Numerical Simulation on the Elastic-Plastic Wave Propagation in Framed Structures

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SUMMARY
The near-field strong motion has the characteristic of pulse-type waveform and long pulse period, which causes the middle-story failure of the framed structures in many great earthquakes. This new failure pattern is difficult to explain well based on the traditional vibration method, which maybe attributes to the initial wave effect. In order to explain the middle-story failure of the framed structures impacted by near-field strong motion, elastic-plastic wave propagation in framed structures is investigated by finite difference method. Firstly, the elastic wave propagation in framed structures by finite difference method is studied. The consideration of longitudinal, torsional and flexural waves in rigid frames yields the finite difference equation for wave propagation in framed structures, The case study shows that the finite difference method can simulate the elastic wave propagation in framed structures effectively. Then, a double-deck difference equation for the elastic-plastic wave propagating in framed structures is structured by a new finite difference method, which only discretizes in the spatial direction and regards the variable quantity in the time direction as acceleration. It is assumed that the constitutive relations of material are independent in three directions, where bilinear restoring force characteristics and Massing rule are adopted. Lastly, the case study shows that the new finite difference method can simulate the plastic wave propagation in framed structures effectively, which will benefit the investigation of the damage mechanism of structures impacted by the near-field strong earthquake motion.

Keywords: near-field, elastic and plastic waves, rigid frame, finite difference method Wilson-θ method

1. INTRODUCTION

The scientific community had paid more and more attention to the near-field problems since the Port Hueneme Earthquake. The near field earthquake is a very special kind of earthquake whose characters are very different from the common far field earthquake because of the focal mechanism, the relationship between the direction of fault rupture and field, the relative orientation of breaking plane. The most notable characters of near-field earthquake is pulse type ground motion which is caused by the orientation effect and sliding effect (S. Li & Xie, 2007; X. L. Li & Zhu, 2004a, 2004b; Tao, Fengxin, & Yushan, 2006).According to the peak ground acceleration, peak ground velocity, and ground displacement around the fault of Chi-Chi Earthquake, Shin(Shin et al., 2001) and Wang(G. Q. Wang, Zhou, Zhang, & Igel, 2002; J. H. Wang, Huang, Chen, Hwang, & Chang, 2002) showed that the main influence factors of acceleration response spectrum are seismic source and field effect; the former has an effect on the low frequency spectrum and the latter influences the high frequency spectrum.

The damage patterns of structures in the near field are very special. The most representative building is a seven floor hotel in the California, which was destroyed in the San Fernando Earthquake and Northridge Earthquake. The main damage area is the forth floor, which is still hard to explain the mechanism of special damaged patterns by vibration-based methods.(Ivanovice, Trifunac, Novikova, Gladkov, & Todorovska, 2000; Trifunac, Ivanovice, & Todorovska, 2001a, 2001b). From the view of waves propagation, Todorovska et al (Todorovska & Trifunac, 1990) investigated the P-delta effects in the soft first floor and Kolher(Kohler, Heaton, & Bradford,

2007) suggested that the system identification from wave-propagation effects should be included into the theoretical dynamic analysis simulations of the building's response. Gicev and Trifunac(Gicev & Trifunac, 2006a, 2006b) introduced a one-dimensional continuum model for the building with bilinear constitutive relationship and the nonlinear response was analyzed by the finite difference method.

In order to explain the middle-story failure of the framed structures impacted by near-field strong motion further, elastic-plastic wave propagation in framed structures is investigated by finite difference method in this paper. The consideration of longitudinal, torsional and flexural waves in rigid frames yields the finite difference equation for wave propagation in framed structures, The case study shows that the finite difference method can simulate the elastic wave propagation in framed structures effectively. Then, a double-deck difference equation for the elastic-plastic wave propagating in framed structures is structured by a new finite difference method, which only discretizes in the spatial direction and regards the variable quantity in the time direction as acceleration. It is assumed that the constitutive relations of material are independent in three directions, where bilinear restoring force characteristics and Massing rule are adopted. Lastly, the case study is taken to validate the new finite difference method.

2.MODLE AND EQUATION

2.1.Global and local coordinate system

For the rigid frame structures, the global coordinate system and the local coordinate system are defined as shown in Fig. 1. the member is expressed by node codes and the global coordinated system is (x, y, z). In the local coordinated systems $(\overline{x}, \overline{y}, \overline{z})^{JK}$, the $\overline{x}^{JK}$-axis is coincided with the shaped mandrel of member bar and the direction is from J to K; $\overline{y}^{JK}$-axis and $\overline{z}^{JK}$-axis are coincided with two principal axes of cross section, which means that plane $\overline{x}^{JK}$-$\overline{y}^{JK}$ and plane $\overline{x}^{JK}$-$\overline{z}^{JK}$ are two bending principal plane. It is assumed that the shear force center is coincided with the cross section center of figure in order to insure that the curvature moment and twist motion are independent.

