Development of 3D Staggered Grid Fourth Order Finite Difference Algorithm for Strong Ground Motion

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SUMMARY:

Development of a 3D fourth order spatial accurate finite-difference algorithm for the simulation of strong ground motion in time domain is documented in this paper. The algorithm is based on staggered grid finite-difference approximation of 3D velocity-stress viscoelastodynamic wave equations with a continuous variable grid size. VGR-stress imaging technique (VGR is acronym for 'vertical grid-size reduction') is used for implementation of free surface boundary condition. The implementation of viscoelastic damping in the time domain finite-difference simulations is validated by comparing the numerically computed phase velocity and quality factor of a viscoelastic homogeneous medium with the phase velocity and quality factor obtained using Futterman's relationship and GMB-EK rheological model. The implementation of viscoelastic damping is also validated by comparing the numerically computed spatial spectral damping with the analytical one. Grid dispersion and numerical stability is studied in details. Continuous variable grid size is used in order to reduce the requirement of computational memory and time. An earthquake source is implemented into the numerical grid based on the moment tensor source formulation.

Keywords: 3D Finite-difference algorithms, fourth order accuracy, viscoelastic damping, strong ground motion simulation, moment tensor source formulation.

1. INTRODUCTION

In the recent past, the simulation of seismic wave field in realistic 3D earth model has been performed by using various numerical methods with the advent of recent advancement in computational facilities. The staggered grid finite-different (FD) scheme was first proposed by Madariaga (1976), which was used subsequently for the simulation of 3D seismic waves by other scientists (Graves, 1996; Pitarka, 1999). The continuous-variable grid size was used by Pitarka (1999) and Oprsal and Zahradnik (2002) for 3D simulation. The incorporation of realistic damping into time-domain simulation was first introduced by Day and Minster (1984) using Pade – approximation. Emmerich and Korn (1987) improved the accuracy and efficiency of this method by considering the rheology of the generalized Maxwell body (GMB), which was later known as GMB-EK rheological model. Carcione et al. (1988) have considered the rheology of generalized Zener body (GZB) and developed an approach in terms of memory variables. Kristek and Moczo (2003) have documented the basic theoretical and algorithmic aspects of memory efficient implementation of realistic damping in time-domain FD simulation with material discontinuities. Kristek et al. (2010) have developed a 3D- fourth order velocity-stress staggered-grid FD algorithm with a spatial discontinuous grid.

In this paper, development of (2, 4) staggered-grid finite-difference algorithm based on velocity-stress viscoelastodynamic 3D-wave equations for a heterogeneous media is documented. The implementation of viscoelastic damping in the time domain finite-difference simulations is validated by comparing the numerically computed phase velocity and quality factor of a viscoelastic homogeneous medium with the phase velocity and quality factor obtained using GMB-EK rheological

model (Emmerich and Korn, 1987; Kristek and Moczo, 2003) and Futterman's relationship (1962). The implementation of viscoelastic damping is also validated by comparing the numerically computed spatial spectral damping with the analytical one. The numerical grid dispersion and stability is studied in details. The accuracy of continuous variable grid size is tested for different grid spacing ratios. Earthquake source is implemented in to numerical grid based on moment tensor source formulation. The snapshots of particle velocity at different times were taken for analysis and verification of implementation of earthquake source in numerical grid.

2. FD APPROXIMATION OF VISCOELASTODYNAMIC 3D WAVE EQUATIONS

In this section, development of a 3D staggered-grid time-domain finite-difference algorithm along with the incorporation of realistic damping is described in brief. The 3D viscoelastic wave equations for heterogeneous medium are given by

$$\rho \frac{\partial U}{\partial t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}$$
(1)

$$\rho \frac{\partial V}{\partial t} = \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}$$
(2)

$$\rho \frac{\partial W}{\partial t} = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}$$
(3)

$$\frac{\partial \sigma_{xx}}{\partial t} = \overline{K}_{u} \left(\frac{\partial U}{\partial x} \right) + \overline{\lambda}_{u} \left(\frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right) - \sum_{l=1}^{m} \left[\overline{Y}_{l}^{\alpha} (\chi_{l}^{xx}) + \overline{Y}_{l}^{\lambda} (\chi_{l}^{yy} + \chi_{l}^{zz}) \right] \qquad \qquad l = 1, 2, ..., m$$
(4)

