Substructural parameters and excitation identification for a high-rise frame model structure

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SUMMARY:
A substructural identification (SSI) approach called weighted adaptive iterative least-squares estimation with incomplete measured excitations (WAILSE-IME) for the simultaneous identification of local structural parameters and unknown dynamic loadings was proposed in this paper. To improve the convergence rate and the identification accuracy, a positive definite weight matrix and an adaptive learning coefficient were introduced. The efficiency and accuracy of the approach were validated via an experimental test with a 20 story high-rise steel frame structure in the lab. Results show that the proposed approach can simultaneously identify the substructural parameters and unknown excitations with acceptable accuracy, and provides a potential way for localized damage detection.

Keywords: substructural identification, partially unknown inputs, weight matrix, learning coefficients

1. INSTRUCTIONS

Structural identification (SI) is to determine the physical parameters of an engineering structure based on the measured vibration data and/or excitation information and can be employed to detect structural damages which may be reflected by the changes in some structural parameters such as stiffness or dynamic characteristics including natural frequencies and/or modal shapes. In the last decades, various analysis methodologies for SI and damage detection of engineering structures in frequency domain, time domain and time-frequency domain have been proposed and further literature review of the related recent works can be found in references (Wu et al, 2003; Carden and Fanning, 2004; Trinh and Koh, 2011; Fan and Qiao, 2011).

Though many SI methods are currently available for structural parameter estimation, a major challenge lies in considerable convergence difficulty when a large number of unknowns are involved. Moreover, identifying a large-scale structure as a whole usually results in expensive evaluation of objective function and extensive computational time for processing huge amount of data. However, if a whole large-scale structure with vast account of unknowns can be decomposed into several smaller substructures and the identification for each substructure is carried out independently, the identification results will be more reliably and accurately (Koh et al, 1991). Furthermore, in many situations, only the critical parts, where damage is most likely to occur, need to be concerned firstly. Koh et al. (1991) proposed a substructural identification (SSI) approach by using the extended Kalman filter with a weighed global iteration algorithm to identify the structural parameters, e.g. stiffness and damping coefficients. In the following years, based on divide-and-conquer strategy, Koh et al (2003) extended the previous approach and proposed a progressive SSI method by employing genetic algorithm (GA). Xu (2005) proposed a neural-network-based SSI method to identify stiffness and damping values of a shear building by the direct use of substructural dynamic response. Tee et al. (2009) presented a novel substructural approach for identifying stiffness matrices and local damage of large systems in a divide-and-conquer manner. More recently, Hou et al (2011) described an effective method of substructure isolation basing on virtual distortion method and force distortions approach for local SHM and damage detection. Weng et al (2011) proposed a substructure-based finite element model updating technique for damage detection. By introducing the exponential window method in the formulation, Trinh and Koh (2011) presented an improved SSI strategy for large structural systems by means of GA and demonstrated its performance in both numerical and experimental studies.

The SSI methods mentioned before can be used to efficiently and accurately identify the considered substructural parameters. However, most of them assumed that the external excitations should be known for identification. When the external excitations are outside the substructure of interest, the parameters of the concerned substructure can be identified solely utilizing the response measurements on the internal degrees-of-freedom (DOFs) and/or interfacial DOFs with namely ‘output-only’ identification approaches. However, there is a need to identify structural dynamic loadings for structural remaining life forecast. A procedure was proposed by Chen and Li (1998) to simultaneously estimate the structural parameters as well as the unmeasured excitations. Zhao et al. (2006) proposed a hybrid identification method for determining parameters of multi-story buildings with unknown ground motion by two separated steps. Yang and Huang (2007) proposed an adaptive sequential non-linear least-square estimate method for the on-line identification of structural damages with unmeasured excitations and unmeasured acceleration responses. Using the cross-power spectral densities between structural floor accelerations and a reference response, a substructure identification method for shear structures was proposed by Zhang and Johnson (2012). Based on a dynamic response reconstruction technique, a substructural damage identification approach under moving vehicular loads was presented only using the finite element model of the intact target substructure and the acceleration measurements of the target substructure in the damaged state (Li and Law, 2012). Law and Li (2012) proposed a substructural damage identification approach without the information of interface responses and forces at interface DOFs. More recently, a SI approach called weighted adaptive iterative least-squares estimation with incomplete measured excitations has been proposed by the authors for simultaneously identifying structural parameters and unknown dynamic loadings of multi-degrees-of-freedoms (MDOFs) structures (Xu et al, 2012).

