Vertical elastic limit strength of damaged low-rise moment-resisting frames after inelastic seismic response

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SUMMARY
In these days, there are a lot of discussions for the establishment of the guideline in order to realize the buildings which have good redundancy and robustness. And there are many concepts and ideas of the design about them. For example, considering the possibility of the sustainable use of damaged buildings, redundancy and robustness are one of the important indexes besides repairability to decide whether the damaged buildings have enough residual performance or not. The purpose of this study is to investigate the residual seismic resistant performance of damaged buildings after the inelastic seismic response. In this paper, the index related to the residual resistance performance is defined. The index is sensitive to the column-to-beam stiffness ratio and strength ratio, the initial maximum moment subjected to fixed-vertical loads, and the maximum ductility factor. It is observed that the predicted index values give lower bound of the analytical index values.

Keywords: Residual resistance performance, Rigid frame, Elastic limit strength, Response analysis, Static incremental analysis

1. INTRODUCTION

In these days, because of the growing interests in social concern, such as resource and environmental problems, there are a lot of discussions for the establishment of the guideline in order to realize the buildings which have good redundancy and robustness. And there are many concepts and ideas of design about them. For example, considering the possibility of the sustainable use of damaged buildings, a redundancy and a robustness are one of the important indexes besides repairability to decide whether the damaged buildings have enough residual performance or not.

Redundancy and robustness are noticed as the peformance to the terrible or unexpected loads. In recent years, many studies on them have been reported in the world. On the other hand, as the studies on repairability, residual deformation which is focused on occupancy, cost and safety etc. has been investigated from the technical or economical viewpoint (Iwata et al., 2005). However, few studies have been done on the states of the stress of damaged buildings. If a building is damaged, its performance should be evaluated not only by the states of residual deformation but also the states of residual stress. As the influence of the residual stress of the damaged structure, it is reported that the moment at the middle point of the beam subjected to the concentrated vertical loads increases by the moment redistribution. And also, the increased moment of the beam is not decreased even after the inelastic seismic response (Ito et al., 2010). In general, a weak-beam-strong-column system is recommended for the design of earthquake-resistant structures, because the associated overall collapse mechanism exhibits a high ductility and energy against to a strong earthquake. From these indications, it is suggested that the residual vertical elastic limit strength of the damaged buildings against to the additional vertical loads, such as snow or over-live loads, would be deteriorated. So, in this paper, the vertical elastic limit strength of damaged low-rise moment-resisting frames after inelastic response is investigated analytically.

2. INDEX RELATED TO REDIDUAL STRENGTH AND EVALUATION METHOD

2.1. The Past Index Related To Redundancy

There are a lot of indexes about redundancy and robustness. For example, Feng et al. (1986) proposed an index relevant to the redundancy of a structure by the following equation:

$$RSDI = \frac{RSD}{R_{ult,s}} \tag{2.1}$$

where, RSD is the residual strength of the structure, $R_{ult,s}$ is the ultimate strength of the structural system.

The residual strength refers to the strength of the structural system after a certain structural component has failed because of an accident or other overload condition.

2.2. The Index Related To Residual Vertical Elastic Limit Strength

In this study, referring to the Eqn.(2.1) , the index related to the residual vertical elastic limit strength after enealstic response is defined as follows:

$$R = \frac{\lambda_d}{\lambda_0} \tag{2.2}$$

where, λ_d is the load carrying capacity in the state in which the structure is damaged by the seismic motion(Fig 2.1.(a)), λ_0 is the load carrying capacity in the state in which the structure is not damaged(Fig 2.1.(b)).

In the state in which the structure doesn't have enough residual vertical elastic limit strength, R is nearly equal to 0. On the other hand, in the state in which the structure has enough residual vertical elastic limit strength, R is nearly equal to 1.