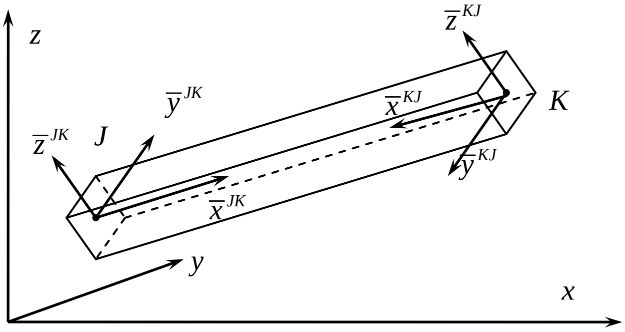


Figure 1. Global and local coordinate system

We consider three kinds of waves, longitudinal wave, torsional wave, and flexural wave in a limited member bar. As a spatial member bar there are six basic parameter for every point of the object, we define that $\overline{u}_{\overline{x}}^{JK}$, $\overline{v}_{\overline{y}}^{JK}$, $\overline{w}_{\overline{z}}^{JK}$ are the linear displacement of $\overline{x}^{JK}$, $\overline{y}^{JK}$, $\overline{z}^{JK}$ axis and $\overline{\theta}_{\overline{x}}^{JK}$, $\overline{\phi}_{\overline{y}}^{JK}$, $\overline{\psi}_{\overline{z}}^{JK}$ are the angular displacement which are caused by the torsional moment in $\overline{x}^{JK}$-axis, the bending moment in $\overline{y}^{JK}$-axis ,and $\overline{z}^{JK}$-axis. Correspondingly, there are six force $\overline{F}_{\overline{x}}^{JK}$, $\overline{F}_{\overline{y}}^{JK}$, $\overline{F}_{\overline{z}}^{JK}$, $\overline{M}_{\overline{x}}^{JK}$, $\overline{M}_{\overline{y}}^{JK}$, and $\overline{M}_{\overline{z}}^{JK}$. The relationship of generalized displacement vector and force vector in the local coordinated systems could be expressed as:

$$\overline{\mathbf{U}}^{JK} = \mathbf{H}\overline{\mathbf{U}}^{JK} \tag{2.1}$$
$$\overline{\mathbf{F}}^{JK} = \mathbf{H}\overline{\mathbf{F}}^{KJ} \tag{2.2}$$

where $\overline{\mathbf{U}}^{JK} = [\overline{u}_{\overline{x}}^{JK} \quad \overline{v}_{\overline{y}}^{JK} \quad \overline{w}_{\overline{z}}^{JK} \quad \overline{\theta}_{\overline{x}}^{JK} \quad \overline{\phi}_{\overline{y}}^{JK} \quad \overline{\psi}_{\overline{z}}^{JK}]^T$, $\overline{\mathbf{F}}^{JK} = [\overline{F}_{\overline{x}}^{JK} \quad \overline{F}_{\overline{y}}^{JK} \quad \overline{F}_{\overline{z}}^{JK} \quad \overline{M}_{\overline{x}}^{JK} \quad \overline{M}_{\overline{y}}^{JK} \quad \overline{M}_{\overline{z}}^{JK}]^T$,

$\mathbf{H} = diag\{-1, \quad -1, \quad 1, \quad -1, \quad -1, \quad 1\}$.The relationship of $\overline{\mathbf{U}}^{JK}$ and $\overline{\mathbf{F}}^{JK}$ between the global and local coordinated systems can be expressed as

$$\mathbf{U}^{JK} = \mathbf{K}^{JK}\overline{\mathbf{U}}^{JK} \tag{2.3}$$

$$\mathbf{F}^{JK} = \mathbf{K}^{JK}\overline{\mathbf{F}}^{JK} \tag{2.4}$$

2.2.Definite solution problem of member bar movement

The equations of motion for the single member bar IJ of three kind of waves and the relationship of generalized displacement and generalized force are

$$\frac{\partial^2 \overline{u}_{\overline{x}}^{JK}}{\partial t^2} = \frac{E^{JK}}{\rho^{JK}} \frac{\partial^2 \overline{u}_{\overline{x}}^{JK}}{\partial(\overline{x}^{JK})^2} \tag{2.5}$$

