Nonlinear Cyclic Modeling of Concentrically Braced Frames

Gang Li
Dalian University of Technology, China

Larry A. Fahnestock
University of Illinois at Urbana-Champaign, USA

Hong-Nan Li & Su-Yan Wang
Dalian University of Technology, China

SUMMARY:
The concentrically braced frame (CBF) is a popular structural system for resisting lateral loads. Steel braces provide high lateral strength and stiffness and participate in seismic energy dissipation by buckling and yielding. Studies have shown that the lateral response of a CBF is dominated by the inelastic cyclic behaviour of the braces. Several types of models have been developed to represent the cyclic behaviour of brace elements. These models can be divided into three broad categories: finite element, phenomenological and physical theory. Finite element models are accurate but computationally expensive, and phenomenological models are computationally efficient but depend heavily on experimental results for calibration. Physical theory models, which are based on fundamental structural behavior, provide a balance of efficiency and accuracy. This paper implements an existing brace physical theory model in the force analogy method (FAM). Two slide plastic mechanisms are used to simulate the axial force- displacement relationship of the brace. The brace model presented here is compared to finite element results. The brace model is also implemented in a frame, where inelastic response occurs in both the frame and the braces.

Keywords: steel braced frame; nonlinear brace behaviour; force analogy method; cyclic loads

1. INTRODUCTION

The steel concentrically braced frame (CBF) is a structural configuration that has been used for many years to resist lateral loads in buildings since steel braces provide high lateral strength and stiffness to the overall system. The cyclic behavior of a CBF is highly dependent on its bracing elements. As the hysteretic behavior of braces is complex, numerous experimental studies have been performed on the inelastic cyclic behavior of brace elements (e.g., Popov and Black 1981; Elchalakani et al. 2003; Shaback and Brown 2003; Goggins et al. 2006). Additionally, analytical methods to simulate the inelastic behavior of brace elements have been developed. Inelastic modeling of steel braces can be classified into three broad categories: finite element, phenomenological nd physical theory (Ikeda and Mahin 1984). Finite element models provide more precise results, but are computationally expensive. Some examples of this approach are presented by El-Tawil (2001a and 2001b) and Jin (2005). Phenomenological models provide a simpler way to simulate the inelastic behavior of braces, but governing relationships depend largely on experimental data. However, these models proved valuable for inelastic analyses (e.g., Fukuta et al. 1989; Maison and Popov 1980). Physical theory models provide a fundamental definition of inelastic brace behavior through brace properties. This method has its own shortcoming, such as assuming plastic behavior is concentrated in hinge zones, and this method still does require specific empirical coefficients (e.g. Soroushian and Alawa 1990; Remennikov and Walpole 1997; Jin and El-Tawil 2003; Dicleli, M. and Calik, E.E. 2008).

The force analogy method (FAM), which was first proposed by Lin (1968), can reduce the stiffness storage space in nonlinear analysis, simplify the computations and enhance the calculation speed. Wong and Yang (1999) first applied the FAM to dynamic elastic–plastic analysis of structures in civil engineering. Further, Wong and Yang (2002, 2003a,b), and Wong and Zhao (2007) built the energy response model of a structure and developed predictive instantaneous optimal control and a stochastic dynamic analysis method using the FAM. Moreover, Chao and Loh (2007) proposed a modified FAM for simulating the inelastic response of reinforced concrete structures. Li et al. (2011) established an approach based on the FAM to analyze the dynamic response of structures with energy dissipation devices. To date, the hysteretic behavior that has been implemented in the FAM at inelastic DOFs is relatively simple, and steel brace behavior has not been considered.

In this paper, an existing physical theory model (Dicleli and Calik 2008) is implemented in the force analogy method. Two sliding plastic mechanisms, which simulate axial displacements produced by transverse brace displacement and the so-called "growth effect," are used to represent the inelastic brace behaviour, and the resulting model is shown to provide good agreement with finite element model response.

