Lumped-Mass Stick Modeling of Building Structures with Mixed Wall-Column Components

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SUMMARY:
This paper presents a new method to develop stick models for building structures having mixed shear walls and columns in two major steps. First, a macro beam member is used to represent a storey, and its sectional properties, including the location of centroid, sectional area, and area moments of inertia are determined using the Euler–Bernoulli beam theory. Second, the translational stiffnesses of each storey are determined based on the rigid floor assumption so that the centre of rigidity, torsional constant, and shear coefficients are found for the macro beam. After deriving the formulas, example analysis is conducted to compare the frequency response of the stick model with that from finite element analysis to validate the method. Results indicate that this type of stick models can consider the effects of the shear-wall out-of-plane stiffness and the eccentricity between the centroid and centre of rigidity of the macro beam. The proposed method can capture the dynamic feature to obtain adequate structural response and can be applied in the seismic analysis of mixed wall-column building structures.

Keywords: Stick model, macro member, seismic analysis

1. INSTRUCTION

Seismic analysis of simple structures using lumped-mass stick models is very cost efficient and can provide direct insights to the overall seismic behaviour of the structure being analyzed. In the past several decades, lumped-mass stick modelling of structures and equipment has been widely used in the seismic qualification of Systems, Structures, and Components (SSCs) of Nuclear Power Plants (NPPs). When developed and applied appropriately, lumped-mass stick models can produce accurate results in a much more economical way by comparison to a full-scale detailed finite element seismic analysis. In the past, when developing a stick model for a building structure, the shear shape factors are often calculated as the ratios of the total area over the effective area taking shear forces in a specific direction. While this simplification may be acceptable for a structure with mostly shear walls, it may not be suitable for a more complex structure consisting of columns and shear walls.

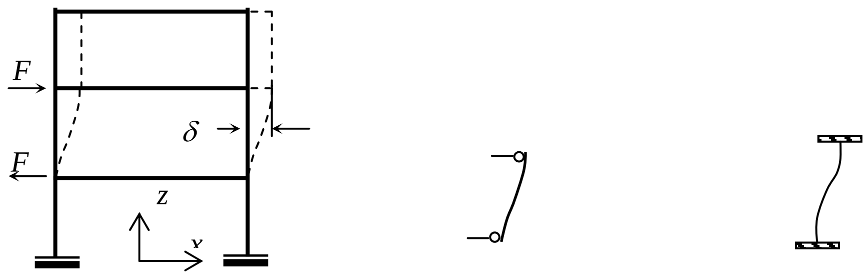
Lumped-mass stick models are simple and efficient to be applied in practice for predicting acceleration response of structures. The basic conception of using a single member to represent a storey of a column-framed building can be found in the literature (Chopra 1995). In nuclear industry, shell and shear-wall structures are often analyzed based on the beam theory that accounts for torsion effect (ASCE 1998). However, it is not completely clear how a lump-mass stick model can be developed for structures with mixed walls and columns. This study proposes a method to deal with such a problem based on the assumption of rigid-floor diaphragms. Each vertical stick member is modeled by a beam associated with shear deformation. Basic formulas are derived for determining the cross-sectional properties, and example analysis illustrates the accuracy of the proposed method.

2. STICK MODELING BASED ON STOREY STIFFNESS

It has been recognized that the stiffness of floor systems in column-framed buildings is considerably higher than the storey stiffness due to columns. Thus, each floor diaphragm composed of slabs and beams is able to be assumed rigid so that the diaphragm is constrained to move as a single unit in the horizontal plane. For column-framed structures, using rigid-floor body assumption one can derive analytical formulas to calculate the storey stiffness. For structures with shear walls, although the in-plane deformation of a diaphragm can be assumed negligible, its out-of-plane stiffness is generally not rigid. This section provides a discussion on how the in-plane and out-of-plane rigid assumption is applied to generate stick models.

2.1. Symmetrical Building Frameworks

In establishing a lumped-mass stick model, the assumption for a floor diaphragm plays a key role in determining the stiffness properties. For an actual frame in Figure 1(a), if the deformation of a floor system is taken into account, no simplified method can be used to determine the stiffness properties for dynamic analysis by hand computation. Even the assumption of in-plane rigid diaphragm is applied as shown in Figure 1(b), it is still hard to determine the member stiffness in a storey based on analytical formulas because of the effects of its upper and low stories.