$$\frac{\partial^2 \overline{\theta}_{\overline{x}}^{JK}}{\partial t^2} = \frac{G^{JK}}{\rho^{JK}} \frac{\partial^2 \overline{\theta}_{\overline{x}}^{JK}}{\partial(\overline{x}^{JK})^2} \tag{2.6}$$

$$\begin{cases} S^{JK} G^{JK} \kappa_{\overline{y}}^{JK} \left(\dfrac{\partial \overline{v}_{\overline{y}}^{JK}}{\partial \overline{x}^{JK}} - \overline{\psi}_{\overline{z}}^{JK} \right) = -E^{JK} I_{\overline{z}}^{JK} \dfrac{\partial^2 \overline{\psi}_{\overline{z}}^{JK}}{\partial(\overline{x}^{JK})^2} + \rho^{JK} I_{\overline{z}}^{JK} \dfrac{\partial^2 \overline{\psi}_{\overline{z}}^{JK}}{\partial t^2} \\ S^{JK} G^{JK} \kappa_{\overline{y}}^{JK} \left(\dfrac{\partial^2 \overline{v}_{\overline{y}}^{JK}}{\partial(\overline{x}^{JK})^2} - \dfrac{\partial \overline{\psi}_{\overline{z}}^{JK}}{\partial \overline{x}^{JK}} \right) = \rho^{JK} S^{JK} \dfrac{\partial^2 \overline{v}_{\overline{y}}^{JK}}{\partial t^2} \end{cases} \tag{2.7}$$

$$\begin{cases} S^{JK} G^{JK} \kappa_{\overline{z}}^{JK} \left(\dfrac{\partial \overline{w}_{\overline{z}}^{JK}}{\partial \overline{x}^{JK}} - \overline{\phi}_{\overline{y}}^{JK} \right) = -E^{JK} I_{\overline{z}}^{JK} \dfrac{\partial^2 \overline{\phi}_{\overline{y}}^{JK}}{\partial(\overline{x}^{JK})^2} + \rho^{JK} I_{\overline{z}}^{JK} \dfrac{\partial^2 \overline{\phi}_{\overline{y}}^{JK}}{\partial t^2} \\ S^{JK} G^{JK} \kappa_{\overline{z}}^{JK} \left(\dfrac{\partial^2 \overline{w}_{\overline{z}}^{JK}}{\partial(\overline{x}^{JK})^2} - \dfrac{\partial \overline{\phi}_{\overline{y}}^{JK}}{\partial \overline{x}^{JK}} \right) = \rho^{JK} S^{JK} \dfrac{\partial^2 \overline{w}_{\overline{z}}^{JK}}{\partial t^2} \end{cases} \tag{2.8}$$

$$\begin{cases} \overline{F}_{\overline{x}}^{JK} = E^{JK} S^{JK} \dfrac{\partial \overline{u}_{\overline{x}}^{JK}}{\partial \overline{x}^{JK}} \\ \overline{F}_{\overline{y}}^{JK} = S^{JK} G^{JK} \kappa_{\overline{y}}^{JK} \left(\dfrac{\partial \overline{v}^{JK}}{\partial \overline{x}^{JK}} - \overline{\psi}^{JK} \right) \\ \overline{F}_{\overline{z}}^{JK} = S^{JK} G^{JK} \kappa_{\overline{z}}^{JK} \left(\dfrac{\partial \overline{w}^{JK}}{\partial \overline{x}^{JK}} - \overline{\phi}^{JK} \right) \\ \overline{M}_{\overline{x}}^{JK} = G^{JK} I_p^{JK} \dfrac{\partial \overline{\theta}_{\overline{x}}^{JK}}{\partial \overline{x}^{JK}} \\ \overline{M}_{\overline{y}}^{JK} = E^{JK} I_{\overline{y}}^{JK} \dfrac{\partial \overline{\phi}^{JK}}{\partial \overline{x}^{JK}} \\ \overline{M}_{\overline{z}}^{JK} = E^{JK} I_{\overline{z}}^{JK} \dfrac{\partial \overline{\psi}^{JK}}{\partial \overline{x}^{JK}} \end{cases} \tag{2.9}$$