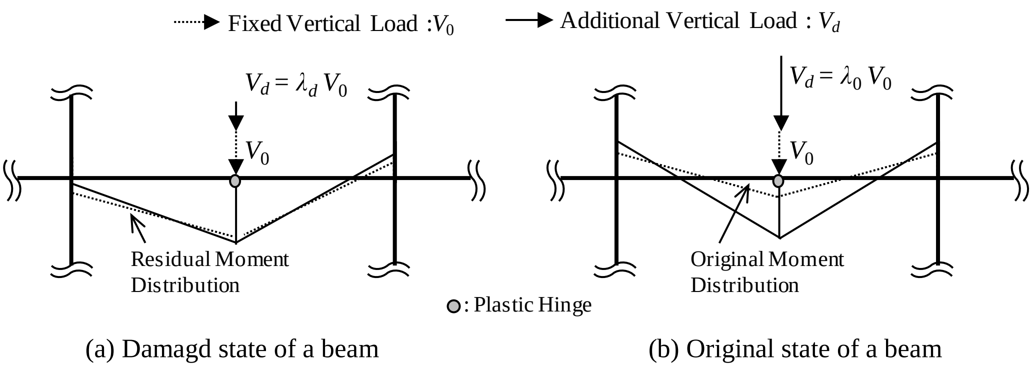


Figure 2.1. Explanation of the moment distributions and load carrying capacities of the each case

2.3. Evaluation Method of Residual Vertical Elastic Limit Strength

In this study, λ_d is defined in two types. One is the predicted values estimated by lateral static incremental analysis and vertical push-down analysis (represented as ${}_s\lambda_d$ in Fig.2.2), and the other is analytical values estimated by the time-history response analysis and vertical push-down analysis (represented as ${}_d\lambda_d$ in Fig.2.2.). So, Eqn.(2.2) is finally expressed as follows:

$$R_s = \frac{{}_s\lambda_d}{\lambda_0} \tag{2.3a}$$

$$R_d = \frac{{}_d\lambda_d}{\lambda_0} \tag{2.3b}$$

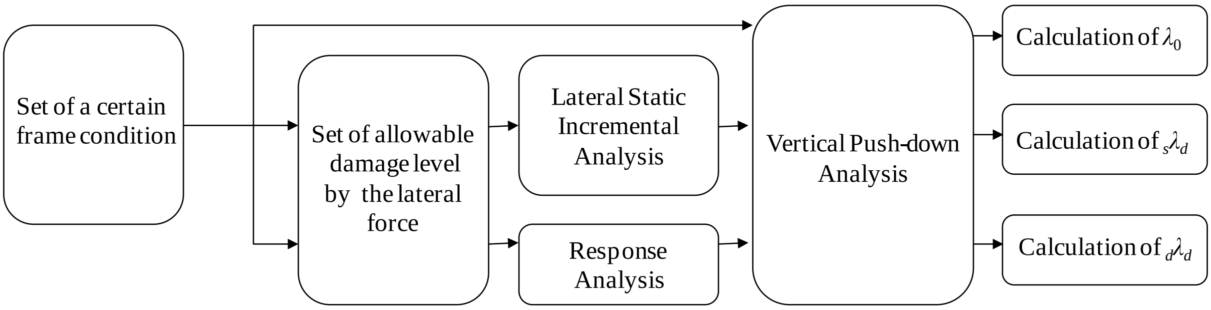
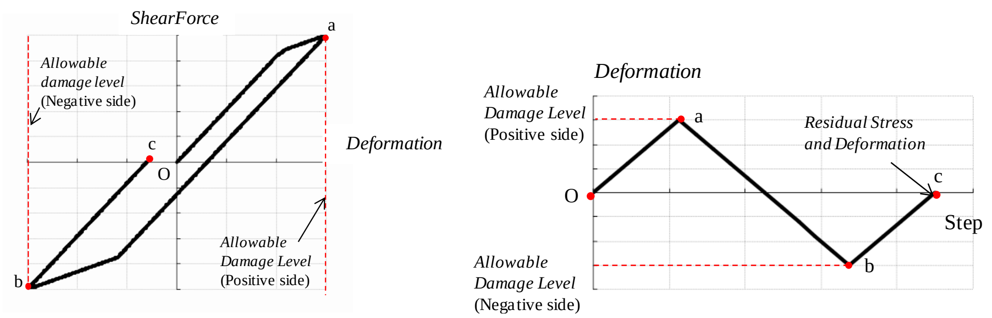


Figure 2.2. Evaluation process of the load carrying capacities

2.4. Loading Condition of the Lateral Static Incremental Analysis

The lateral static incremental analysis is done as shown in Fig.2.3. The point O represents the original state of the structure, the point a and b are the allowable damage level at each side. Finally, the point c is obtained by the elastic unloading from the point b. In this paper, the allowable damage level is considered as the ductility factors.