(a) Actual frame (b) In-plane rigid (c) Rigid-floor body

Figure 2.1. Assumptions of floor diaphragms

When a rigid-floor body is assumed (i.e., both in-plane and out-of-plane floor stiffnesses are assumed rigid), a wall or column member in a storey can be assumed fixed at its two ends as shown in Figure 2.1(c). In this case if a pair of force F in Figure 2.1 (a) is applied in a storey, the force and deformation are limited to the storey only, because of the rigid-floor body assumption. The lateral stiffness k_{xj} of the jth member in the storey due to relative unit displacement in direction x can be expressed as

$$k_{xj} = \frac{12EI_{yj}}{H^3(1+\varphi_{xj})} \tag{2.1}$$

where E, I_{yj}, H are respectively the Young's modulus, moment inertia of the member, and the storey height. In Eqn. 2.1, term φ_{xj} is a factor accounting for shear deformation in direction x, and is given by

$$\varphi_{xj} = \frac{12EI_{yj}}{H^2GA_{sxj}} = \frac{Ef_{xj}}{G}\left(\frac{b_j}{H}\right)^2 \tag{2.2}$$

where A_{sxj} and f_{xj} are the shear area and shear coefficient defined for wall j in direction x. For rectangular cross sections, the shear coefficient f_{xj} =1.2 (Gere 2001). Upon applying Eqn. 2.1 to each column/wall member in the storey, the storey stiffness contributed by all the vertical members can be expressed as

$$k_x = \sum_j k_{xj} = \sum_j \frac{12EI_{yj}}{H^3(1+\varphi_{xj})} = \frac{12E}{H^3}\sum_j \frac{I_{yj}}{(1+\varphi_{xj})} \tag{2.3}$$

in which it is assumed that all members in the storey have the same height H and material property E. It is of interest to discuss the following special situations associated with f_{xj} =1.2 and the ratio E/G about 2.3 for concrete materials. First, from Eqn. 2.2 for a column with ratio b_j/H less than 0.1, the value of the factor φ_{xj} in Eqn. 2.2 is less than 0.05, which can be ignored in computing the stiffness in

Eqn. 2.1. Thus, when the effect of member shear deformation is negligible, the following storey stiffness is obtained

$$k_x = \sum_j k_{xj} = \sum_j \frac{12EI_{yj}}{H^3} = \frac{12E}{H^3}\sum_j I_{yj} = \frac{12EI_{ye}}{H^3} \tag{2.4}$$

In this special case, an equivalent moment of inertia I_{ye} is introduced

$$I_{ye} = \sum I_{yj} \tag{2.5}$$

which indicates that for a storey having no rotation at its two ends, the equivalent storey member can be treated as a macro "beam" with moment of inertia I_{ye}. It should be noted that this equivalence is only for the purpose of dynamic analysis because this moment of inertia is not determined using the beam theory for the macro member. In the second special case, if the ratio of b_j/H in Eqn. 2.2 approaches or even exceeds 1 for a wall member in the storey, the factor φ_{xj} will dominate the storey stiffness. In an extreme case, say b_j/H>2, the flexural deformation can be ignored, and the stiffness in Eqn. 2.3 reduces to

$$k_x = \sum_j \frac{GA_{sxj}}{H} = \frac{G}{H}\sum_j A_{sxj} = \frac{G}{H}A_{sx} \tag{2.6}$$

where A_{sx} is an equivalent shear area equal to the summation of the shear areas of all shear walls parallel to direction x. Eqn. 2.6 indicates that for a storey of building structures with pure shear walls, the storey stiffness is proportional to the storey shear force. Note that if the storey has mixed columns and walls, judgment should be exercised to determine the contribution of columns and walls to the storey stiffness. In general, the storey stiffness should be determined using Eqn. 2.3 to reduce the uncertainty of prediction.