$$\frac{\partial \sigma_{yy}}{\partial t} = \overline{K}_{u} \left(\frac{\partial V}{\partial y} \right) + \overline{\lambda}_{u} \left(\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} \right) - \sum_{l=1}^{m} \left[\overline{Y}_{l}^{\alpha} \left(\chi_{l}^{yy} \right) + \overline{Y}_{l}^{\lambda} \left(\chi_{l}^{xx} + \chi_{l}^{zz} \right) \right] \qquad l = 1, 2, ..., m$$
(5)

$$\frac{\partial \sigma_{zz}}{\partial t} = \overline{K}_{u} \left(\frac{\partial W}{\partial z} \right) + \overline{\lambda}_{u} \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) - \sum_{l=1}^{m} \left[\overline{Y}_{l}^{\alpha} \left(\chi_{l}^{zz} \right) + \overline{Y}_{l}^{\lambda} \left(\chi_{l}^{xx} + \chi_{l}^{yy} \right) \right] \qquad l = 1, 2, ..., m$$
(6)

$$\frac{\partial \sigma_{xy}}{\partial t} = \bar{\mu}_{u} \left[\left(\frac{\partial U}{\partial y} \right) + \left(\frac{\partial V}{\partial x} \right) \right] - \sum_{l=1}^{m} \left[\overline{Y}_{l}^{\mu} (\chi_{l}^{xy}) \right]$$

$$l = 1, 2, ..., m$$
(7)

$$\frac{\partial \sigma_{xz}}{\partial t} = \bar{\mu}_{u} \left[\left(\frac{\partial U}{\partial z} \right) + \left(\frac{\partial W}{\partial x} \right) \right] - \sum_{l=1}^{m} \left[\overline{Y}_{l}^{\mu} (\chi_{l}^{xz}) \right]$$

$$l = 1, 2, ..., m$$
(8)

$$\frac{\partial \sigma_{yz}}{\partial t} = \bar{\mu}_{u} \left[\left(\frac{\partial V}{\partial z} \right) + \left(\frac{\partial W}{\partial y} \right) \right] - \sum_{l=1}^{m} \left[\overline{Y}_{l}^{\mu} (\chi_{l}^{yz}) \right]$$
 $l = 1, 2, ..., m$ (9)

where U, V and W are the particle velocity components along the x, y and z-axes, respectively. σ_{xx} , σ_{yy} and σ_{zz} are normal stress components and σ_{xy} , σ_{xz} , and σ_{yz} are shear stress components. \overline{K}_u , $\overline{\lambda}_u$ and $\overline{\mu}_u$ are modified elastic parameters and \overline{Y}_1^{α} , \overline{Y}_1^{λ} and \overline{Y}_1^{β} are the modified anelastic coefficients. χ_1^{xx} , χ_1^{yy} , χ_1^{zz} and χ_1^{xy} , χ_1^{xz} , χ_1^{yz} are the anelastic functions for normal and shear stress components, respectively. λ_u is the unrelaxed Lame's parameter and K_u is equal to sum of λ_u and twice of μ_u . $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, $\frac{\partial}{\partial z}$, and $\frac{\partial}{\partial t}$ are the differential operators and 'm' is the number of relaxation frequency. Four relaxation frequencies are used for the present study. Figure 1 show the staggering technique, where normal stress components σ_{xx} , σ_{yy} and σ_{zz} , unrelaxed elastic parameters K_u and λ_u , anelastic coefficients Y^{α} and Y^{λ} and anelastic functions χ^{xx} , χ^{yy} and χ^{zz} are defined at the center of the grid. The particle velocity components U, V and W, and density ρ are defined at the center of the faces. Shear stresses σ_{xy} , σ_{xz} , σ_{yz} , unrelaxed modulus of rigidity μ_u , anelastic coefficient Y^{β} , and anelastic functions χ^{xy} , χ^{xz} , χ^{yz} are defined at the mid of the edges as shown in figure 1.