In this paper, a SSI approach called weighted adaptive iterative least-squares estimation with incomplete measured excitations (WAILSE-IME) was proposed for the simultaneous identification of the physical parameters and the unknown dynamic loading applied in the interior DOFs of substructures of large scale structures. The efficiency and accuracy of the approach were validated via an experimental test with a 20 story high-rise steel frame structure in the lab

2. THE PROPOSED WAILSE-IME APPROACH

The equation of motion of a complete MDOFs structural system can be expressed as,

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \tag{2.1}$$

where *M*, *C*, and *K* are the mass, damping, and stiffness matrix, respectively, $\ddot{x}(t)$, $\dot{x}(t)$, and $x(t)$ are the structural acceleration, velocity, and displacement responses, and *f*(*t*) is the external excitation. According to the concept of substructuring, Eqn. 2.1 can be rearranged using partitioned matrices and shown as follows (Koh et al, 1991)

$$\begin{bmatrix} M_{ii} & M_{is} & 0 \\ M_{si} & M_{ss} & M_{sr} \\ 0 & M_{rs} & M_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{x}_i(t) \\ \ddot{x}_s(t) \\ \ddot{x}_r(t) \end{Bmatrix} + \begin{bmatrix} C_{ii} & C_{is} & 0 \\ C_{si} & C_{ss} & C_{sr} \\ 0 & C_{rs} & C_{rr} \end{bmatrix} \begin{Bmatrix} \dot{x}_i(t) \\ \dot{x}_s(t) \\ \dot{x}_r(t) \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{is} & 0 \\ K_{si} & K_{ss} & K_{sr} \\ 0 & K_{rs} & K_{rr} \end{bmatrix} \begin{Bmatrix} x_i(t) \\ x_s(t) \\ x_r(t) \end{Bmatrix} = \begin{bmatrix} f_i(t) \\ f_s(t) \\ f_r(t) \end{bmatrix} \tag{2.2}$$

where the subscript *i*, *s*, and *r* denotes the interior, interfacial and external DOFs of the concerned substructure, respectively. Then, the equations of motion for the concerned substructure can be extracted from the full system as

$$\begin{bmatrix} M_{ii} & M_{is} \end{bmatrix} \begin{Bmatrix} \ddot{x}_i(t) \\ \ddot{x}_s(t) \end{Bmatrix} + \begin{bmatrix} C_{ii} & C_{is} \end{bmatrix} \begin{Bmatrix} \dot{x}_i(t) \\ \dot{x}_s(t) \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{is} \end{bmatrix} \begin{Bmatrix} x_i(t) \\ x_s(t) \end{Bmatrix} = f_i(t) \tag{2.3}$$

Assuming the mass distribution is known, Eqn. 2.3 can be rearranged as

$$\begin{bmatrix} C_{ii} & C_{is} \end{bmatrix} \begin{Bmatrix} \dot{x}_i(t) \\ \dot{x}_s(t) \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{is} \end{bmatrix} \begin{Bmatrix} x_i(t) \\ x_s(t) \end{Bmatrix} = f_i(t) - \begin{bmatrix} M_{ii} & M_{is} \end{bmatrix} \begin{Bmatrix} \ddot{x}_i(t) \\ \ddot{x}_s(t) \end{Bmatrix} \tag{2.4}$$

This equation can be also represented as follows,

$$H_{(m\times n)\times L}\theta_{L\times 1} = P_{(m\times n)\times 1} \tag{2.5}$$

where H = the response matrix composed of the velocity and displacement measurements of the concerned substructure and the interfacial DOFs; θ = the substructural physical parameters which will be identified, i.e. damping and stiffness coefficients; m = the number of sample points; n = the total number of the DOFs including the DOFs in the concerned substructure and the associated interfaces; L = the total number of the structural parameters; and force vector P = an $(m\times n)\times 1$ vector composed of the external excitations and inertia forces at time t. Here, the force vector P can be expressed as

$$P = \begin{bmatrix} P(t_1) & P(t_2) & \dots & P(t_m) \end{bmatrix}^T \tag{2.6}$$

where $P(t_l) = \left[f_i(t_l) - M_{ii}\ddot{x}_i(t_l) - M_{is}\ddot{x}_s(t_l) \right]^T$ (l=1, 2, …, m).