2. BACKGROUND

2.1 Force Analogy Method

The fundamental framework of the FAM has been presented in detail by Wong and Yang (1999). The main governing equations are introduced here for a single-degree-of-freedom (SDOF) system representing a framed structure. In the FAM, the total displacement *x(t)*is defined as the sum of elastic displacement *x'(t)* and inelastic displacement *x"(t),* where the inelastic displacement *x"(t)* is produced from the plastic hinges at the beam-column joint:

$$x(t) = x'(t) + x''(t) \tag{1}$$

Then, the force of the frame $F_s(t)$ can be expressed as:

$$F_s(t) = k_s x'(t) = k_s (x(t) - x''(t)) \tag{2}$$

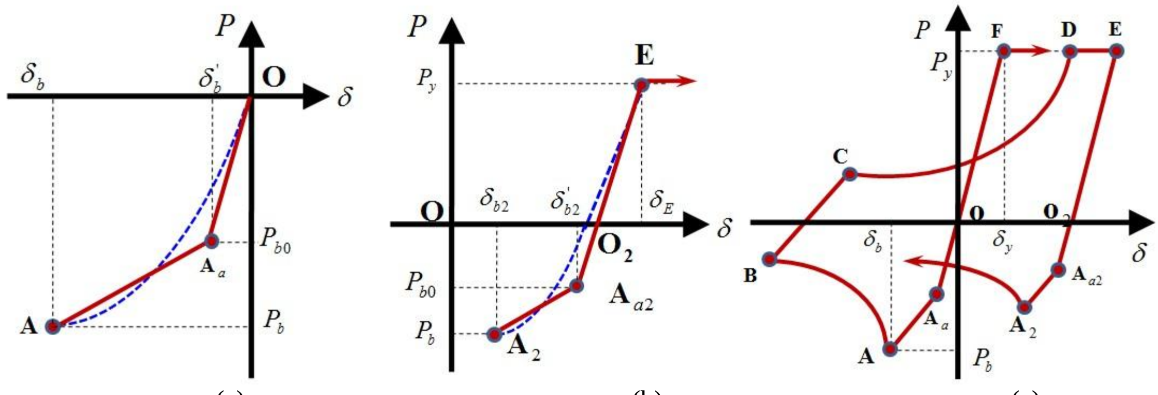
where k_s is the initial elastic stiffness of the frame. After a series of derivations (Wong 1999), $F_s(t)$ and the moment at the plastic hinge $M_s(t)$ can be written as:

$$\begin{cases} F_s(t) = k_s x(t) - k_p \theta''(t) \\ M_s(t) = k_p^T x(t) - k_R \theta''(t) \\ M_s(t) = g(\theta''(t)) \end{cases} \tag{3}$$

Where $\theta''(t)$is the rotation angle of the plastic hinge; k_p is the matrix that relates the plastic rotation at the hinge with the applied force; k_p^T is the matrix transposition of k_p; k_R is the matrix that relates the plastic rotation with the moments at the plastic hinge; g(·) is the relation between moment and plastic rotation. It can be seen from equation (3) that $F_s(t)$ can be solved for a given *x(t)*.

2.2 Physical Theory Model

In this paper, a physical theory model is used to simulate the inelastic behavior of a brace element under cyclic load. The model used here is an extension of existing model developed by Dicleli and Calik (2008). Relationships between axial force *P(t)* and axial displacement $\delta(t)$ of the brace have been defined for several zones to express the relationship between axial force and axial displacement or transverse displacement of the brace (Dicleli and Calik 2008). In this paper, a slightly modified version of the Dicleli and Calik (2008) model is used to simulate the inelastic behavior of braces under cyclic load. Since the FAM requires a linear initial elastic stiffness, region OA (compression) shown in Fig. 1(a) was divided into two parts: (1) line OA_a, which is the opposite extension of line OF such that the initial stiffness is the same in tension and compression, and (2) line AA_a. Axial displacement δ'_b at Point A_a is set equal to a constant parameter β times the axial displacement δ_b that occurs at Point A. The behavior of this region is similar between initial loading in compression and subsequent cycles in compression after tension load, as shown in Fig. 1(b). The schematic framework for the physical theory model used in this research is shown in Fig. 1(c).



(a) (b) (c)

Figure 1. Relationship between axial force and displacement of a brace under cyclic load: (a) Local modified phases (OA); (b) local modified phases (EA_2) and (c) modified hysteretic curve.

3. BRACE ELEMENT IN FORCE ANALOGY METHOD

In the FAM, the axial elastic displacement is equal to the difference between the total displacement and the inelastic displacement *δ''(t)* :

$$\delta'(t) = \delta(t) - \delta''(t) \tag{4}$$

The axial force can be written as:

$$P(t) = k_b(\delta(t) - \delta''(t)) \tag{5}$$

Where $k_b = EA/L$; and L, E and A are the length, modulus of elasticity and area of the brace.