Figure 1. Staggering technique for 3D-wave modeling with fourth order spatial accuracy

The material independent anelastic functions has been computed at four relaxation frequencies using the following equations (Kristek and Moczo, 2003; Moczo et al., 2005).

$$(\chi_l^{xx})^{n+\frac{1}{2}} = \frac{2 - \Delta t\omega_l}{2 + \Delta t\omega_l} (\chi_l^{xx})^{n-\frac{1}{2}} + \frac{2\Delta t\omega_l}{2 + \Delta t\omega_l} \frac{\partial U}{\partial x}$$

$$(\chi_l^{xy})^{n+\frac{1}{2}} = \frac{2 - \Delta t\omega_l}{2 + \Delta t\omega_l} (\chi_l^{xy})^{n-\frac{1}{2}} + \frac{2\Delta t\omega_l}{2 + \Delta t\omega_l} \left[\frac{1}{2} \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \right]$$

$$l = 1, 2, ..., m$$

$$(10)$$

Similarly, the anelastic functions χ_l^{yy} , χ_l^{zz} and χ_l^{xz} , χ_l^{yz} can be computed. In order to compute the input parameters namely unrelaxed modulus and anelastic coefficients to the FD numerical grid, phase velocity and quality factor at reference frequency (ω_r) is used. The P-wave anelastic coefficients Y_l^{α} , l = 1, 2, ..., m and S-wave anelastic coefficients Y_l^{β} , l = 1, 2, ..., m have been computed with the help of following equations using Futterman's equation (Futterman, 1962) and least square technique for optimization.

$$Q_{\alpha}^{-1}(\omega_{k}) = \sum_{l=1,2,...,m}^{m} \frac{\omega_{l}\omega_{k} + Q_{\alpha}^{-1}(\omega_{k})\omega_{l}^{2}}{\omega_{k}^{2} + \omega^{2}} Y_{l}^{\alpha} \qquad l = 1,2,...,m \qquad k = 1,2,...,2m-1$$
(12)

$$Q_{\beta}^{-1}(\omega_{k}) = \sum_{l=1}^{l=1}^{m} \frac{\omega_{l}\omega_{k} + Q_{\beta}^{-1}(\omega_{k})\omega_{l}^{2}}{\omega_{l}^{2} + \omega_{k}^{2}} Y_{l}^{\beta} \qquad \qquad l = 1, 2, ..., m \qquad k = 1, 2, ..., 2m - 1$$
(13)

The logarithmically distributed considered values of relaxation frequencies ω_1 , ω_2 , ω_3 and ω_4 are 0.1257, 1.257, 12.57 and 125.7, respectively. The Q(ω_k) values obtained using the Q at reference frequency and Futterman's equation has been used for optimization. The ω_k values are also logarithmically distributed. ω_k is defined at ω_l as well as at the mid of two consecutive ω_l values. Further, $\omega_{k=1} = \omega_{l=1}$ and $\omega_{k=2m-1} = \omega_{l=m}$.

The anelastic coefficients Y_1^{λ} , l = 1, 2, ..., m were obtained using the following relationship

$$Y_1^{\lambda} = \frac{K_u Y_1^{\alpha} - 2\mu_u Y_1^{\beta}}{\lambda_u}$$
(14)

Further, the unrelaxed elastic parameters K_u and μ_u for P-wave and S-wave have been obtained using phase velocity of P-wave (V_{P,ω_r}) and S-wave (V_{S,ω_r}) at reference frequency $(f_r = 1Hz)$ and the following equations (Moczo et al., 1997).

$$\mu_{u} = \rho V_{S,w_{r}}^{2} \frac{R + \theta_{1}}{2R^{2}} \qquad \qquad K_{u} = \rho V_{P,w_{r}}^{2} \frac{R + \theta_{1}}{2R^{2}} \qquad \qquad R = \sqrt{\theta_{1}^{2} + \theta_{2}^{2}}$$
(15)

where

$$\theta_{1} = 1 - \sum_{l=1}^{m} \left[Y_{l} \frac{1}{1 + {\binom{\omega_{r}}{\omega_{l}}}^{2}} \right]; \qquad \theta_{2} = \sum_{l=1}^{m} \left[Y_{l} \frac{\frac{\omega_{r}}{\omega_{l}}}{1 + {\binom{\omega_{r}}{\omega_{l}}}^{2}} \right] \qquad l = 1, 2, ..., m$$
(16)