Based on the available external excitations and the corresponding responses of the concerned substructure as well as the interfaces, the physical parameters can be obtained directly according to Eqn. 2.5 by means of least-squares estimation (LSE) as follows,

$$\tilde{\theta} = \left[H^T H \right]^{-1} H^T P \tag{2.7}$$

It can be seen that the traditional LSE requires that all the input data should be measured or available, which may not be the case for many practical situations. It is sometimes inevitable to encounter such a situation in which part of the excitations involved in the concerned substructure are unknown for identification.

Here, it is assumed that the force vector P is composed of the known part (P_k) and unknown part (P_u) and can be expressed as follows,

$$P = \begin{bmatrix} P_k & P_u \end{bmatrix}^T \tag{2.8}$$

where the subscript k and u = subset consisting of the DOFs of the substructure on which the known excitations are applied, and the DOFs on which the unknown excitations are applied, respectively. Consequently, the estimated force vector on the j-th iteration are composed of two corresponding parts and shown below,

$$\tilde{P}^j = \begin{bmatrix} \tilde{P}_k^j & \tilde{P}_u^j \end{bmatrix}^T \tag{2.9}$$

where the symbol '~' indicates estimated values, superscript j means the j-th iteration, and subscript k and u are defined before. Here, the above estimated force vector is firstly updated by replacing $\tilde{P}_k^j$ with the known part P_k as shown in Eqn. 2.10,

$$\hat{P}^j = \left[P_k^j \quad \tilde{P}_u^j \right]^T \tag{2.10}$$

where the symbol '^' indicates the updated values at the *j*-th iteration. In order to accelerate the convergence of the unknown excitations, the increment of the unknown external excitation identification results at the previous two iterations is employed to re-update the unknown part ($\tilde{P}_u^j$) as shown below,

$$\overline{P}^j = \left[P_k^j \quad \overline{P}_u^j \right]^T \tag{2.11}$$

in which $\overline{P}_u^j = \tilde{P}_u^j + \beta(\tilde{P}_u^{j-1} - \tilde{P}_u^{j-2})$, the symbol '-' indicates the re-updated values, and β is a learning coefficient taking the value $\beta \in [0,1)$. To prevent the ultra-iteration, the learning coefficient β employed here can be variable values. The estimated unknown excitations are getting close to the actual values when the iteration process carried out, and then the learning coefficient should take smaller values accordingly. For simplicity, β takes the value of β / j in this paper.

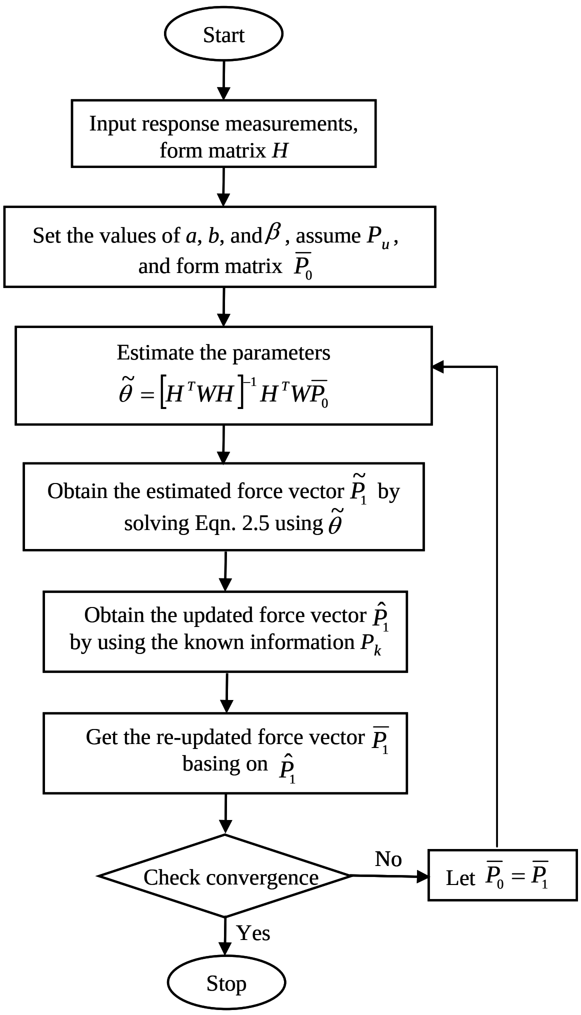

Figure 2.1. Flowchart for the proposed WAILSE-IME-LDD approach

Moreover, a positive definite weight matrix defined in Eqn. 2.12 is introduced to the objective function of the traditional LSE for the purpose of improving the convergence rate and the accuracy of the identification results,

