpha_{cri}$  depends on the angle of shear strength of the soil,  $\varphi$ , and seismic horizontal,  $k_h$ , and vertical,  $k_v$ , coefficients (Kramer, 1996). Under earthquake condition, the shear wave velocity  $V_s = (G/\rho)^{1/2}$  and primary wave velocity  $V_p = [G(2-2\upsilon)/\rho (1-2 \upsilon)]^{1/2}$  are assumed to act within the reinforced soil structure. For most geological materials,  $V_p/V_s=1.87$  is considered. The period of lateral shaking,  $T=2\pi/\omega=4H/V_s$  is considered in the analysis.

## 2.1. Tensile force generated in the reinforcement

The Free body diagram of the failure wedge and its acting forces of the reinforced soil-wall system which considered in the analysis, are schematically shown in Figure. 2.



Figure 2. Forces acting on the wedge failed of reinforced soil-wall system.

In this figure S and N are, respectively, the shear (tangential) and normal forces acting on the failure plan of reinforced soil.  $\sum_{i=1}^{n} T_i$  is the sum of the forces needed to maintain the stability of the reinforced retaining wall, Ti is the tension force generated in the i<sup>th</sup> reinforcement layer located at the soil failure wedge horizontally and n is the number of reinforcements. For the whole failure wedge, using the dynamic equilibrium condition in X and Y direction subjected to the horizontal and vertical simultaneously, the sum of the values of T<sub>i</sub> obtains by the following expression:

$$\sum T_i = (W + Q_{v \to w}) \tan(\phi - \alpha) + Q_{h \to w}$$
<sup>(1)</sup>

W is weight of the soil failure wedge,  $Q_{h-w}$  and  $Q_{v-w}$  are respectively the total horizontal and vertical inertia force acting on the reinforced soil wall. After simple calculation, Eqn. 1 becomes:

$$K = (\Gamma + \frac{Q_{\nu - w}}{0.5\gamma H^2}) \tan(\phi - \alpha) + \frac{Q_{h - w}}{0.5\gamma H^2}$$
(2)

Where  $\Gamma = 2W/\gamma H^2$ ,  $K = 2\Sigma T_i / \gamma H^2$  are dimensionless quantities.

## 2.2. Pseudo-dynamic inertia forces

If the base is subjected to harmonic horizontal and vertical seismic accelerations of amplitudes  $a_h$  and  $a_v$ , the accelerations at depth z below the top of the wall can be expressed as

$$a_h(z,t) = a_h \sin \omega (t - \frac{H - z}{V_s})$$
(3)

$$a_{v}(z,t) = a_{v}\sin\omega(t - \frac{H-z}{V_{p}})$$
(4)

For a thin elemental slice of thickness dz at depth of z, the mass of the elemental slice is given by

$$m_z = \frac{\gamma}{g} \left(\frac{H-z}{\tan\alpha}\right) dz \tag{5}$$

The total vertical inertia force Q<sub>v-w</sub> acting on the reinforced soil wall can be expressed as

$$Q_{h-w} = \int_0^H m(z) a_h(z,t) = \frac{\lambda \gamma k_h}{4\pi^2 \tan \alpha} \left( 2\pi H \cos \omega \zeta_1 + \lambda (\sin \omega \zeta_1 - \sin \omega t) \right)$$
(6)

The total vertical inertia force Q<sub>v-w</sub> acting on the reinforced soil wall can be expressed as

$$Q_{\nu-\nu} = \int_0^H m(z) a_{\nu}(z,t) = \frac{\eta \gamma k_{\nu}}{4\pi^2 \tan \alpha} \left( 2\pi H \cos \omega \zeta_2 + \lambda (\sin \omega \zeta_2 - \sin \omega t) \right)$$
(7)

Where  $\lambda$ =TV<sub>s</sub> and  $\eta$ =TV<sub>p</sub> are respectively the wavelength of the vertically propagating shear wave and primary wave.  $\zeta_1$  and  $\zeta_2$  define as  $\zeta_1$ =t-H/V<sub>s</sub> and  $\zeta_2$ =t-H/V<sub>p</sub>.

## 2.3. Factor of Safety, FS

Calculating of safety factor is carried out assuming full mobilization of shear resistance along the reinforcement-soil interfaces considering the parameters defined in Figure 1. The shear resistance is considered only due to axial pullout of reinforcement. Hence, the tension mobilized in the reinforcement layers over the effective length of reinforcement in the stable soil mass,  $\sum_{i=1}^{n} t_i$  obtains by the following expression:

$$\sum_{i=1}^{n} t_{i} = \sum_{i=1}^{n} 2(1+k_{v}) \gamma h_{i} L_{i} \tan \phi_{r}$$
(8)