3.1 Sliding Plastic Mechanisms

In the physical theory model, axial plastic displacement *δ''(t)* is composed of two components: (1) the transverse bending deformation due to the presence of eccentricity and compression loading; (2) the "growth effect" due to degradation (Dicleli and Calik 2008). Therefore, two sliding hinge (SH) plastic mechanisms, called SH1and SH2, are used for modeling the effects of these two critical inelastic behaviors. The basic framework for these sliding hinges is illustrated in Fig. 2, where the brace element outside the hinges is elastic.

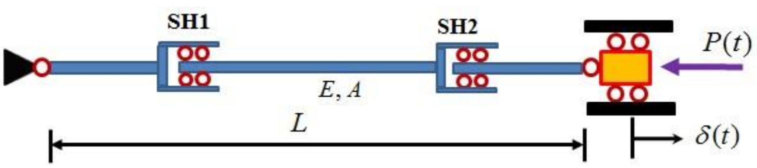


Figure 2. Plastic deformation mechanisms of brace

The internal forces in the sliding hinges must be equal to each other are related to the applied force:

$$P_1(t) = P_2(t) = -P(t) \tag{6}$$

The plastic displacement can be written as:

$$\delta''(t) = \delta_1(t) + \delta_2(t) \tag{7}$$

Where $\delta_1(t)$ is the plastic axial displacement caused by transverse deformation and $\delta_2(t)$ is the plastic axial deformation caused from the "growth effect."

To develop the governing behavior for SH1, it is assumed that the axial displacement $\delta_1(t)$ arises from bending deformation only, and the axial elastic deformation is neglected. SH2 is locked, E is assumed to be infinite and axial displacement is only generated from SH1. Based on the physical theory brace model, the axial inelastic displacement $\delta_1(t)$ is related to the transverse deformation $\Delta(t)$ at any time. The relationship between axial displacement and transverse displacement can be written as:

$$\delta_1(t) = L - \sqrt{L^2 - 4\left[\Delta(t)\right]^2} \tag{8}$$

In the physical theory brace model, $\Delta(t)$ is a function of the axial force $P_1(t)$:

$$\Delta(t) = f_1\left[P_1(t)\right] \tag{9}$$

Here $f_1[P_1(t)]$ represents a set of functions depending on the zone of behavior, where the various zones are based on prior work by Dicleli and Calik (2008). The axial force $P_1(t)$ can be determined using Eq. (8) and (9) as:

$$P_1(t) = g_1\left[\delta(t)\right] \tag{10}$$

Where $g_1[\delta(t)]$ represents a set of functions that are based on the zone of behavior and are inversions of $f_1[P_1(t)]$rewritten in terms of the brace total axial displacement. Thus, Eq. (10) governs the behavior of SH1.

Sliding hinge SH2 is used to simulate the growth effect in the present research. To determine the relationship between the axial force $P_2(t)$ and the associated axial displacement $\delta_2(t)$, SH1 is locked and the axial elastic deformation is ignored. The inelastic behavior, which is only related to the status of point B and the tensile force, is defined by Dicleli and Calik (2008) and the axial force $P_2(t)$ of SH2 can be expressed as:

$$P_2(t) = g_2\left[\delta_2(t)\right] = \frac{AE}{L\delta_{Gn}(t)}\delta_2(t) + P_B(t) \tag{11}$$

Where $P_B(t)$ is the axial force at Point B in Fig. 1(c), the normalized brace growth $\delta_{Gn}(t)$ is expressed as a function of normalized cumulative plastic deformation (Dicleli and Calik 2008) and $g_2[\delta_2(t)]$is a compact representation of the function defining the behavior of SH2.

3.2 Governing Brace Element Equations

Combining Eq. (4), Eq. (5), Eq. (6), Eq. (7), Eq. (10) and Eq. (11), the governing equation set for a brace element within the FAM is:

$$
\begin{cases}
\delta(t) = \dfrac{P(t)L}{AE} + \delta_1(t) + \delta_2(t) \\
P(t) = -P_1(t) \\
P(t) = -P_2(t) \\
P_1(t) = g_1[\delta_1(t)] \\
P_2(t) = g_2[\delta_2(t)]
\end{cases}
\tag{12}
$$

Thus, for a given $\delta(t)$, the five unknowns $P(t)$, $P_1(t)$, $P_2(t)$, $\delta_1(t)$ and$\delta_2(t)$ can be solved using the five equations in the set of Eq. (18).