$$W = \begin{bmatrix} aI & 0 \\ 0 & bI \end{bmatrix} \tag{2.12}$$

where I = identity matrix, and a, b = weight coefficients ($a \in [1,+\infty)$, $b \in (0,1]$). The dimension of aI and bI depends on the dimension of P_k and P_u defined before, respectively. By introducing the weight matrix to the objective function of LSE, the physical parameters identification of the substructure can be carried out by the following equation,

$$\tilde{\theta} = \left[H^T W H\right]^{-1} H^T W \overline{P} \tag{2.13}$$

When the parameters of the substructure are updated iteratively, the unknown excitations can be identified accordingly with Eqn. 2.5.

The flowchart of the approach shown in Fig. 2.1 helps to illustrate the procedure.

3. EXPERIMENTAL VALIDATION

To validate the efficiency and feasibility of the proposed approach with test measurements, a dynamic test is carried out on a 20-story steel frame building model structure shown in Fig. 3.1(a). The frame structure is 1-bay by 2-bay as shown in Fig. 3.1(b) with a total height of 4.5m and a plan of 0.5m × 0.75m. The model is designed with six 30mm × 30mm × 3mm angle steel columns, and seven 15mm × 15mm × 3mm angle steel beams for each story. Four steel plates with the thickness of 5mm are fixed on each floor of the model as additional mass. Four steel inter-story diagonal braces with diameter of 2mm are fixed by bolts in the Y direction on each floor of the structure. According to the design of model and due to the existence of the diagonal braces, the inter-story stiffness of the model in the Y direction is much larger than that in X direction. Here, suppose the substructure of interest includes the 7th, 8th, and 9th floor as shown in Fig. 3.1(c).

In this dynamic test, an impact hammer is employed to excite the structure in X direction. The impact force is assumed to be applied on the 8th floor and considered as unknown information for identification. For the purpose of comparison, the excitation force is measured directly by a piezoelectric force gauge located in the head of the hammer. The corresponding acceleration responses of the concerned substructure and the associated interface are measured with a sampling frequency of 1,024Hz. The velocity and displacement responses are obtained by numerical integration using the measured accelerations. Since there are no forces applied on the 7th and 9th floor, the values of the external excitations on these two floors are zero and can be utilized as known information for updating the estimation inputs during the iteration. Since the impact force is applied in the X direction, the whole structure is considered as a lumped-mass model with one horizontal DOF in X direction on each floor. The mass distribution of the structure is estimated and considered as known for identification. The lumped mass values for 1st to 19th floors and 20th floor are 11.86kg and 10.96 kg, respectively.

In this study, the weight coefficients of a and b, and learning coefficient β are set to be 10, 0.1, and 0.8, respectively. The unknown excitation applied on the 8th floor is initially assumed to be 50 N which is different from the actual values. The dynamic response measurements of the two boundaries and the three stories of the substructure from 1 sec to 4 sec with a total of 3,072 sampling points are employed to identify the parameters and unknown dynamic loading. The identified results of the damping and stiffness coefficients are shown in Table 3.1. To shown the efficiency of the proposed iteration approach, the convergence performance of the identified parameters is plotted in Fig. 3.2. It is clear from Fig. 3.2 that the results can converge stably only through a very limited iterations.

Moreover, the unknown impact force applied on the 8th floor in the dynamic test from 1 to 4 sec can be also simultaneously obtained. The comparison between the identified external excitation with the measurement is shown in Fig. 3.3. It is clear from Fig. 3.3 that the identified excitation force is very close to the measurement. The mean difference between the measured peak values and those given by the proposed approach is 6.72%.