Eqn. 8 includes the soil mass over the effective length of reinforcement in the stable soil mass which provides the total tension resistance force in the reinforcement layers. Where  $h_i = (i-0.5) S_v$  and  $L_i = L_i (H-h_i)$  cotgacri are, respectively, depth of embedment and effective length of i<sup>th</sup> layer of reinforcement beyond critical failure plane.  $t_i$  is the tensile force mobilized due to bond resistance in the i<sup>th</sup> layer of reinforcement, n is the number of reinforcement layers and  $\varphi_r$  is the angle of interface friction between the soil and reinforcement ( $\varphi_r = 0.67\varphi$ ). The conventional safety factor, FS is the ratio of the total mobilized bond resistance,  $\sum_{i=1}^{n} t_i$  to the maximum tensile forces generated in the reinforcement

layers,  $\sum_{i=1}^{n} T_i$  which can be written as:

$$FS = \sum_{i=1}^{n} t_i / \sum_{i=1}^{n} T_i$$
(9)

## **3. RESULT**

A computer program has been developed for all the formulation presented. It is used to evaluate the value of  $L_c/H$  (which its variation not reported in this paper) and the maximum required geosynthetic forces in terms of K=K<sub>max</sub>. Also the factor of safety, FS is evaluated considering the length of reinforcement, Lr and the number of reinforcement layers, n. The analysis considered for soil wall of height, H=5 m (Figure 1) and unit weight of  $\gamma$ =18 kN/m<sup>3</sup>, and for four different coefficients of horizontal seismic acceleration, k<sub>h</sub> (=0, 0.1, 0.2, 0.3) and five different coefficients of vertical seismic acceleration in terms of k<sub>v</sub>/k<sub>h</sub> (= -1, -0.5, 0, 0.5, 1). The soil friction angle; $\phi$ , varies from 25° to 45°. The shear wave velocity, Vs, the primary wave velocity, V<sub>p</sub> and period of lateral shaking are considered 100m/s, 187m/s and 0.3 sec, respectively. The non-dimensional parameters used in the analysis are H/ $\lambda$ =0.167 and H/ $\eta$ =0.09. Obviously, the presentation of all the result figures would have made the paper lengthy, so only a selection to illustrate the observed trends in terms of K<sub>max</sub> and FS is presented.

## 3.1. Verification of present approach

The maximum total tensile forces,  $(\Sigma t_j)_{max}$  generated in the reinforcement layers has been compared with those obtained using Reslop (Leshchinsky, 1997) and HSM procedures (Shahgholi et al, 2001) in Table 1 for different values of  $\varphi$  and  $k_h$  ( $k_v$ =0). ( $\Sigma t_j$ )<sub>max</sub> values of the present approach show a satisfactory agreement with those obtained by Leshchinsky (1997) and Shahgholi et al. (2001) as the maximum difference in estimating the values of ( $\Sigma t_j$ )<sub>max</sub> is insignificant (less than 8%). Hence, the results obtained from the present study could be pioneer to use for estimating the parameters such as  $K_{max}$ ,  $L_c/H$  and the factor of safety, FS for pseudo dynamic analysis of the vertical reinforced retaining wall subjected to accelerations.

|                | $\varphi = 20 \text{ degree}$ |                         |               |
|----------------|-------------------------------|-------------------------|---------------|
| k <sub>h</sub> | Leshchinsky, (1997)           | Shahggoli et al. (2001) | Present study |
| 0.0            | 110                           | 110                     | 110           |
| 0.1            | 128                           | 137                     | 127           |
| 0.2            | 151                           | 164                     | 150           |
| 0.3            | 187                           | 196                     | 183           |
|                | $\varphi = 25 \text{ degree}$ |                         |               |
| 0.0            | 95                            | 91                      | 91            |
| 0.1            | 110                           | 113                     | 107           |
| 0.2            | 126                           | 135                     | 126           |
| 0.3            | 153                           | 160                     | 150           |
|                | $\varphi = 30 \text{ degree}$ |                         |               |
| 0.0            | 74                            | 75                      | 75            |
| 0.1            | 90                            | 93                      | 89            |
| 0.2            | 106                           | 105                     | 105           |
| 0.3            | 128                           | 130                     | 126           |

**Table 1.** Comparison of  $(\Sigma t_j)_{max}$  calculated by Shahggoli et al. (2001), Leshchinsky (1997) and present study in the case of horizontal seismic acceleration ( $k_v$ =0).

#### 3.2. The influence of the internal angle of friction of backfill

Figure 3 shows the variation of  $K_{max}$  with soil internal angle of friction,  $\phi$  for different values of

horizontal seismic coefficient. Figure 3 shows that with an increase in the soil internal angle of friction,  $\varphi$ ; the values of K<sub>max</sub> considerably decreases, irrespective of the values of k<sub>h</sub>. For example, for a typical value of k<sub>h</sub>=0.2, the values of K<sub>max</sub> is 0.55 when  $\varphi$ =25°, but its value reduces to about 0.32 when  $\varphi$ =40°. It is apparent with increase in the value of k<sub>h</sub> the variation of K<sub>max</sub> with internal angle of friction,  $\varphi$  changes from linear to nonlinear. It is clear that the value of K<sub>max</sub> increases remarkably for low values of soil shear strength ( $\varphi$ <30°) and for high values of seismic acceleration (k<sub>h</sub>≥0.2) which confirms the results obtained by Nouri et al. (2008).