In case of P-wave Y_l will be replaced by Y_l^{α} and that in case of S-wave will be replaced by Y_l^{β} in equation (16). The unrelaxed Lame's parameter λ_u is obtained using the following relationship

$$\lambda_{\rm u} = K_{\rm u} - 2\mu_{\rm u} \tag{17}$$

The modified elastic parameters \overline{K}_u , $\overline{\mu}_u$ and $\overline{\lambda}_u$ are computed using the following relationships.

$$\overline{K}_{u} = K_{u} \left[1 + \sum_{l=1}^{m} G_{1l} Y_{l}^{\alpha} \right], \quad \overline{\mu}_{u} = \mu_{u} \left[1 + \sum_{l=1}^{m} G_{1l} Y_{l}^{\beta} \right], \quad \overline{\lambda}_{u} = \lambda_{u} \left[1 + \sum_{l=1}^{m} G_{1l} Y_{l}^{\lambda} \right] \qquad l = 1, 2, \dots, m \quad (18)$$

Similarly, the modified anelastic parameters \overline{Y}_1^{α} , \overline{Y}_1^{β} and \overline{Y}_1^{λ} are computed using the following relationships.

$$\overline{Y}_{1}^{\alpha} = G_{2l}K_{u}Y_{1}^{\alpha}, \qquad \overline{Y}_{1}^{\beta} = 2G_{2l}\mu_{u}Y_{1}^{\beta}, \qquad \overline{Y}_{1}^{\lambda} = G_{2l}\lambda_{u}Y_{1}^{\lambda}, \qquad l = 1, 2, ..., m$$
(19)

The constants G_{11} and G_{21} are given by.

$$G_{11} = \frac{\Delta t \omega_l}{2 - \Delta t \omega_l}$$
 and $G_{21} = \frac{2}{2 - \Delta t \omega_l}$ $l = 1, 2, ..., m$ (20)

The effective value of the unrelaxed modulus of rigidity μ_u at the center of the side is obtained using harmonic mean of μ_u at the node points in order to incorporate the material discontinuity. The λ_u , and K_u at the centre of grid cell are obtained using harmonic mean and effective values of the density ρ at the center of the face are obtained using arithmetic mean of ρ at the node points (Moczo et al., 2000). The 3D-viscoelastodynamic wave eqns. (1-9) are discretized using an explicit 4th- order in space, 2ndorder in time finite difference scheme (Graves, 1996; Pitarka, 1999 and Moczo et al., 2000, Narayan and Kumar, 2008). In order to avoid the soil thickness discrepancy arising due to the use of images of stress components across the free surface (Levander 1988, Graves 1996), VGR-stress imaging technique proposed by Narayan and Kumar (2008) was implemented at the free surface. Both the sponge boundary (Israeli and Orszag 1981) and absorbing boundary condition of Clayton and Engquist (1977) were implemented on the model edges to avoid the edge reflections.

3. VALIDATION OF DEVELOPED ALGORITHM

3.1 Numerical grid dispersion

The accuracy of developed algorithm is verified by comparing the numerical and analytical dispersion curves for P-wave for a homogeneous unbounded elastic model. The P-wave velocity, S-wave velocity

and density for the homogeneous model were taken as 5150 ms⁻¹, 3090 ms⁻¹ and 2.8 gcm⁻³, respectively. Ricker wavelet with 4.0 Hz dominant frequency was used as an excitation function. The seismic responses computed at distances of 500 m and 580 m, were used for the computation of phase velocity. Figure (2) shows the comparison of normalized phase velocity $\alpha^{\text{grid}}/\alpha$ for P wave versus sampling ratio s (= h/ λ) with the same obtained by analytical relation given by Moczo et al. (2000). Analysis of figure (2) reveals that the error in the numerically computed dispersion curve is within the permissible limit when the number of grids per shortest wavelength is more than 6.