2.2. Torsion Effect

Although seismic design requires symmetrical configuration of floor plan and symmetrical arrangement of vertical supporting members, the centre of mass may not coincide with the centre of force resistance. Thus, even for biaxial symmetrical buildings, accidental eccentricity of mass is required to be considered (NBCC 2011). For buildings in nuclear industry, the structural components and the mounted equipment cannot be arranged exactly in symmetry. Torsion effect on structural response has to be considered in developing the stick models. It is a common practice to make the assumption of rigid floors in dealing with torsional effect (ASCE 1998). To account for torsion, the storey stiffnesses in both horizontal x and y directions should be determined. The resultant shear forces, passing through a so-called centre of stiffness, are proportional to these two stiffnesses. Similar to the storey stiffness in Eqn. 2.3 for direction x, the storey stiffness in y direction is given by

$$k_y = \sum_j k_{yj} = \sum_j \frac{12EI_{xj}}{H^3(1+\varphi_{yj})} = \frac{12E}{H^3}\sum_j \frac{I_{xj}}{(1+\varphi_{yj})} \tag{2.7}$$

Thus, the centre of stiffness can be determined by storey shear force in each of the two perpendicular directions. Assume each member in the storey has the same displacement δ_x in direction x as shown in Figure 2.1, the shear force in member j is $F_{xj} = k_{xj}\delta_x$ as shown in Figure 2.2. The centre of the shear forces in direction x of the storey is defined by y_s as

$$y_s = \sum F_{xj} / F_x = \sum y_j k_{xj} / k_x \tag{2.8}$$

In a similar manner, when the displacement δ_y is imposed in direction y alone, the centre of the shear forces in the direction is defined by x_s as

$$x_s = \sum x_j k_{yj} / k_y \tag{2.9}$$

In Eqns. 2.8 and 2.9, coordinate (x_j, y_j) is the centre of shear force of the vertical member j. It is interesting to note that the determination of the centre of stiffness may be relevant to the moments of inertia I_{xj} and I_{yj} alone for column-framed structures based on Eqn. 2.5, while relevant to the shear areas A_{sxj} and A_{syj} alone for shear-wall-framed structures from Eqn. 2.6. On the basis of the storey shear forces and the centre of stiffness defined previously, the torsional stiffness K_z can be determined by the torques about the centre of stiffness. Assume a rotational angle θ about the centre of stiffness as shown in Figure 2.3, the two translations along x and y directions can be found to be

$$\delta_{xj} = -r_j \theta \sin \alpha_j = -\theta(y_j - y_s) \tag{2.10a}$$

$$\delta_{yj} = r_j \theta \cos \alpha_j = \theta(x_j - x_s) \tag{2.10b}$$

where r_j is the distance between the centre of stiffness and the force centre of member j, and α_j is the angle between the r_j line and x axis. From Eqns. 2.10 the shear forces in x and y directions can be expressed in terms of the corresponding displacement and stiffness. Thus, the torque due to these shear forces is expressed as

$$T = \sum [-\delta_{xj} k_{xj} (y_j - y_s) + \delta_{yj} k_{yj} (x_j - x_s)] = K_z \theta \tag{2.11}$$

$$K_z = \sum [k_{xj} (y_j - y_s)^2 + k_{yj} (x_j - x_s)^2] \tag{2.12}$$

where K_z is defined as torsional stiffness**.** Again from the two special cases defined in Eqns. 2.5 and 2.6, the torsional stiffness in Eqn. 2.12 may be relevant to the moments of inertia I_{xj} and I_{yj} alone for column-framed structures, while relevant to the shear areas A_{sxj} and A_{syj} alone for shear-wall-framed structures. It is noted that no warping torsion is considered in the above derivation, and the effect of warping on the stiffness of stick models and on the structural response may be an interesting topic in the future research work.