(a) Load-displacement curve (b) Transition of the displacement

Figure 2.3. Loading condition on the lateral static incremental analysis

3. EXPLANATION OF THE FUNDAMENTAL RELATION BETWEEN RESIDUAL MOMENT AND RESIDUAL VERTICAL ELASTIC LIMIT STRENGTH

3.1. General Description

It depends on some factors, such as structural condition or damage level and so on, whether the damaged structure has enough residual strength or not. Thus, in explanation of the fundamental relation between residual moment and residual vertical elastic limit strength, a 1st-story moment-resisting rigid frame model is supposed (as shown in Fig.3.1).

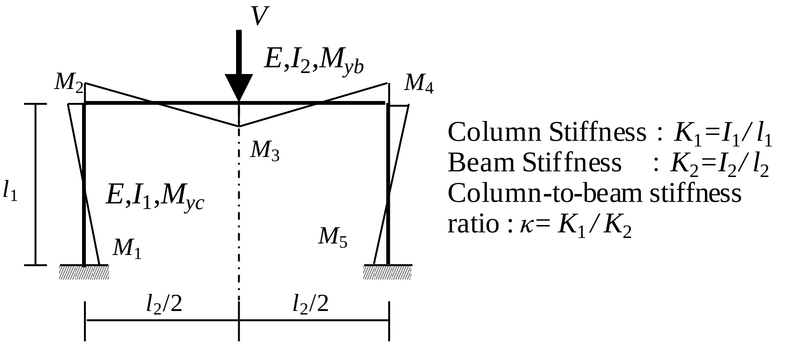


Figure 3.1. 1st-story moment-resisting rigid frame models

Where, l_1 and l_2 are the length of the columns and beam, E is the young's modulus, $I_1$and I_2 are the geometrical moment of inertia of the columns and beam, M_{yc} and M_{yb} is the elastic limit strength of the columns and beam, V is the fixed vertical load, moment at each joint is represented as M_1~M_5 as shown in Fig.3.1.

3.2. Analytical Variables

Herein, to consider the simple example, the structural conditions as below are considered as fixed:

1) The column-to-beam strength ratio (M_{yc} / M_{yb}) is 1.2
2) The damping constant is 0.2
3) Each member is assumed to be a perfectly elastic-plastic body

The moment distribution of the structures depends on V and column-to-beam stiffness ratio (as shown in Fig.3.1). In this case, λ_0 and ${}_s\lambda_d$ are caluculated by the follwing equations:

$$\lambda_0 = \frac{M_{yb} - M_3}{M_3} \tag{3.1a}$$

$$\lambda_d = \frac{M_{yb} - M_{3r}}{M_3} \tag{3.1b}$$

where, M_{3r} is the residual moment at the middle point of the beam after inelastic response.
In addition, defining the following equations, R_s is finally represented as follows:

$$\chi = \frac{M_3}{M_{yb}} \quad , \quad \chi_r = \frac{M_{3r}}{M_{yb}} \tag{3.2}$$

$$R_s = \frac{1-\chi_r}{1-\chi} \tag{3.3}$$

The index χ represents the degree of the magnitude of the vertical load to the beam's strength. Thus, R_s is influenced by the χ. Then, in this paper, χ is adopted as one of the analytical variables.

Table 3.1. shows the analytical variables used in this chapter.