4. NUMERICAL SIMULATION

4.1 Brace Element Simulation

A steel rectangular tube as shown in Figure 3 is subjected to axial cyclic force *P(t)*. Two sets of geometric parameters shown in Table 1 are given here to demonstrate the FAM. The steel yield stress is 275 Mpa. Figure 4(a) to 4(d) show the comparison of hysteretic curves using the finite element method (FEM) and the FAM. These two figures demonstrate good agreement between these two modeling approaches.

Table 1. The geometric parameters of braces

Sets	*b*	*h*	*t*	*e*	*L*	*E*
1	20	20	2	8	600	2.1*Gpa*
2	25	25	2	10	1200	

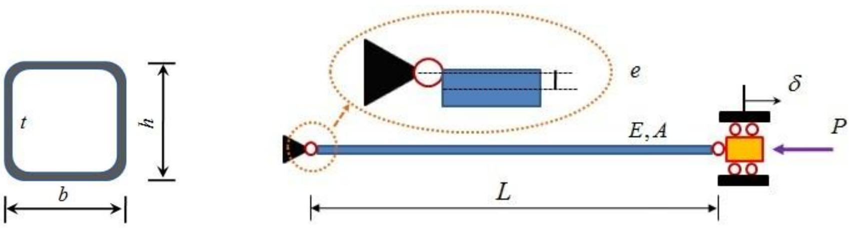
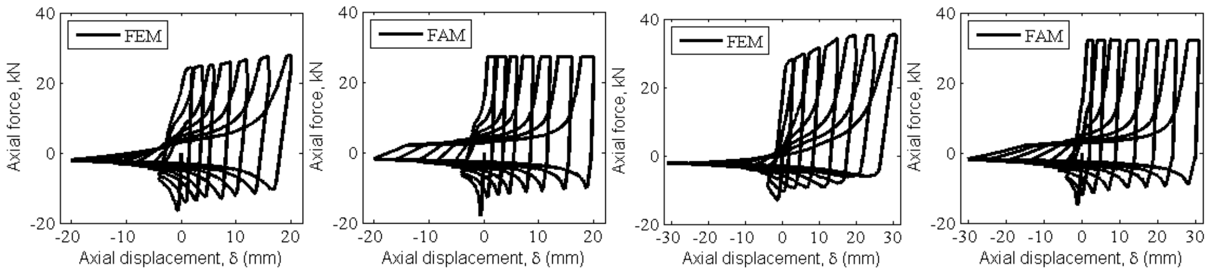


Figure 3. Geometric parameters of brace

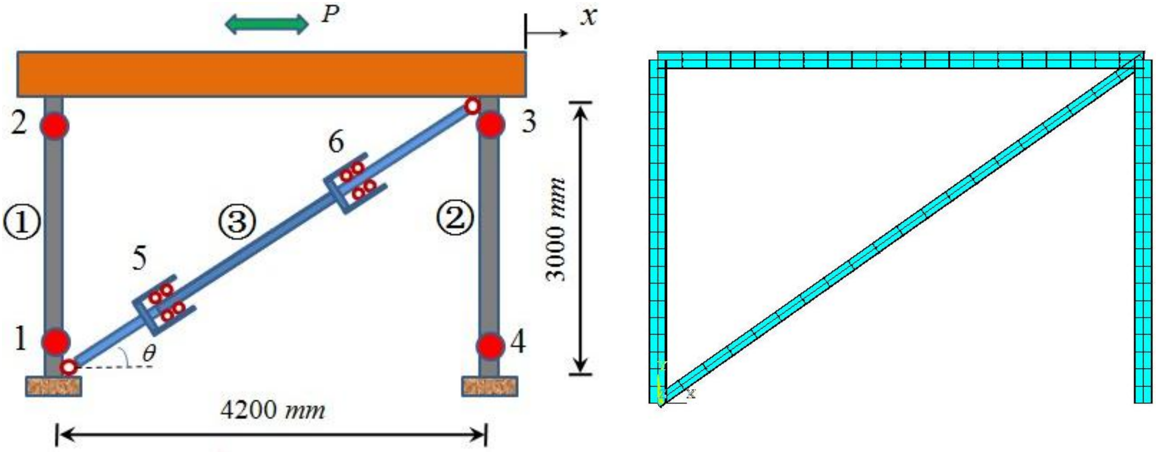


(a) (b) (c) (d)