**Figure 3.** Variation of  $K_{max}$  with soil internal angle of friction.

The variation of FS with soil internal angle of friction,  $\varphi$  for different horizontal seismic coefficient,  $k_h$  and for n=5,  $L_r/H=0.8$  is shown in Figure 4. This figure depicts that the value of FS increases nonlinearly with an increase in angle of friction,  $\varphi$ , owing to the increase in bond resistance due to mobilization of friction resistance. For example, for  $k_h=0.2$ , the value of FS increases from 2.54 to 6.10 (around 140% increase in safety factor) with increase in  $\varphi$  from 25 to 35.



Figure 4. Variation of FS with soil internal angle of friction.

## 3.3. The influence of the horizontal and vertical seismic acceleration

Figure 5 shows the variation of  $K_{max}$  with horizontal seismic coefficient,  $k_h$  for five different values of  $k_h/k_v$  and  $\phi=30^\circ$ . Positive vertical seismic coefficient  $k_v$  was assumed to act downwards at the center of gravity of the sliding wedge. Figure 5 reveals the value of  $K_{max}$  increases with increase in  $k_h$ , irrespective of the value of  $k_h/k_v$ . It can be deduced that the effect of the vertical seismic coefficient,  $k_v$ 

 $(k_v/k_h)$  on the value of  $K_{max}$  is more remarkable for high value of horizontal seismic coefficient  $(k_h>0.1)$  and poor quality backfill ( $\varphi<30^\circ$ ).



Figure 5. Variation of  $K_{max}$  with  $k_h$  for different values of  $k_v/k_h$ .

In addition Figure 5 illustrates that the vertical component of seismic coefficient acting downwards (positive values of  $k_v$ ) tends to increase the maximum sum of the force needed to maintain the stability of the reinforced retaining wall,  $K_{max}$ . A similar finding of the influential role of the value of the horizontal seismic coefficient,  $k_h$  and vertical seismic acceleration,  $k_v$  (and its direction) on the value of  $K_{max}$  has also been reported by Nouri et al. (2008).

For the reinforced wall with  $\varphi=30^{\circ}$ , n=5 and L<sub>r</sub>/H=0.8 the factor of safety decreases significantly with increase in k<sub>h</sub>, irrespective of the value of k<sub>v</sub>/k<sub>h</sub> as can be seen in Figure 6. For example in the case of k<sub>v</sub>/k<sub>h</sub>=0, the value of FS decreases from 5.29 to 2.87 for k<sub>h</sub> increasing from 0.1 to 0.3. The influence of k<sub>v</sub> (k<sub>v</sub>/k<sub>h</sub>) on the FS results is insignificant for the low value of horizontal seismic coefficient, (k<sub>h</sub>≤0.1) while there are more severe for high values of the horizontal seismic coefficient, k<sub>h</sub> (k<sub>h</sub>≥0.2). For example, in the case of k<sub>h</sub>=0.1 and 0.3, respectively, the value of FS varies from 5.05 to 5.33 (6.62 % increase) and from 1.69 to 3.67 (186 % increase) for k<sub>v</sub>/k<sub>h</sub> increasing from -1 to +1. In addition, this figure shows that the vertical component of seismic acceleration acting upwards (negative values of k<sub>v</sub>) tends to decrease the safety factor, FS and the controls the value of safety factor.



**Figure 6.** Variation of FS with  $k_h$  for different values of  $k_v/k_h$ .

## 3.4. The influence of the number and the length of reinforcement layers on factor of safety

Figure 7 and Figure 8 depict the value of FS considerably increases linearly with increase in number of reinforcement layers, *n* and length of reinforcement layers,  $L_r/H$  for different values of  $k_h$  for  $\varphi=30^\circ$ . It is due to increase in bond resistance between the soil and reinforcement with increase in the values of *n* and  $L_r/H$ . For an illustration from Figure 7 the value of *FS* increases about 140% with increase in number of reinforcement layers from 3 to 7 for  $k_h = 0.2$ .



Figure 7. Variation of FS with n for different values of k<sub>h</sub>.



Figure 8. Variation of FS with L<sub>r</sub>/H for different values of k<sub>h</sub>.

## 4. CONCLUSION

Based on the results obtained from the present study, the following conclusions can be drawn:

- (1) The values of  $K_{max}$  considerably decrease with increase in the soil internal angle of friction,  $\varphi$  and increase with increase in the horizontal seismic acceleration coefficient,  $k_h$ . Also safety factor increases significantly due to increase in soil shear strength,  $\varphi$ , particularly, for low values of the coefficient of horizontal seismic acceleration ( $k_h \leq 0.1$ ).
- (2) Factor of safety considerably increases with increase in the number and length of the reinforcement layers in the backfill due to increase in bond resistance between the soil and

reinforcement.

(3) Comparisons of the results of the present formulation, for the reinforced backfill without surcharge, with those obtained by Shahgholi et al. (2001) and Leshchinsky (1997) are in a good agreement

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