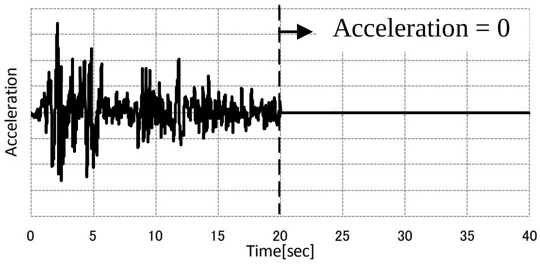
Table 3.1. Analytical variables in this chapter

Variables	Symbols	Values
Column-to-beam stiffness ratio	κ	1.0 , 2.0 , 3.0
Initial maximum moment	χ	0.1 , 0.2 , 0.3 , 0.4

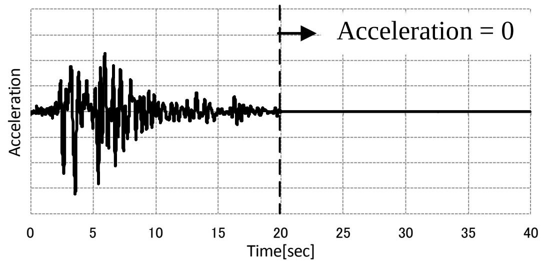
And, in this chapter, the natural period of the analytical models are assumed to be 0.12 seconds in the case of κ = 3.0, χ = 0.1, 0.19 seconds in the case of κ = 1.0, χ = 0.4.

3.3. Input Ground motions

The observed earthquake records of the Imperial Valley El CentroNS component in 1940 and Hyogoken-Nambu Earthquake FukiaiNS component in 1995 are used in this paper. Principal shock suration of each earthquake motion is 20 seconds, with the enougth time for free viblation (Fig.3.2.).



(a) El CentroNS Component



(b) FukiaiNS Component

Figure 3.2. Input earthquake's time history records

3.4. Allowable Damage Level

Herein, as the allowable damage level of the structure under the lateral force, lateral force for the lateral static incremental analysis and the accelerations of the input ground motion for the response analysis are scaled so that the ductility factor of the structure is equals to 1.5.

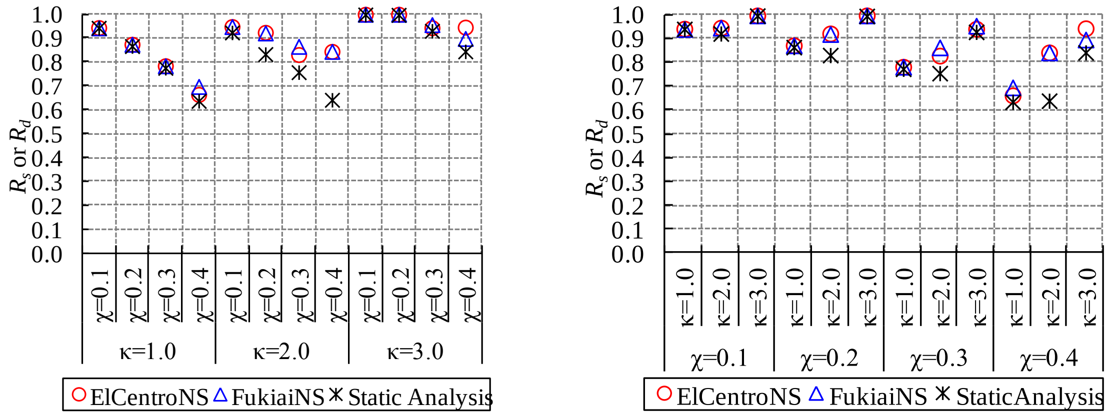
3.5. Analytical Results and Consideration

Fig.3.3 shows the comparisons of the R_s and R_d viewed from κ and χ . Fig.3.4 shows the time-history of the M_3 in the case κ = 1.0, χ = 0.4 and κ = 3.0, χ = 0.4. Fig.3.5. shows the shearforce - deformation curves of the structure under the lateral static incremental analysis and response analysis in the case κ = 1.0, χ = 0.4 and κ = 3.0, χ = 0.4. Where, Q is the shearforce of the story, Q_y is the yield-shearforce of the story, δ is the deformation of the story, δ_y is the yield-deformation of the story.

From Fig.3.3(a), as χ increases, the R_d and R_s tend to be deteriorated. From Fig.3.3(b), on the other hand, as κ increases, the R_d and R_s tend to be increased. It is confirmed that the analytical values (R_d) exceed the predicted values (R_s) in all the cases.