Figure 2. A comparison of numerical and analytical grid dispersion curves for the P-wave

3.2 Anelastic damping

In order to verify the accuracy of implementation of anelastic damping in time domain FD simulation, seismic responses of a homogeneous viscoelastic unbounded model were computed for different values of quality factor at reference frequency. The velocities and quality factors for P- and S-waves at reference frequency (1.0 Hz) and computed unrelaxed moduli are given in table 1. A horizontal line source was inserted into numerical grid using a number of point sources at the same depth, and the responses were computed on a vertical array.



Figure 3. Comparison of phase velocity and quality factor of P-wave computed numerically with the same computed by the GMB-EK model and theoretical Futterman relation

First, FFT of two traces at an offset of Δx were used to find out the phase difference ($\Delta \phi$) and spectral amplitude (v) and then were used to compute the phase velocity and spectral quality factors using the following relations.

$$V(\omega) = \frac{\omega \Delta x}{\Delta \phi(\omega)} \qquad (21) \qquad \qquad \frac{1}{Q(\omega)} = -\frac{2 V(\omega)}{\omega \Delta x} [\ln|v(x + \Delta x, \omega)| - \ln|v(x, \omega)|] \qquad (22)$$

Figure (3) shows the comparison of numerically obtained phase velocity and quality factor for P-wave with the same obtained using GMB-EK model and Futterman's relation. The good agreement between numerical and analytical results proves the authenticity of the present algorithm.

	Velocity at reference		Quality	factor at	ρ	Unrelaxed moduli		
Model	frequency		reference frequency		(g/cc)			
	$V_p (ms^{-1})$	$V_{s} (ms^{-1})$	Qp	Qs		µu(GPa)	$\lambda_u(GPa)$	K _u (GPa)
MVQ1	5000	3000	50	50	2.8	26.73	20.79	74.25
MVQ2	5000	3000	100	100	2.8	25.95	20.18	72.08

Table 1: The velocities and quality factors for P- and S-waves and unrelaxed moduli.

3.3 Spatial spectral damping

In order to further confirm the accuracy of implementation of viscoelastic damping in time domain FD simulation the spatial spectral damping in the form of spectral amplitude ratio (A/A_0) was computed for different distance travelled. Seismic responses were computed at difference locations using the same source and receiver configuration and model parameters as in previous the case.



Figure 4. (a) Time response at reference receiver (continuous line) and responses at distance of 450 m (dash line) and 3000 m (dash-dot line) and their respective spectral amplitudes, (b) Comparison of numerically computed spectral amplitude ratios with the same computed by analytical relation for S-wave

Figure 4(a) shows the response at reference receiver point (continuous line) and responses at distance of 450 m (dash line) and 3000 m (dash-dot line). It also shows the respective spectral amplitudes. The

spectral amplitude ratios were computed for 450 m and 3000 m distance travelled. The analytical spectral amplitude ratio was computed with the help of well known equation $\frac{A}{A_o} = e^{-\alpha x}$, using phase velocity and quality factor obtained from GMB-EK rheological model. Figure 4(b) shows the comparison of numerically and analytically computed spectral amplitude ratios for S-wave. The excellent matching betweem analytical and numerical spectral amplitude ratios also confirm the accuracy of procedure of implementation of viscoelastic damping.

3. PERFORMANCE OF NON-UNIFORM GRID MODELS

The computation of seismic response of a model containing a lateral geometrical variations or a very soft soil layer with uniform grid, requires very large computational memory and time. This can be reduced by using a continuous grid mesh with a variable grid size. The performance of grid spacing ratios as 1:2, 1:2.67, 1:4 and 1:5 in one direction only is analyzed. The responses of homogeneous elastic model (11 km \times 8 km \times 8 km) with V_P=5000 m/s, V_S=3000 m/s and ρ =2.8 g/cc was computed with larger size of a grid cell as 40 m and different smaller size of a grid cell as 20 m, 15 m, 10 m, and 8 m along x-direction. In the x-direction, the size of the grid cell was 40 m up to a distance of 7 km and after that variable (20 m, 15 m, 10 m and 8 m). The number of grids in small grid zone were increased accordingly to keep the model size same. A fixed grid size as 40 m was used along y and zdirections. Seismic responses on a vertical array at a distance of 200 m from source were computed using plane wave front normal to x-axis. The distance of grid discontinuity zone from the source is 6.0 km. The seismic response using uniform grid size of 40 m was also computed for the comparison. Figure 5(a) shows the comparison of responses computed using homogeneous grid size and various variable grid size for P-wave (upper) and S-wave (lower) at receiver point in the centre of vertical array. In figure 5(a), the signals reflected from the grid discontinuity zone are shown inside circle, which are negligible for P-wave and very minutely visible for S-wave. The reflected signals (inside circle in figure 5a) from grid discontinuity zone corresponding to grid spacing ratios as 1:2, 1:2.67, 1:4 and 1:5 are shown in figure 5(b) with a 100 times larger amplitude scale. The computed average spectral reflectance is 0.64, 0.69, 0.7, and 0.72 percent for P-wave and 0.82, 0.84, 0.92, and 0.94 percent for S-wave for grid spacing ratios 1:2, 1:2.67, 1:4 and 1:5, respectively.