Equations (5)~(9) could be written in matrix form

$$\mathbf{A}^{JK} \frac{\partial^2 \overline{\mathbf{U}}^{JK}}{\partial(\overline{x}^{JK})^2} + \mathbf{B}^{JK} \frac{\partial \overline{\mathbf{U}}^{JK}}{\partial \overline{x}^{JK}} = \mathbf{C}^{JK} \frac{\partial^2 \overline{\mathbf{U}}^{JK}}{\partial t^2} + \mathbf{D}^{JK} \overline{\mathbf{U}}^{JK} \tag{2.10}$$

$$\overline{\mathbf{F}}^{JK} = \mathbf{M}^{JK} \frac{\partial \overline{\mathbf{U}}^{JK}}{\partial \overline{x}^{JK}} + \mathbf{N}^{JK} \overline{\mathbf{U}}^{JK} \tag{2.11}$$

It is assumed that the initial displacements and velocities of all members are zero, which can be expressed as

$$\begin{cases} \overline{\mathbf{U}}^{JK}(L^{JK},t)|_{t=0} = \mathbf{0} \\ \dfrac{\partial \overline{\mathbf{U}}^{JK}(L^{JK},t)}{\partial \overline{x}^{JK}}|_{t=0} = \mathbf{0} \end{cases} \tag{2.12}$$

There are three kinds of boundary conditions which could be written as the common form:

$$\begin{cases} a_1 \dfrac{\partial \overline{\mathbf{U}}^{JK}(0,t)}{\partial \overline{x}^{JK}} + a_2 \overline{\mathbf{U}}^{JK}(0,t) = g_1^{JK}(t) \\ a_3 \dfrac{\partial \overline{\mathbf{U}}^{IJ}(l^{JK},t)}{\partial \overline{x}^{JK}} + a_4 \overline{\mathbf{U}}^{JK}(l^{JK},t) = g_2^{JK}(t) \end{cases} \tag{2.13}$$

The inside joint conditions are the composition of forces of all the member bars at joint J must be zero in the global coordinate and the displacement vector of all member bars at joint I must be the same. The function $\varphi(J,K_i)$ is defined as :

$$\varphi(J,K_i) = \begin{cases} \mathbf{E}_{6\times6}(J < K_i) \\ \mathbf{H}(J > K_i) \end{cases} \tag{2.14}$$

The inside connectable condition could be expressed as

$$\sum_{i=1}^{n} \mathbf{K}^{JK_i} \varphi(J,K_i) \overline{\mathbf{F}}_I^{JK_i} = 0 \tag{2.15}$$

$$\overline{\mathbf{U}}_I^{JK_1} = \overline{\mathbf{U}}_I^{JK_2} = \cdots\cdots = \overline{\mathbf{U}}_I^{JK_{n-1}} = \overline{\mathbf{U}}_I^{JK_n} \tag{2.16}$$

2.3.Elastic Finite Difference Method

Equation (2.10) could be expressed as the below by the finite difference method:

$$[\overline{\mathbf{U}}^{JK}]_j^{k+1} = -[\overline{\mathbf{U}}^{JK}]_j^{k-1} + \mathbf{E}^{JK}[\overline{\mathbf{U}}^{JK}]_{j-1}^k + \mathbf{F}^{JK}[\overline{\mathbf{U}}^{JK}]_j^k + \mathbf{G}^{JK}[\overline{\mathbf{U}}^{JK}]_{j+1}^k \tag{2.17}$$

Where $\mathbf{E}^{JK} = \tau^2[\mathbf{C}^{JK}]^{-1}(2\mathbf{A}^{JK} - \mathbf{B}^{JK}h^{JK})/[2(h^{JK})^2]$, $\mathbf{G}^{JK} = \tau^2[\mathbf{C}^{JK}]^{-1}(2\mathbf{A}^{JK} + \mathbf{B}^{JK}h^{JK})/[2(h^{JK})^2]$, $\mathbf{F}^{JK} = [4h^{JK}\mathbf{C}^{JK} - 4\tau^2\mathbf{A}^{JK} - 2\tau^2\mathbf{D}^{JK}(h^{JK})^2][\mathbf{C}^{JK}]^{-1}/[2(h^{JK})^2]$.The h^{JK} and τ^2 are separate the space distance and the time distance of member bar JK. The i and j means the distance point and time point of finite difference method , respectively.