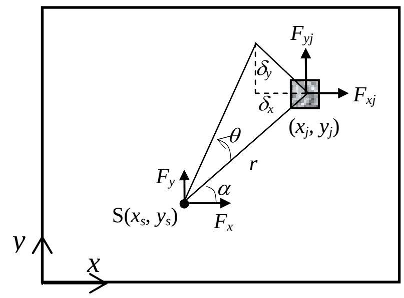


Figure 2.2. Centre of stiffness and torsion

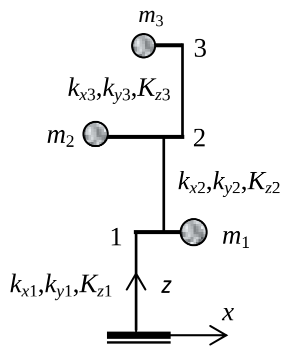


Figure 2.3. Idealized stick model

2.3. Computer-based analysis

Based on the formulation derived above, a lumped-mass stick model can be established as shown in Figure 2.3 for the dynamic analysis of structures. In this model, each vertical member with storey stiffness (k_x, k_y, K_z) is located at its centre of stiffness determined by Eqns. 2.8 and 2.9. A rigid link is used to connect the two ends of the adjacent two vertical members at the same elevation, and the concentrated mass m located at its centre of mass is rigidly connected to one of the centres of stiffness. It is seen that the eccentricity between mass and stiffness centres is considered. A stiffness matrix [K] can be formed using Eqns. 2.3, 2.7, and 2.12. A diagonal lumped-mass matrix [M] can then be determined by static equivalence to concentrate the distributed mass and mass moments of inertia on the corresponding floor elevations that correspond to the structural stiffness matrix. As such, dynamic analysis can be conducted to find the structural response of vibration modes, frequencies, accelerations, and floor response spectra etc.

If the storey stiffnesses and lumped masses can be directly input into a computer program, the dynamic analysis of structure can be automatically completed. However, conventional commercial software (e.g., ANSYS) requires inputting member cross section *A* and moment of inertia *I* for each beam member rather than the storey stiffness. Thus, the above storey-based stiffnesses have to be transformed into the cross-sectional member properties. This is difficult to conduct such a transformation except for a special case, where the structure is framed with columns alone. In this case, the cross-sectional area *A* can be obtained by algebraic summation of the areas for all the vertical members in the storey, and the equivalent moment of inertia I_{ye} is determined by Eqn. 2.5, which is the algebraic summation of the moments of inertia for all the vertical members in the storey. For the computer-based analysis, the torsion constant *J* of the cross-sectional area is also required to account for torsion. Given the torque-rotation relation defined in Eqn. 2.11 and the torsional stiffness in Eqn. 2.12, similar to the torsion of members with circular cross sections, the torsion constant can be equivalently expressed by

$$J = HK_z / G \tag{2.13}$$

For the special case when the storey members consist of walls alone, substituting Eqns. 2.6 and 2.7 into Eqn. 2.12, then Eqn. 2.13 can be expressed as

$$J = \sum [A_{sxj}(y_j - y_s)^2 + A_{syj}(x_j - x_s)^2] \tag{2.14}$$

It is important to note that to match the assumption of rigid-floor body as indicated in Figure 2.1(c), the rocking rotations at both ends of each vertical member in Figure 2.3 should be rigidly restrained to meet the rigid-floor assumption. However, in current practice, such a rocking restraint may not be applied in the computer-based stick model. This may cause error to some degree. For instance, when rigid-floor body assumption is applied for a single storey frame, a system of single degree of freedom is obtained. The frequency *f*, using the stiffness defined in Eqn. 2.3, is found to be

$$f = \sqrt{k_x / m} / 2\pi = \sqrt{3EI_{ye} / mH^3} / \pi \tag{2.15}$$

If the computer-based model with top infinite-rocking restraint is used, the same result will be obtained as that given in Eqn. 2.15. However, when the computer-based model is adopted without the top rigid-rocking restraint, the frequency will have the following expression

$$f = \sqrt{k_x / m} / 2\pi = \sqrt{3EI_{ye} / mH^3} / 2\pi \tag{2.16}$$

which is 50% lower than that provided in Eqn. 2.15. One may argue that because a floor diaphragm system is not rigid exactly in actual frameworks, releasing the rocking restraints at floor levels can rectify the original assumption. However, analysis results show that compared to those from the finite element analysis for frames with columns, the rigid-floor body can lead to much accurate predictions (Lu and Lin 1999). Therefore, it is suggested that the rigid-rocking restraints be imposed on each floor elevation of column-framed structures.

3. STICK MODEL USING MACRO BEAMS

To improve the stick modeling discussed in the previous section, a new method is discussed in this section. The key issue is that the rigid-diaphragm assumption may not be appropriate for the storey with shear walls because the lateral storey stiffness due to shear walls may be considerably stronger than the out-of-plane stiffness of the floor diaphragms. Research has indicated that using rigid-floor assumption could cause erroneous results because of the strong storey stiffness of shear walls

(Boppana and Naiem 1985). Thus, this section provides a method to develop stick models based on the beam theory that accounts for shear deformation.