Figure 5 a & b. Computed responses of a homogeneous half space model for P and S-waves and only reflected signals from the grid discontinuity zone on scale 100 times larger scale, respectively

P-wave responses were also computed for difference grid spacing ratios but keeping the discontinuous grid zone between the source and receiver array. The distance of discontinuous grid zone and receiver

array was 600 km and 1.8 km from the source. Figure 6 shows the comparison of the computed responses using grid spacing ratios as 1:2, 1:2.67, 1:4 and 1:5 with the response computed using uniform grid at receiver point in the center of the vertical array. Analysis of figure 6 reveals that amplitude discrepancy arising due to the presence of discontinuous zone between the source and receiver is almost insignificant in all the cases. The percentage root mean square error with respect to the homogeneous grid size is 0.75, 0.79, 0.91 and 0.96 for grid spacing ratios 1:2, 1:2.67, 1:4 and 1:5, respectively. A good resemblance of responses for grid spacing ratios up to 5 with the response computed using uniform grid reveals that the maximum grid spacing ratio up to of the order of 5 can be used.



Figure 6. Comparison of computed responses of a homogeneous half-space model using different grid spacing ratio with response computed using uniform grid

4. POINT SOURCE IMPLEMENTATION

A point source was implemented into the numerical grid based on moment tensor formulation (Coutant et. al., 1995; Pitarka, 1999; Narayan, 2001). In moment tensor source formulation, the ratio of moment tensor component to volume of grid was used as a stress tensor component (Aki and Richard, 1980).

$$(\sigma_{xx})_{i+\frac{1}{2}i+\frac{1}{2}k+\frac{1}{2}}^{n} = (\sigma_{xx})_{i+\frac{1}{2}i+\frac{1}{2}k+\frac{1}{2}}^{n} - \Delta t \, \dot{M}_{xx}(t)/V \qquad (\sigma_{xy})_{i,k+\frac{1}{2}}^{n} = (\sigma_{xy})_{i,k+\frac{1}{2}}^{n} - \Delta t \, \dot{M}_{xy}(t)/V \qquad (23)$$

$$(\sigma_{yy})_{i+\frac{1}{2}j+\frac{1}{2}k+\frac{1}{2}}^{n} = (\sigma_{yy})_{i+\frac{1}{2}j+\frac{1}{2}k+\frac{1}{2}}^{n} - \Delta t \, \dot{M}_{yy}(t)/V \qquad (\sigma_{xz})_{i,j+\frac{1}{2}k}^{n} = (\sigma_{xz})_{i,j+\frac{1}{2}k}^{n} - \Delta t \, \dot{M}_{xz}(t)/V \qquad (24)$$

$$(\sigma_{zz})_{i+\frac{1}{2}j+\frac{1}{2}k+\frac{1}{2}}^{n} = (\sigma_{zz})_{i+\frac{1}{2}j+\frac{1}{2}k+\frac{1}{2}}^{n} - \Delta t \, \dot{M}_{zz}(t)/V \qquad (\sigma_{yz})_{i+\frac{1}{2}j,k}^{n} = yz)_{i+\frac{1}{2}j,k}^{n} - \Delta t \, \dot{M}_{yz}(t)/V \qquad (25)$$