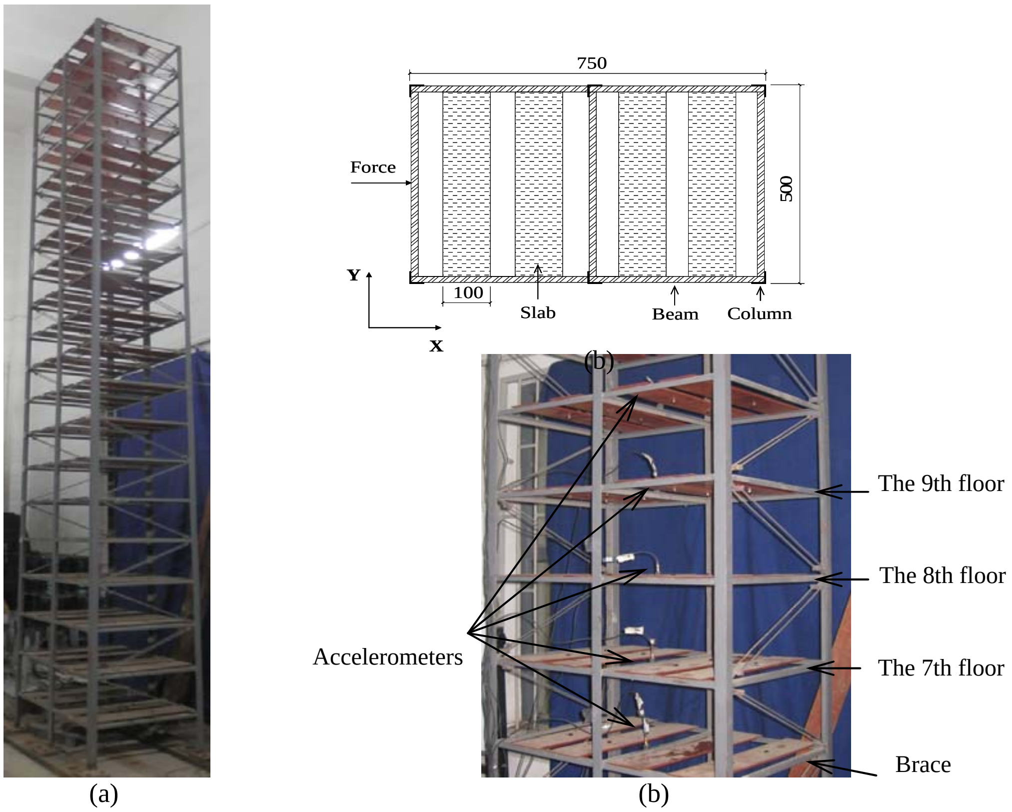

Figure 3.1. The high-rise steel structure in the lab: (a) the whole 20-story model, (b) the plan of the model of the 8th floor, (c) the concerned substructure

Table 3.1. Identified parameters of the concerned substructure

Physical parameters	Identified values (N/m)	Physical parameters	Identified values (Nm/s)
k_7	2.77×10^6	c_7	117.53
k_8	3.04×10^6	c_8	103.85
k_9	3.13×10^6	c_9	111.08
k_{10}	3.09×10^6	c_{10}	97.03