For the single member bar, it is assumed that the element is divided as m finite difference points and the serial number is from 1 to m, so we could get the expression of a single member bar JK :

$$[\overline{\mathbf{U}}^{JK}]_{in}^{k+1} = -[\overline{\mathbf{U}}^{JK}]_{in}^{k-1} + \mathbf{H}^{JK}[\overline{\mathbf{U}}^{JK}]_{in}^k + \mathbf{I}^{JK}[\overline{\mathbf{U}}^{JK}]_{bn}^k \tag{2.18}$$

where the subscript in and bn mean the boundary and the inside difference points, the matrixes $\mathbf{H}^{JK}$ and $\mathbf{I}^{JK}$ are square matrixes of $m\times6$ rank $\mathbf{I}^{JK}(1,1) = \mathbf{E}^{JK}$, $\mathbf{I}^{JK}(m,m) = \mathbf{G}^{JK}$, $\mathbf{H}^{JK}(q,q) = \mathbf{F}^{JK}$ $\mathbf{H}^{JK}(q,q+1) = \mathbf{G}^{JK}$, $\mathbf{H}^{JK}(q-1,q) = \mathbf{E}^{JK}$, $(q = 1\cdots\cdots m)$ .The other element of $\mathbf{H}^{JK}$ and $\mathbf{I}^{JK}$ is zero.

For the rigid frame, the final finite difference format could be shown as:

$$[\overline{\mathbf{U}}]_{it}^{k+1} = -[\overline{\mathbf{U}}]_{it}^{k-1} + \mathbf{L}[\overline{\mathbf{U}}]_{it}^k + \mathbf{P}[\overline{\mathbf{U}}]_{bbt}^k + \mathbf{P}[\overline{\mathbf{U}}]_{ibt}^k \tag{2.19}$$

The $\mathbf{L}$ and $\mathbf{P}$ are square matrixes of $\sum_{i=1}^{n} m_i \times 6$ rank, $\mathbf{L} = diag\{\mathbf{H}^{JK} \cdots \mathbf{H}^{RT}\}$, $\mathbf{P} = diag\{I^{JK} \quad \cdots \quad I^{RT}\}$.

The forward finite difference method could be used for Eq. (2.14), and we can get:

$$\sum_{i=1}^{n}\mathbf{K}^{JK_i}\varphi(J,K_i)(\mathbf{M}^{JK_i}\frac{[\bar{\mathbf{U}}_I^{JK_i}]_1^k-[\bar{\mathbf{U}}_I^{JK_i}]_0^k}{h^{JK_i}}+\mathbf{N}^{JK_i}\bar{\mathbf{U}}_I^{JK_i})=0 \tag{2.20}$$

By Eq. (2.14)~ (2.16), the relationship of $[\bar{\mathbf{U}}_I^{JK_i}]_0^k$ and $[\bar{\mathbf{U}}_I^{JK_i}]_1^k$ could be expressed as:

$$[\bar{\mathbf{U}}_I^{JK_i}]_0^k=\sum_{i=1}^{n}\varphi(J,K_i)[\mathbf{K}^{JK_i}]^T[\mathbf{MM}^{Jn}]^{-1}\mathbf{NN}^{JK_i} \tag{2.21}$$

where $\mathbf{MM}^{Jn}=\sum_{i=1}^{n}[\mathbf{K}^{JK_i}]\varphi(J,K_i)(\mathbf{M}^{JK_i}/h^{JK_i}-\mathbf{N}^{JK_i})\varphi(J,K_i)[\mathbf{K}^{JK_i}]^T$, $\mathbf{NN}^{JK_i}=\mathbf{K}^{JK_i}\varphi(J,K_i)\mathbf{M}^{JK_i}/h^{JK_i}$.
According to Eq. (2.22), the relationship between inside joint points of rigid frame and inside finite difference points of element could be shown as:

$$\bar{\mathbf{U}}_{ibt}^k=\mathbf{S}\bar{\mathbf{U}}_{it}^k \tag{2.22}$$

According to Eqs. (2.19) and (2.22), the final form of elastic problem is :

$$[\bar{\mathbf{U}}]_{it}^{k+1}=-[\bar{\mathbf{U}}]_{it}^{k-1}+(\mathbf{L}+\mathbf{PS})[\bar{\mathbf{U}}]_{it}^k+\mathbf{P}[\bar{\mathbf{U}}]_{bbt}^k \tag{2.23}$$

Based on Eq. (2.11), Eq.(2.13) could be shown as the finite difference form:

$$\begin{cases} a_1[\mathbf{M}^{JK}]^{-1}(\bar{\mathbf{F}}_i^{JK}-\mathbf{N}^{JK}\bar{\mathbf{U}}_1^{JK})+a_3\bar{\mathbf{U}}_1^{JK}=\mathbf{g}_1^{JK}(t) \\ a_3[\mathbf{M}^{JK}]^{-1}(\bar{\mathbf{F}}_i^{JK}-\mathbf{N}^{JK}\bar{\mathbf{U}}_m^{JK})+a_4\bar{\mathbf{U}}_m^{JK}=\mathbf{g}_1^{JK}(t) \end{cases} \tag{2.24}$$

By Eqs. (2.23) and (2.24), the problem elastic wave propagation in rigid frame could be solved.