$$\sum_{j=1}^{n} \sum_{i+\frac{1}{2},j+\frac{1}{2},k+\frac{1}{2}}^{n} - \Delta t \, \dot{M}_{zz}(t)/V \qquad (\sigma_{yz})_{i+\frac{1}{2},j,k}^{n} = yz)_{i+\frac{1}{2},j,k}^{n} - \Delta t \, \dot{M}_{yz}(t)/V \qquad (25)$$

where $\dot{M}_{xx}(t)$, $\dot{M}_{yy}(t)$, $\dot{M}_{zz}(t)$, $\dot{M}_{xy}(t)$, $\dot{M}_{xz}(t)$, and $\dot{M}_{yz}(t)$ are the time derivatives of moment tensor components and V is the volume of a finite difference cell. The radiation patterns were computed for double couple point shear dislocation source with focal mechanism as dip 90°, rake 0° and strike 0° in a homogeneous model of size $500 \times 500 \times 500$ grids. The source was kept at the center of considered model. The snapshots were computed after 2.4 s. The grid size, time step and dominant frequency were taken as 60 m, 0.0055 s, and 4.0 Hz, respectively. Figures 7(a), 7(b) and, 7(c) shows the U, V, and W components of radiation patterns, in XY, XZ and YZ planes, respectively. In figure (7) different lobes of P-wave, SV-wave and SH-wave are visible in different components of radiation patterns, which are found to be in agreement with the used focal mechanism. According to the used focal parameters only σ_{xy} component of stress will be effective. Hence, from the wave equations, the amplitude of SH-wave should be more dominating than P-wave and SV-wave, which is visible in the radiation patterns. Maximum positive amplitude in the snapshots is shown in orange and yellow and negative amplitude is shown in blue color.



Figure 7 a, b &c. Snapshots for U, V and W components in XY, XZ and YZ- planes, respectively

5. DISCUSSION AND CONCLUSIONS

A new (2, 4) staggered-grid finite-difference algorithm has been developed using 3D viscoelastodynamic wave equations based on GMB-EK rheological model in order to incorporate the realistic damping in time domain simulation in a heterogeneous medium. The excellent matching of numerically computed phase velocity and quality factor of a viscoelastic medium with the same computed analytically based on GMB-EK rheological model (Emmerich and Korn, 1987; Kristek and Moczo, 2003) and Futterman's relationship (1962) confirmed the accuracy of procedure of implementation of viscoelastic damping in the time domain finite-difference simulations. The spectral amplitude ratios were computed for different distances and compared with the analytically computed spectral amplitude ratios using GMB-EK rheological model. A good matching of numerical and analytical spectral amplitude ratios for different distances further confirms the accuracy of procedure. To study grid dispersion, the grid dispersion curve for P-wave was computed numerically and compared with the grid dispersion curve obtained by using analytical solution given by Moczo et al. (2000). The good agreement between numerically and analytically computed dispersion curves revealed the requirement of 5-6 grid points per wavelength, which is in agreement with Moczo et al. (2000). Based on iterative numerical experiments, it was inferred that the required stability condition is the same as reported by Moczo et al. (2000). In order to reduce the computational time and memory in case of a model containing very soft soil layer or geometry of local site, continuous variable grid size is used. The responses were computed with grid spacing ratios 1:2, 1:2.67, 1:4 and 1:5. The computed average spectral reflectance is 0.64, 0.69, 0.7, and 0.72 percent for P-wave and 0.82, 0.84, 0.92, and 0.94 for S-wave for grid spacing ratios 1:2, 1:2.67, 1:4 and 1:5, respectively. The percentage root mean square error with respect to the homogeneous grid size is 0.75, 0.79, 0.91 and 0.96 for grid spacing

ratios 1:2, 1:2.67, 1:4 and 1:5, respectively, for P-wave. These results reveal that the error from the grid discontinuity zone less than 1% even in case of grid spacing ratio 5. Hence, the maximum grid spacing ratio of the order of 5 can be used (Pitarka, 1999 and Oprsal and Zahradnik, 2002). A double couple point source was implemented in to computational grid using moment tensor source formulation (Pitarka, 1999; Narayan, 2001). The radiation patterns of P and S-waves are in agreement with the basic theory of source mechanism, which proves the accuracy of procedure for source implementation.

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