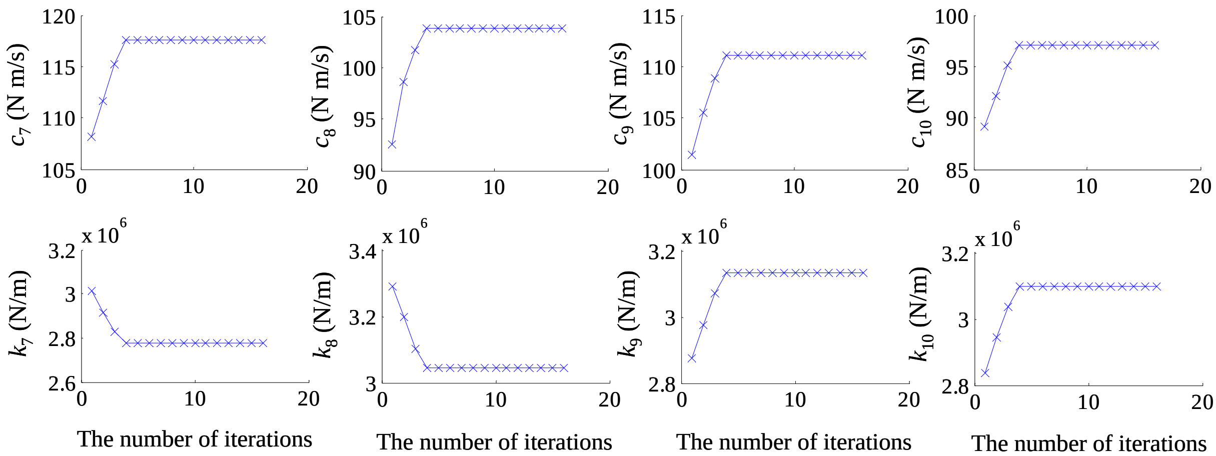

Figure 3.2. The identified substructural parameters during the iteration

In this test, the 7th and the 9th floors of the high-rise model structure are not excited and the excitation forces on these two floors are zero and are treated as known information. Here, the residual errors of the known excitation $\widetilde{P}_k^j$ shown in Eqn. 2.11 on the 7th and the 9th floors when the iteration ends are considered. The final results of the excitations on the 7th and 9th floors at the end of the iteration are shown in Fig. 3.4. Because the excitation forces at these two floors are zero, these results can be considered as the residual error of the identified forces. It can be observed from Fig. 3.4 that the residual error of the identified excitation forces for the unexcited floors are mostly less than 80N and are relatively very small as compared with the excitation on the 8th floor. Such findings mean that the dynamic excitation identification results are acceptable.

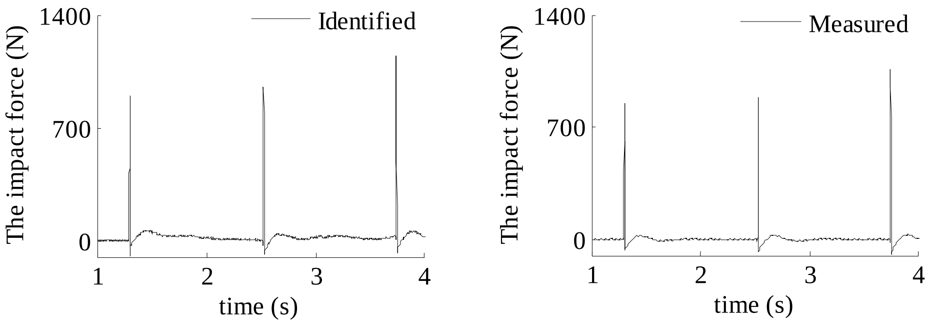

Figure 3.3. The comparison of the external excitation applied on the 8th floor

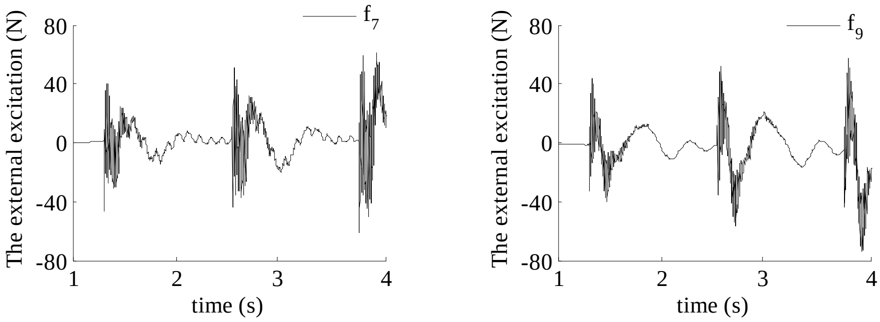

Figure 3.4. The estimated external excitations on the 7th and 9th floor obtained from the iteration

4. CONCLUDING REMARKS

In this paper, a updated substructural iterative time-domain identification approach, referred to as weighted adaptive iterative least-squares estimation with incomplete measured excitations (WAILSE-IME), was proposed for simultaneously identifying the substructural parameters and the unknown dynamic loadings which were applied on the concerned substructures. To improve the efficiency of the convergence and the accuracy of the identification results, a positive definite weight matrix and an adaptive learning coefficient were introduced during the iteration. By dividing a large-scale or complex structure into convenient substructures containing the parameters of interest, the proposed approach could be utilized for structural identification basing on the response measurements of the internal DOFs and the interfacial DOFs of the concerned substructure.

A dynamic test on a substructure in a 20 story high-rise steel frame building structure in lab was carried out to demonstrate the reliability and robustness of the proposed approach. Results show that the proposed approach is capable of simultaneously identifying the substructural physical parameters as well as the unknown dynamic loadings with acceptable accuracy, and provides a potential way for structural identification, damage detection and dynamic loading profile analysis which plays key roles in the remaining life forecast of large-scale and complex engineering structures.

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