2.4.Elastic-Plastic Difference Method

The new difference technique is based on the traditional difference method and $Wilson-\theta$ method, of which the space distance is fixedness, and the distance of time is changed. Because of the modulus and coefficient matrixes is not constant, the matrixes and vectors should add the superscript k to means the time point in the equation. The new form of equation (2.10) and (2.11) are:

$$[\mathbf{AP}^{JK}]^k\frac{\partial^2[\bar{\mathbf{U}}^{JK}]^k}{\partial(\bar{x}^{JK})^2}+[\mathbf{BP}^{JK}]^k\frac{\partial[\bar{\mathbf{U}}^{JK}]^k}{\partial\bar{x}^{JK}}=[\mathbf{CP}^{JK}]^k[\bar{\mathbf{a}}^{JK}]^k+[\mathbf{DP}^{JK}]^k[\bar{\mathbf{U}}^{JK}]^k \tag{2.25}$$

$$[\bar{\mathbf{F}}^{JK}]^k=[\mathbf{MP}^{JK}]^k\frac{\partial[\bar{\mathbf{U}}^{JK}]^k}{\partial\bar{x}^{JK}}+[\mathbf{NP}^{JK}]^k[\bar{\mathbf{U}}^{JK}]^k \tag{2.26}$$

The new finite difference form of Eq. (2.25) is:

$$[\mathbf{DD}^{JK}]_j^k[\bar{\mathbf{a}}^{JK}]_j^k=[\mathbf{AA}^{JK}]_j^k[\bar{\mathbf{U}}^{JK}]_{j+1}^k+[\mathbf{BB}^{JK}]_j^k[\bar{\mathbf{U}}^{JK}]_j^k+[\mathbf{CC}^{JK}]_j^k[\bar{\mathbf{U}}^{JK}]_{j-1}^k \tag{2.27}$$

where $[\mathbf{AA}^{JK}]_j^k=2[\mathbf{AP}^{JK}]_j^k+[\mathbf{BP}^{JK}]_j^k h^{JK}$, $[\mathbf{BB}^{JK}]_j^k=-4[\mathbf{AP}^{JK}]_j^k-2(h^{JK})^2[\mathbf{DP}^{JK}]_j^k$, $[\mathbf{CC}^{JK}]_j^k=2$ $[\mathbf{AP}^{JK}]_j^k-[\mathbf{BP}^{JK}]_j^k h^{IJ}$, $[\mathbf{DD}^{JK}]_j^k=2(h^{JK})^2[\mathbf{CP}^{JK}]_j^k$.

In comparison with Eqs. (2.18) ~(2.23), the difference equation for elastic-plastic problem is expressed as

$$[\mathbf{LL}]^k[\bar{\mathbf{a}}]_{it}^k+[\mathbf{RR}]^k[\bar{\mathbf{U}}]_{it}^k=[\mathbf{QQ}]^k[\bar{\mathbf{U}}]_{bbt}^k \tag{2.28}$$

The structural rule of coefficient matrix of Eqs. (2.25) ~ (2.28) is the same with the elastic part.

Eq. (2.28) could be further discretized in time domain based on $Wilson-\theta$ method

$$[\Delta\bar{\mathbf{U}}]_{it}^{k} = (A1[\mathbf{LL}]^{k} + [\mathbf{RR}]^{k})^{-1}[\mathbf{LL}]^{k}(A2[\bar{\mathbf{V}}]_{it}^{k} + 3[\bar{\mathbf{a}}]_{it}^{k}) + \theta[\mathbf{QQ}]^{k}([\bar{\mathbf{U}}]_{bbt}^{k+\Delta t} - [\bar{\mathbf{U}}]_{bbt}^{k}) \tag{2.29}$$

$$\begin{cases} [\bar{\mathbf{a}}]_{it}^{k+\Delta t} = A3[\Delta\bar{\mathbf{U}}]_{it}^{k} + A4[\bar{\mathbf{V}}]_{it}^{k} + A5[\bar{\mathbf{a}}]_{it}^{k} \\ [\bar{\mathbf{V}}]_{it}^{k+\Delta t} = [\bar{\mathbf{V}}]_{it}^{k} + A6([\bar{\mathbf{a}}]_{it}^{k} + [\bar{\mathbf{a}}]_{it}^{k+\Delta t}) \\ [\bar{\mathbf{U}}]_{it}^{k+\Delta t} = [\bar{\mathbf{U}}]_{it}^{k} + \Delta t[\bar{\mathbf{V}}]_{it}^{k} + A7(2[\bar{\mathbf{a}}]_{it}^{k} + [\bar{\mathbf{a}}]_{it}^{k+\Delta t}) \end{cases} \tag{2.30}$$

where $A1 = 6/(\theta\Delta t)^2$, $A2 = 6/(\theta\Delta t)$, $A3 = 6/\theta(\theta\Delta t)^2$, $A4 = -6/\theta(\theta\Delta t)$, $A5 = (1-3/\theta)$, $A6 = \Delta t/2$, and $A7 = (\Delta t)^2/6$.