Figure 4. Hysteretic curves of two braces under axial force using: (a) set 1 with FEM; (b) set 1 with FAM; (c) set 2 with FEM; (d) set 2 with FAM

4.2 Concentrically Braced Frame Simulation

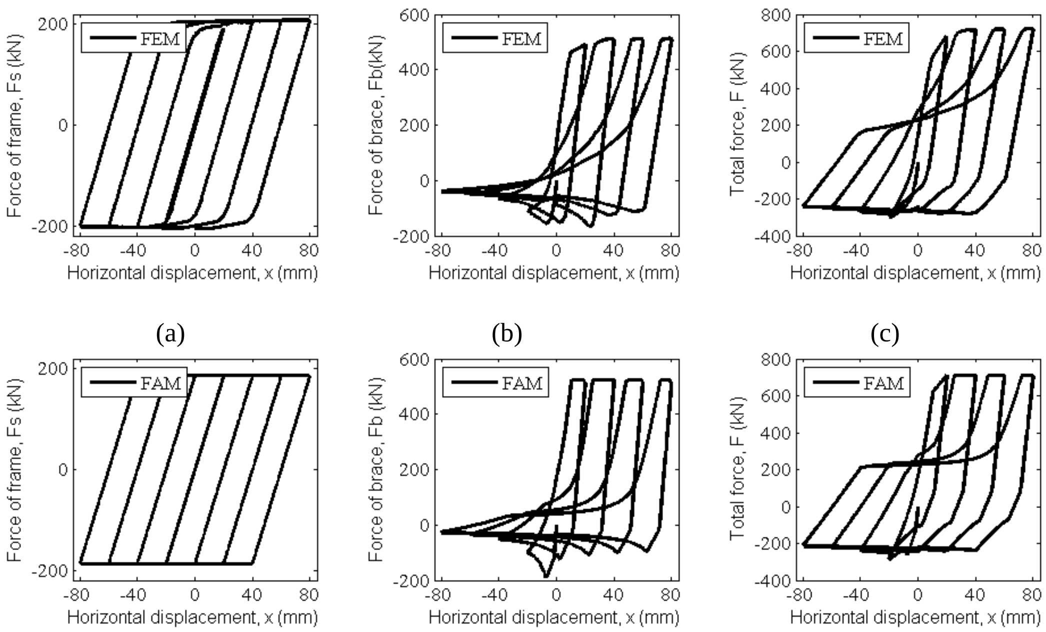
An example one-story frame with a diagonal brace is used to illustrate the FAM method presented here. The number of structural elements and plastic mechanisms are shown in Figure 5(a). The columns are tubes sections 250×250×6*mm*. The stiffness of the beam is idealized as infinite. The section properties of the brace are the same as set 1 in section 4.1. The mesh for the finite element model is shown in Figure 5(b).



(a) (b)

Figure 5. An example of one-storey CBF: (a) FAM model and (b) FEM model

Figures 6(a) to 6(f) show the comparison of the hysteretic behavior of the CBF, brace and frame using the FAM and FEM. It can be seen from these figures that the results using the FAM agree reasonably well with the FEM.



(a) (b) (c)

(b) (e) (f)

Figure 6. Comparison of the hysteretic behavior of the frame, brace and CBF using FAM and FEM: (a) frame (FEM); (a) brace (FEM); (a) CBF (FEM); (a) frame (FAM); (a) brace (FAM); (a) CBF (FAM).

5. CONCLUSIONS

Steel concentrically braced frames have complex inelastic behavior under cyclic loads due to brace yielding and buckling. In this paper, a minor modification was made to an existing physical theory model, making the initial stiffness in compression equal to the initial stiffness in tension so that it would be compatible with the force analogy method. This small modification does not adversely affect the accuracy of the model. Two sliding plastic mechanisms, which simulate axial deformations produced by transverse deformation and the so-called “growth effect,” were used to represent the inelastic brace behavior. The behavior of these sliding mechanisms is defined by tracking the brace axial displacement through a series of discrete zones. Within each zone, the brace force-displacement behavior is defined by a unique function and the displacement is always decomposed into elastic and inelastic components such that the initial stiffness can be used throughout the response to determine the brace force. The use of sliding mechanisms is a new feature for the force analogy method and provides the framework for future implementation of inelastic axial behaviors in other contexts. The resulting brace model was shown to represent the salient features of realistic brace behavior and to provide good agreement with the results using finite element models.

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