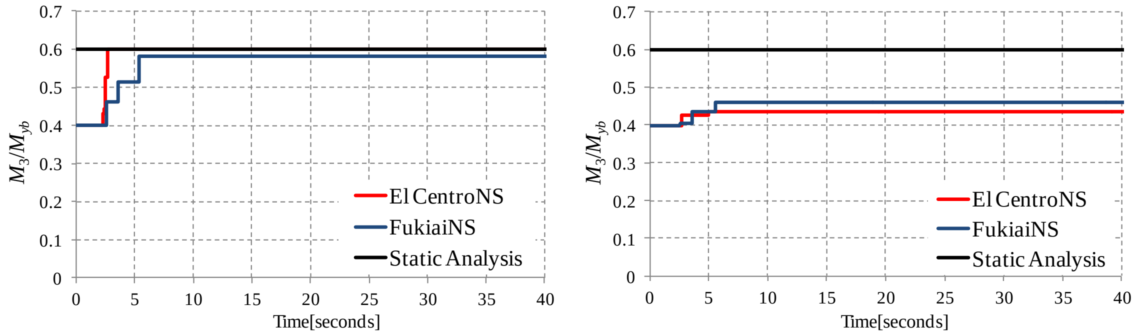
From Fig.3.4(a), in the case κ = 1.0, χ = 0.4, the M_3 is increased to the value which is estimated by the lateral static incremental analysis. As κ increses, the columns are subjected to the moment compared with the beam. Therefore, in the case κ = 3.0, χ = 0.4, the M_3 is increased little during inelastic response. However, the increased moment of the beam is not unloaded even after inelastic seismic response in each case.

From Fig.3.5, the Q-δ curves of the lateral static incremental analysis exceed that of response analysis even in the negative side. This is the one of the reasons that the analytical values (R_d) exceed the predicted values (R_s)



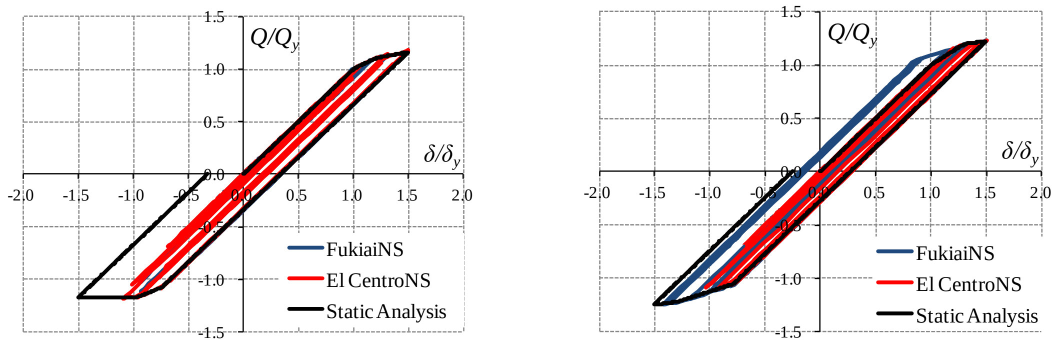
(a) Relation between R and χ (b) Relation between R and κ

Figure 3.3. Comparisons of the R_s and R_d



(a) In the case κ = 1.0, χ = 0.4 (b) In the case κ = 3.0, χ = 0.4

Figure 3.4. Time history of the M_3



(a) In the case κ = 1.0, χ = 0.4 (b) In the case κ = 3.0, χ = 0.4

Figure 3.5. Shearforce-displacement curves of the structure

4. A EXERCISE IN THE CASE OF LOW-RISE MOMENT-RESISTING FRAME MODEL

4.1. General Description

Herein, a 3-story 3-span low-rise moment-resisting frame model shown in Fig.4.1 is considered. At first, the design condition and the analytical variables of the frame model are discribed. And then, analytical results and consideration are discribed.