3.EXAMPLES

To validate the finite difference method to simulate the process of wave propagation in structures, a rigid frame is shown as figure 2. The space frame is composited by four member bars with the same E, G, ρ, and L. The plastic modulus are $0.4E$ and $0.4G$. The velocities of longitudinal and flexural wave of member bar are c_1 and c_2. All the boundary points are fixed. The displacement pulse $u_0\sin(10\times pi\times t),(0<t<0.1)$ is input along the z-axis at node "1" for initial condition. The dimensionless time is defined as $T = t/(L/c_1)$. We consider two cases, the purely elastic and elastic-plastic.

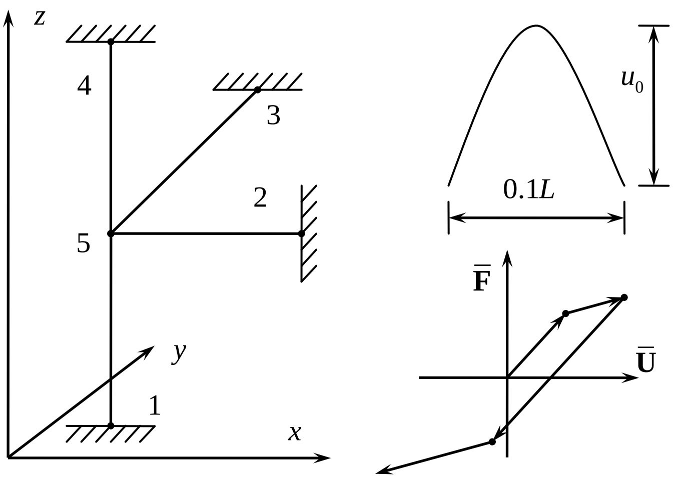
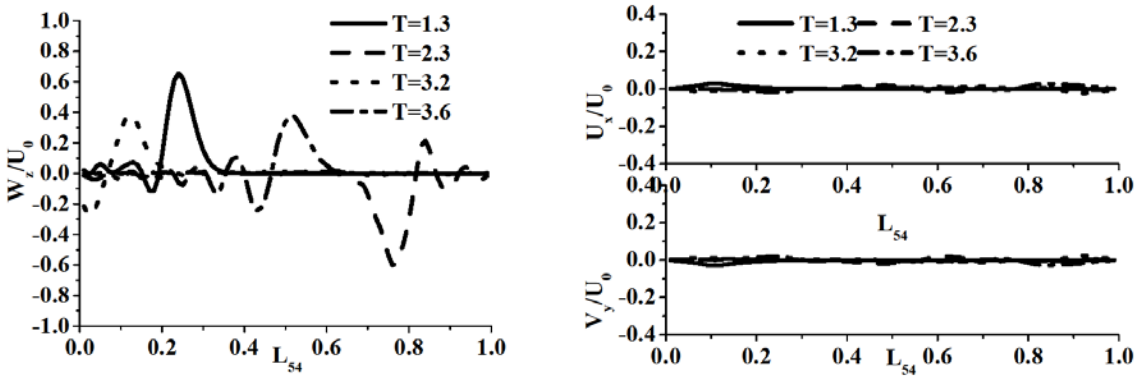