It is helpful to recall the method of developing stick models for pure shear-wall structures. The nuclear reactor containment often has enclosed-tube section, and a storage tank has hallow-rectangular section. For this type of structures, a stick model can be developed using a macro beam to represent a storey. The member sectional properties include the cross-sectional area A, moments of inertia I_x and I_y, and polar moment of inertia or torsional constant J, which is often equal to I_x+I_y. Compared to a conventional beam, the macro beam has significant shear deformation. The two shear coefficients f_x and f_y should be taken into account in the modeling. Compared with the results of finite element analysis, the stick model can produce very accurate results as long as the number of nodes is not less than twice of the highest mode number of interest (ASCE 1980). Similar concept can be applied to determine the flexural properties of stick models for structures with walls and columns.

3.1. Member Properties Based on Beam Theory

It is desirable to use one stick to capture the bending, shearing, and axial loading of a wall-column structure subjected to earthquake. The section properties relating to axial and bending loading of the macro beam are determined as done for an ordinary beam member. To that end, the section properties, including the values of cross-sectional area A, centroid x_c and y_c of the area, moments of inertia of area I_x and I_y, and polar moment of inertia about the coordinate system with origin at the centroid of the stick for each storey with height H are presented in this subsection. Following the procedure for the case of sections with closed walls/shells, the coordinate of the centroid or geometrical centre C (x_c, y_c) of the cross-sectional area A is determined for the macro beam and is given by (Gere 2001)

$$x_c = \sum x_j A_j / A \,;\; y_c = \sum y_j A_j / A \qquad (3.1a, b)$$

where A_j is the area of the jth wall/column component with centroid (x_j, y_j), and A is the total area of the storey cross section. If the cross section has one symmetric axis, the centre is located at that axis; if the section has two axes of symmetry, the centre is at the intersection of the axes of symmetry. Once the coordinate (x_c, y_c) of centre of cross section is found, the bending behaviour of member is estimated based on the coordinate system with origin (x_c, y_c). Thus by using parallel-axis theorem, the second moments of area are expressed as

$$I_x = \sum [I_{xj} + (y_j - y_c)^2 A_j] \,;\; I_y = \sum [I_{yj} + (x_j - x_c)^2 A_j] \qquad (3.2a, b)$$

where I_x and I_y are the moments of inertia about y_c and x_c axes; I_{xj} and I_{yj} are the moments of inertia about the principle centroid axes of member j in the storey. Normal stresses and rotations of the cross section due to bending are dependent on the moments of inertia. The polar moment of inertia I_0 can be obtained by (I_x+I_y).

Note that the resultant force of axial forces passes through this centre, and this is important to vertical vibration of the stick model. At the same time, the moments of inertia, I_x and I_y about the principal coordinate system x_1Cy_1 are calculated. It is seen that the axial and flexural loading is coupled and considered simultaneously.

3.2. Modelling account for torsion

The torsion properties are determined to capture torsion effect. It is well-known that when the centriod, centre of shear, and centre of mass are coincident, the St. Venant torsion is uncoupled with bending and axial loading. However, when eccentricities of the three centres occur, the coupled torsion effect should be considered. The following two ways may be applied to deal with such an effect. The first way follows the thin-wall theory, and the centre of shear is determined based on the shear flow. The corresponding torsional constant is determined by

$$J = \sum b_j t_j^3 / 3 \tag{3.3}$$

where t_j is the wall thickness, which is far less than the length b_j of the wall. Because the value of J in Eqn. 3.3 is considerably less than that of the moment of inertia I_x+I_y, torsional buckling is significant for member with thin-walled section. Once the centre V is determined, any transverse force passed through this centre will not cause torsion, and the analysis of conventional beam can be performed. Note that for complicated cross sections, it is difficult to find the centre of shear. It has been shown by comparing the results from finite element analysis that using Eqn. 3.3 to account for torsion could lead to significant error for shear-wall building structures. As well, a storey of building structures is not similar to a thin-wall member. Thus, the following storey-based method is adopted.