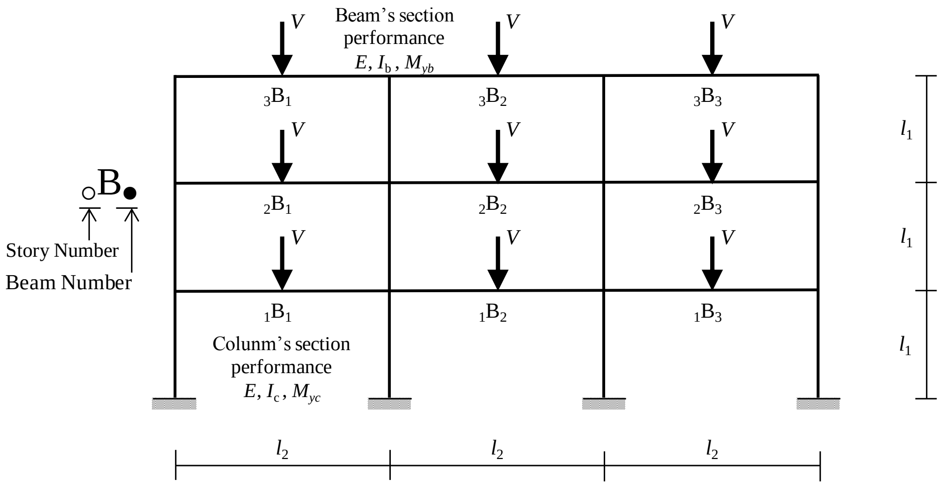


Figure 4.1. 3-story 3-span low-rise moment-resisting frame model

4.2. Design Conditions

In this chapter, the analytical frame model (Fig.4.1.) is designed by the follow conditions:

1) each column has the same section property and length
2) each beam has the same section property and length
3) every floor has the same column-to-beam stiffness ratio(shown as κ) and the same column-to-beam strength ratio(shown as τ) except for the roof level
4) lateral-load pattern used in the lateral static incremental analysis is subjected to the *Ai* distribution according to the Japanese Building Standard Law
5) each member is assumed to be a perfectly elastic-plastic body
6) weight of each story is the same value

κ and τ are calculated by use of the following equations;

$$\kappa = \frac{\sum K_c}{\sum K_b} = \frac{\sum \frac{I_c}{l_1}}{\sum \frac{I_b}{l_2}} \tag{4.1}$$

$$\tau = \frac{\sum M_{y\,c}}{\sum M_{y\,b}} \tag{4.2}$$

where K_c and K_b stand for a column's and a beam's stiffness, I_c and I_b are the column's and beam's geometrical moment of inertia, h is the floor height, l is the length of the span, M_{yc} and M_{yb} are the column's and beam's elastic limit strength.

4.3. Analytical Variables

In this chapter, analytical variables as below are considered:

(1) κ : the column-to-beam stiffness ratio
(2) τ : the column-to-beam strength ratio
(3) χ : the initial maximum moment ratio of the beam by the fixed vertical load
(4) μ : the maximum ductility factor of the structure

Then, κ and τ are shown as Eqn. (4.1.) and (4.2.), χ and μ are difined as following equations:

$$\chi = max(M_j / M_{yb}) \tag{4.3}$$

$$\mu = \max(|_{\max}\delta_i| / \delta_{yi}) \tag{4.4}$$

where M_j is the maximum value of the moment at the middle point of the beams subjected to the fixed vertical loads when the structure is not damaged, $_{max}\delta_i$ is the maximum relative story-deformation of the structure, δ_{yi} is the yield-deformation of the story.
Tablel 4.1. shows the analytical variables used in this chapter.

Table 4.1. Analytical variables in this chapter

Variables	Symbols	Values
Column-to-beam stiffness ratio	κ	1.0 , 1.5 , 2.0
Column-to-beam strength ratio	τ	1.2 , 1.8 , 2.4
Initial maximum moment ratio	χ	0.1 , 0.2 , 0.4
The maximum ductility factor	μ	1.5 , 3.0

And, in this chapter, the first natural period of the analytical models are assumed to be 0.28 seconds in the case in which κ = 2.0, χ = 0.1, 0.42 seconds in the case in which κ = 1.0, χ = 0.4.