Figure 2. Frame structure, pulse, restoring force diagram

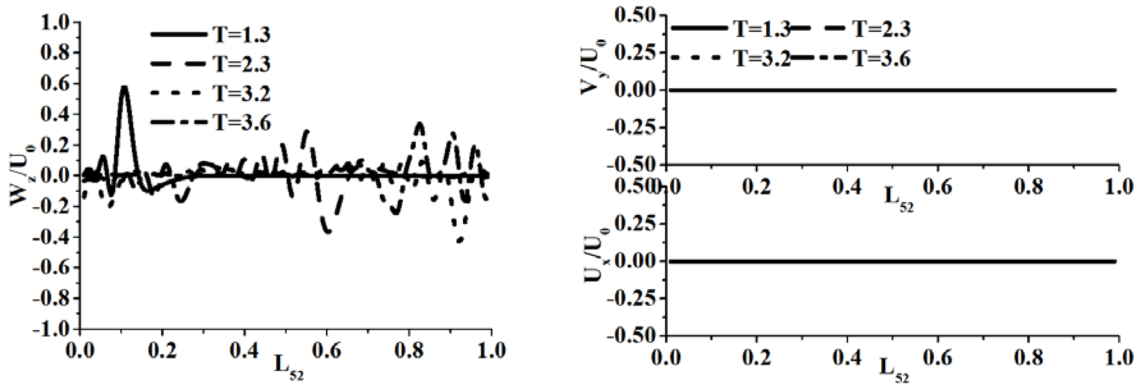
Figure 3(a) shows the displacement along z-axis of member bar 54 at the moment $T = 1.3, 2.3, 3.2, 3.6$. The initial longitudinal wave pass the joint 5 at $T = 1$ and then the longitudinal amplitude of pole 54 is reduced to 60% of the initial value. When $T = 2$, the reflection leads to the change of direction, but the amplitude is remain unchanged. The direction and amplitude are all different between $T = 2.3$ and $T = 3.2$ as a result of the influence of joint 5 at the moment of $T = 3$. From $T = 3.2$ to $T = 3.6$, there are no influence factors, so the direction and amplitude of longitudinal wave are not changed. Figure 3(b) shows that there are flexural waves in x-axis and y-axis of member bar 54 after $T = 1$, because of the transmission effect of the inside connected joint 5. In the direction of x and y, the member bars 52 and 53 block the movement of joint 5 so that the amplitude is too small to distinguish. We could consider that there is no flexural in member bar 54 approximately.



(a) (b)

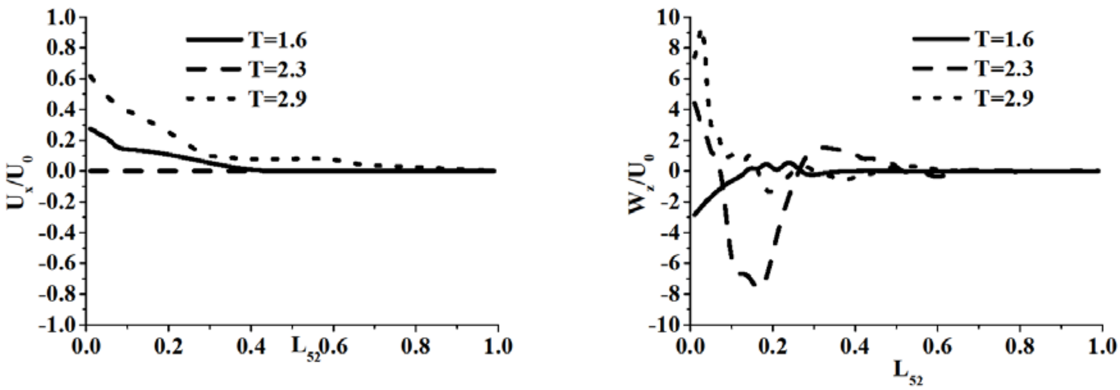
Figure 3. The elastic wave motion response of member bar 54

It is shown in the Fig.4(a) that the line displacement along z-axis of member bar 52 at the moment $T = 1.3, 2.3, 3.2, 3.6$. Because of the joint 5 is the common node, the amplitude of flexural wave in member bar 52 and longitudinal wave in member bar 54 are the same when $T = 1.3$. The frequency dispersion characteristics of flexural wave is well expressed in Fig 4(a). By contrast with the four moment, the amplitude becomes smaller and the width of pulse is increasing. Figure 4(b) shows the longitudinal wave in x-axis and flexural wave in y-axis. As a result of the direction of incident pulse is along z-axis and the x-axis and y-axis are blocked by member bars 52 and 53,the line displacement amplitude is almost zero.



(a) (b)

Figure 4. The elastic wave motion response of member bar 52



(a) (b)

Figure 5. The elastic-plastic wave motion response of member bar 52

Figure 5 shows the elastic-plastic wave spread in frame. It is assumed that the member bar 51 is elastic and all the other member bars are elastic-plastic. In spite of the vibration errors of numerical calculation is large and the theoretical defect, the figures could also show the tendency of wave spread.

Compared with Figs. 4 and 5, the amplitude of plastic wave is bigger than the elastic, which shows that the process of plastic is more serious but the velocity of propagation is slower.

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