The storey-based stiffness method, described in the previous section, can lead to better results in the dynamic analysis of structures. In this method, the storey stiffness k_x and k_y are computed first, and then the centre of stiffness S is determined using Eqns. 2.8 and 2.9 and accounting for shear deformation. And finally, the torsional constant J can be determined using Eqn. 2.13. The first modification is using the equivalent torsional constant J defined in Eqn. 2.13 to replace the polar moment of inertia I_0 (I_x+I_y). In other words, the lateral storey stiffnesses k_x and k_y are used to determine the coordinate x_s and y_s of the centre of stiffness, and then torsional stiffness K_z is used to determine the equivalent torsional constant J.

If the way based on thin-wall theory is referred to as shear centre method, the way based on storey stiffness can be referred to as stiffness centre method to deal with torsional effect. It has been shown that the stiffness centre method is better than the shear centre method to account for torsion (Chokshi and Lee 1976). Thus, this study recommends the stiffness centre method be applied to analyze shear-wall type of structures.

3.3. Modelling account for shear deformation

It is well-known that shear deformation is significant for both shear walls and column-framed structures. Although the shear coefficient for shear-wall structures can be approximately determined based on current experience, it is hard to determine the shear effects in the case of a macro member with mixed columns and walls. Because shear stress is not uniformly distributed over the cross section, a shear coefficient f or its reciprocal κ is commonly introduced to obtain an effective shear area

$$A_s = A / f = \kappa A \tag{3.4}$$

where A is the total cross-sectional area as determined before. Upon using this approach, it is convenient to use an equivalent shear force from the uneven shear stress over the section. There are some ways to calculate the shear coefficient, and for beam member the formula can be readily found in the literature (Gere 2001). Because this coefficient is an integral integrating over the section, the shape of the section has to be defined. For irregular cross sections of structures with walls and columns, it is hard to define the section shape of the macro beam and the factor f in Eqn. 3.4 cannot be easily found.

For pure shear-wall structures, the shear area A_s is approximately equal to the summation of all the areas of shear walls parallel to the direction under consideration so that the shear coefficient in Eqn. 3.4 is determined. It is seen that when this method is applied, the out-of-plane stiffness of the wall is ignored. Furthermore, this method cannot be applied for a column since it belongs to the two horizontal directions. Therefore, a reasonable method should be employed to define the shear coefficients. If the area A and area moments of inertia I_x and I_y of the macro beam for an actual storey are determined using Eqn. 3.2, the following equivalence of storey stiffness can be applied to determine the shear coefficients.

It is assumed that the macro beam for a storey has the same area moments of inertia about x and y axes. The lateral stiffness accounting for bending and shear deformations of the macro beam with fixed ends is equal to the corresponding lateral storey stiffnesses obtained from Eqns. 2.6 and 2.7 for the storey. Thus, the equivalence of lateral stiffnesses can be expressed as

$$\frac{12EI_y}{H^3(1+\varphi_x)} = k_x \; ; \; \frac{12EI_x}{H^3(1+\varphi_y)} = k_y \tag{3.5a, b}$$

It is important to point out that the storey stiffness k_x and k_y are determined based on the rigid floor assumption, while determining the area moments of inertia I_x and I_y of the macro beam does not use the rigid floor assumption. Thus, Eqn. 3.5 only serves as a way to determine shear coefficients of the macro beam. The two terms φ_x and φ_y in Eqn. 3.5 have similar definition as that in Eqn. 2.2

$$\varphi_x = \frac{12 f_x EI_y}{GAH^2} \; ; \; \varphi_y = \frac{12 f_y EI_x}{GAH^2} \tag{3.6a, b}$$

where only parameters f_x and f_y are unknown. From Eqn. 3.5, terms φ_x and φ_y can be expressed in terms of E, I_x, I_y, k_x, k_y, and H, which are known for a specified storey. Upon using Eqns. 3.5 and 3.6, the two unknown shear coefficients f_x and f_y are obtained and given by

$$f_x = \frac{GAH^2}{12EI_y}\left(\frac{12EI_y}{k_x H^3} - 1\right) ; \; f_y = \frac{GAh^2}{12EI_x}\left(\frac{12EI_x}{k_y h^3} - 1\right) \tag{3.7a, b}$$

which are obtained from the combined beam theory and storey-based structural analysis. Thus, a macro beam is developed to model a storey of building structures accounting for storey shear deformation. Note that this method is still somehow approximate since the rigid floor assumption is applied in the derivation.