4.4 Analytical Results and Consideration

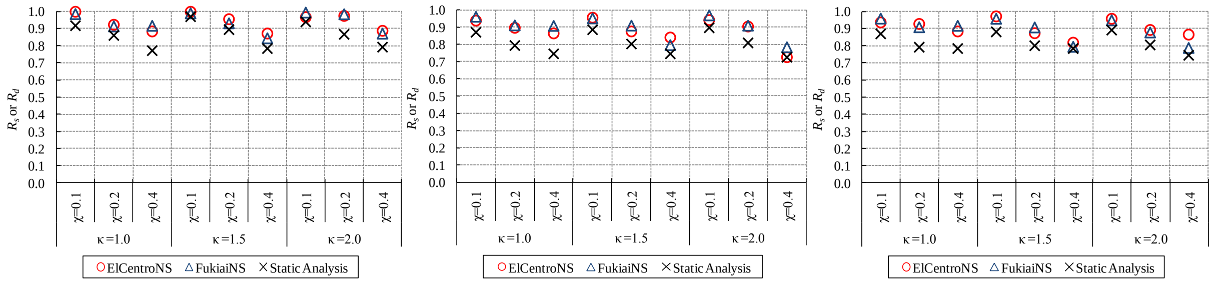
Fig.4.2. shows the comparisons of the R_s and R_d in the cases of μ = 1.5. And, Fig.4.3. shows the comparisons of the R_s and R_d in the cases of the μ=3.0. These figures show the distributions of the R in the cases that τ is constant. Fig.4.4. shows the examples of the relation between R and other variables except for χ. Fig.4.5. shows the time-history of the moment of the $_1B_1$ in the case of μ=3.0, κ = 2.0, χ = 0.4 , τ = 1.8 as an example of the beams' behavior during the inelastic response. Fig.4.6. shows the shearforce-deformation curve of the 1st story under the lateral static incremental analysis and response analysis in the case of μ=3.0, κ = 2.0, χ = 0.4, τ = 1.8.

From Fig.4.2. and Fig.4.3., as χ increases, the R_d and R_s tend to be deteriorated. Moreover, as μ increases, the R_d and R_s also tend to be deteriorated.

From Fig.4.4(a), as τ increases, the R_d and R_s tend to be deteriorated. From Fig.4.4(b), on the other hand, as κ increases, the R_d and R_s tend to be a littele increased. And, it is confirmed that the relation between R and the other variables has the same tendency to the cases in Fig.4.4.

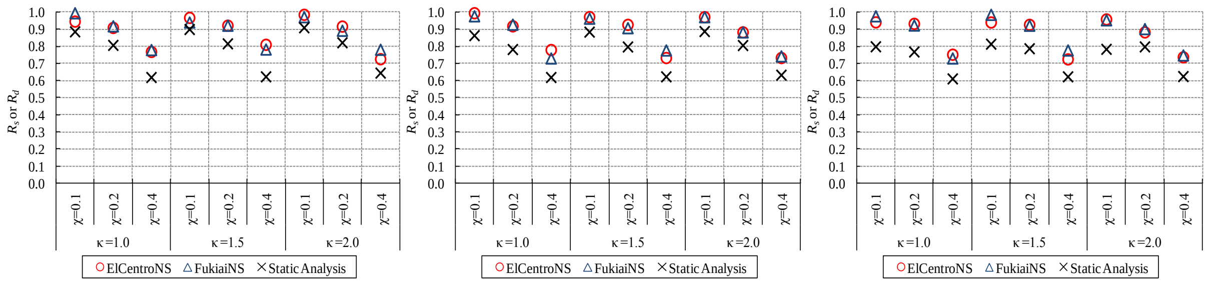
From Fig.4.5., although the moments of the $_1B_1$ increase in each case, the time-histories of the moment vary during the response analysis. This is because that the inflection point of the moment distribution by the lateral forces doesn't correspond with the middle point of the beam. But, the residual moment which is estimated by the lataral static incremental analysis exceeds that of response analysis.

From Fig.4.6., the Q-δ curve of the lateral static incremental analysis exceeds that of response analysis even in the negative side. Thus, evaluating the allowable damage level of the both side of the deformation by use of the static analysis can generally simulate the lower bound of the residual vertical elastic limit strength.