3.4. Procedure of Developing Stick Models

It is seen from the discussion in the previous subsections that a stick model for a mixed wall-column structure can be developed using macro beam members capable of modeling storey shear deformation. Two major steps are taken in the process: a storey is represented by a macro beam, and its area, centriod, and moments of inertia are determined following conventional beam theory; the effects of torsion and shear deformation are considered based on storey stiffness on the basis of storey-based structural analysis. Thus, the established macro beams for the stick model behave like Timoshenko beams, which capture both bending and shear deformations. Based on the derivation above, the procedure of developing lumped-mass stick models using the proposed method can be summarized as follows:

(1) Compute the cross-sectional A and centriod (x_c, y_c) for each storey using Eqn. 3.1;
(2) Calculate moments of inertia I_x and I_y about the principle axes at centriod using Eqn. 3.2;
(3) Compute storey stiffness k_x and k_y for each storey using Eqns. 2.4 and 2.7;
(4) Determine the centre of stiffness (x_s, y_s) based on Eqns. 2.8 and 2.9;
(5) Calculate torsional stiffness K_z and torsional constant J using Eqn. 2.13;
(6) Compute the shear coefficients f_x and f_y using Eqn. 3.7;
(7) Determine the mass m, its centre (x_m, y_m), and mass moment of inertia.

Upon using the procedure, an example of two-storey building will illustrate the analysis results in the following section.

4. EXAMPLE ANALYSIS

To show the computation results from the proposed method above, a two-storey building as shown in Figure 4.1 is adopted in this example analysis. In the first storey shown in Figure 4.1(a) , all columns have square cross section of 0.4 by 0.4 m, while all walls have cross section of 0.3 by 2 m. The second storey in Figure 4.1(b) has only columns with cross section of 0.3 by 0.3 m. As shown in Figure 4.1(c), the storey heights are 6 m and 4 m for the first and second storeys. In developing the stick and FEA model, the Young's modulus and Poisson's ratio of the applied concrete material are 2.8E10 MPa and 0.15 for all walls and columns. For simplicity, the distributed masses are assumed to concentre to the two floor elevations, where the slabs have thickness of 0.4 m and 0.3 m for the first and second floors respectively and have density of 2400 kg/m^2.

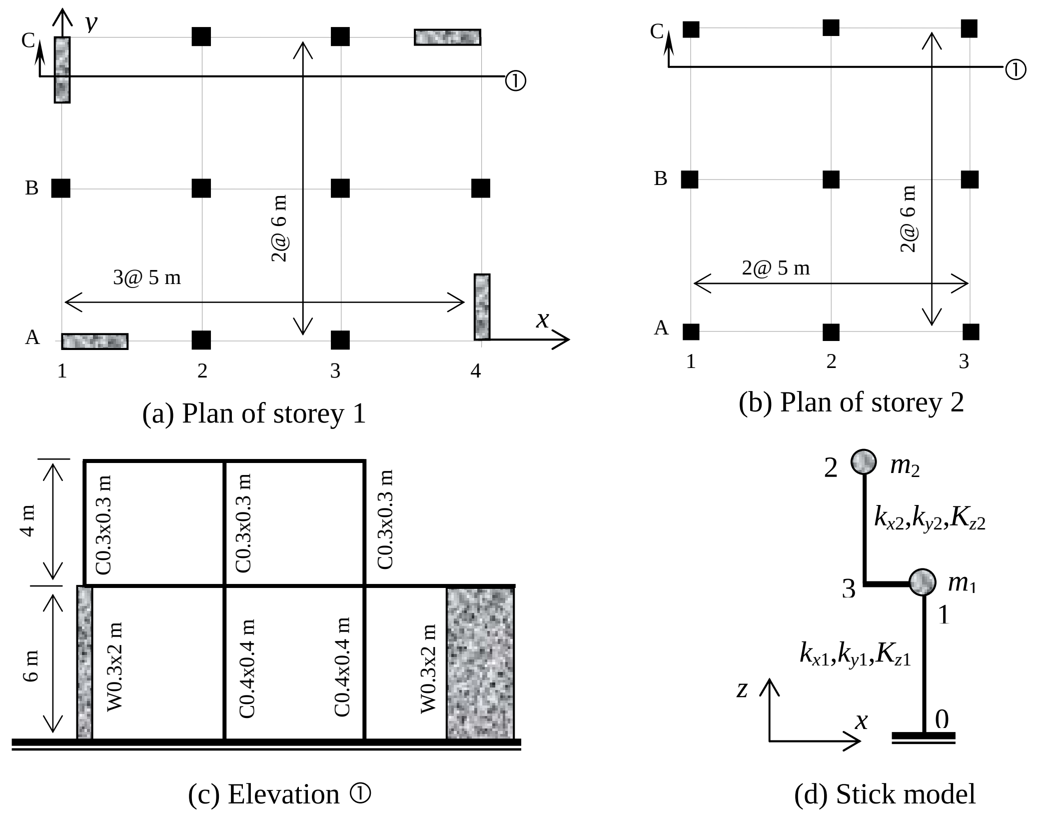


Figure 4.1. A two-storey wall-column structure

Upon using the dimensions and material properties as well as the corresponding formulas, the lumped-mass stick model is developed and shown in Figure 4.1(d). The two concentred masses m_1= 172.8 ton and m_2= 86.4 ton, which are located at the corresponding centres of stiffness in Figure 4.1(d). After conducting modal analysis for this stick model, the first six frequencies are obtained and shown in Table 1. For the purpose of comparison and validation of the proposed method, a finite element model is developed using beam and shell elements. Four beam elements are divided for each column, and sixty-four shell elements for each wall panel in a storey. For each slab panel in a bay, at least 64 shell elements are applied for the finite element analysis (FEA). After conducting modal analysis for this FEA model, the first six frequencies are obtained and provided in Table 1 as well. To assess the influence of floor rigidity, it is assumed that all other properties are remained unchanged except that the Young's modulus of the slabs is assumed as 2.8E10, 2.8E11, 2.8E12, and 2.8E13.

It is expected that the first six frequencies determined using the stick model shown in Figure 4.1(d) are close to those obtained by the finite element analysis when the floor slabs are rigid with E_{slab} = 2.8E13. No error occurs for the first two frequencies, and less than 0.6% error for other frequencies. This indicates that upon using the assumption of rigid floor slabs, the proposed stick model can lead to almost the same results from the FEA. It is interesting to note that in reality the floor slabs are not rigid and if the actual E_{slab} = 2.8E10 is applied; the lumped-mass stick model will overestimate 20% of the first frequency, and 18% for the second frequency. Meanwhile, the torsional frequency (f_3) is 20% overestimated. Much more significant error happens for the high frequencies. It is observed form Table 4.1 that, with the increase of slab stiffness, the error from stick model decreases. Note that the floor diaphragm is not rigid in reality and a method applied in un-braced frames (Liu and Xu 2005) may be helpful in considering the flexibility effect to improve the accuracy.

Table 4.1 Comparison of frequency response from stick model and FEA model

Method		f_1	f_2	f_3	f_4	f_5	f_6
Stick modeling		2.923	2.930	4.230	8.951	8.974	15.037
FEA E_{slab}=	2.8E+10	2.438	2.482	3.555	5.982	6.043	9.619
	2.8E+11	2.807	2.827	4.093	7.377	7.420	12.115
	2.8E+12	2.907	2.916	4.201	8.664	8.681	14.442
	2.8E+13	2.923	2.930	4.243	9.003	9.015	15.108

5. CONCLUSIONS

A method is proposed in this study to develop stick models for building structures with mixed walls and columns. A macro beam is applied to represent a storey, whose sectional properties are determined using beam theory. To capture the torsional and shear effects, the storey stiffness based on rigid-floor assumption is used to determine the equivalent torsion constants and shear coefficients. Basic formulas are derived for determining the cross-sectional properties, and example analysis is conducted to illustrate the proposed method. Although the derivation in this paper focuses on the mixed wall-column structures, the proposed method is universal. The method can be applied to analysis of structures with pure columns, pure shear walls, or a combination of them. The determination of the shear coefficients is important to improve the accuracy in predicting structural response. This method can reasonably simulate horizontal and torsional vibration of structures. The vertical response close to the vertical wall/column member at a floor elevation can be predicted by the analysis. However, the response at a location far away from the vertical members on a floor cannot be captured because of the rigid-floor assumption.

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