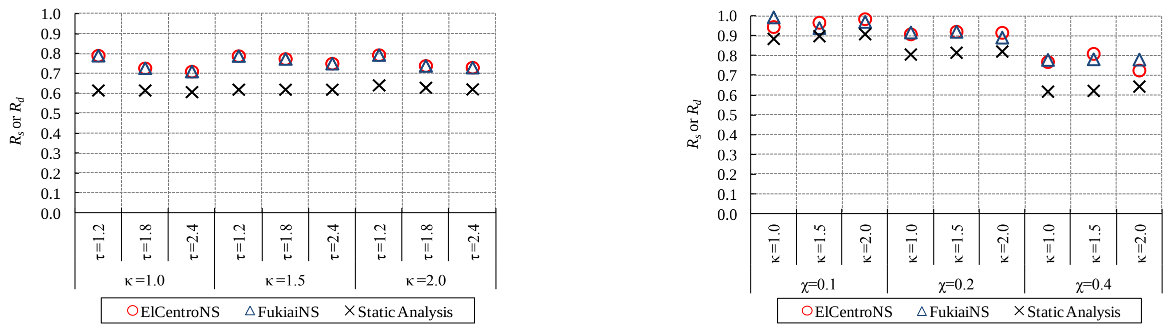
(a) In the cases τ = 1.2 (b) In the cases τ = 1.8 (c) In the cases τ = 2.4

Figure 4.2. Comparisons of the R_s and R_d in the cases of the μ=1.5 (relation with R and χ)



(a) In the cases τ = 1.2 (b) In the cases τ = 1.8 (c) In the cases τ = 2.4

Figure 4.3. Comparisons of the R_s and R_d in the cases of the μ=3.0 (relation between R and χ)



(a) In the cases χ = 0.4(relation between R and τ) (b) In the cases τ = 0.4(relation between R and τ)

Figure 4.4. Examples of the relation between R and other variables in the cases of μ =3.0

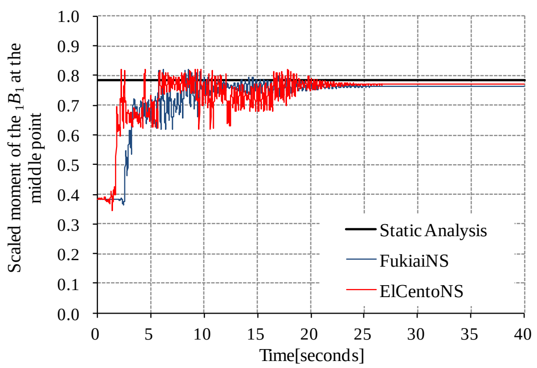


Figure 4.5. Example of the time-history of the moment at the $_1B_1$

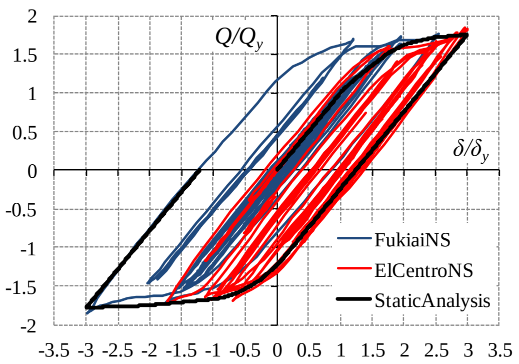


Figure 4.6. Example of the shearforce-deformation curve of the 1st story

5. CONCLUSONS

In this paper, the R indexes related to the residual vertical resistance performance of the damaged buildings after inelastic response are defined. Based on the allowable damage level of the structure under the lateral forces, the evaluation methods of the R indexes are proposed. And, the R indexes are calculated analytically. The following conclusions are drawn:

1. The lower bounds of the residual vertical elastic limit strength of the damaged structures can be estimated on the safe side by use of the static incremental analysis.

2. As the column-to-beam strength ratio (τ), the initial maximum moment ratio of the beam by the fixed vertical load (χ) and the maximum ductility factor of the structure (μ) increse, the R_d and R_s tend to be deteriorated.

3. As the column-to-beam stiffness ratio (κ) increses, the R_d and R_s tend to be incresed.

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