Optimum Design of Triple Friction Pendulum Bearing Subjected by Near-Field Ground Motions

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SUMMARY:

Triple Friction Pendulum Bearing (TFPB) as a novel seismic isolator provides different combination of stiffness and damping during earthquake. Its adaptive behaviour is one of the practical solutions for doubtful performance of seismic isolations systems under near-field ground motions. Hence selecting its design parameters is complicated process and their optimum combination depends on input motion characteristics and seismic performance objective of superstructure.

Here in, specific numerical optimization method based on Genetic Algorithms (GA) has been applied to determine the optimum values of design parameters. In this process, near-field ground motions have been employed with range of pulse periods and hazard levels as input excitations. According to GA results, the optimum design parameters had significantly different optimum intervals for different target responses. So different response targets were combined, to make a multi-objective fitness function. The derived optimum design parameters can be used for different types of superstructures with the same behaviour.

Keywords: Triple Friction Pendulum Bearing, near field, genetic algorithm

1. INTRODUCTION

After extensive damages observed in engineering designed structures at vicinity of seismic sources (Bertero, et al., 1978; Hall, et al., 1995; Alavi & Krawinkler, 2001), many researchers have been conducted towards studying on nature of ground motions in close distance of causative fault. The variety in characteristics of these kinds of motions, have encouraged engineers to find advance technology to improve seismic resistance of structures against such kinds of vibrations. One of these technologies is seismic isolation with lots of well known variety and application in different construction. However, large amplitudes with long periods of near-field motions, made some doubts in efficiency of the isolations systems that only focus on gathering the displacement in base level. This problem can be solved by using supplemental dampers in isolation level whilst it increases floor acceleration and damages of sensitive equipments even in low level earthquakes. This dilemma can be solved by newly developed class of multiple pendulums isolators called Triple Friction Pendulum Bearings (TFPB-Figure 3.1.a). TFPB exhibits improved hysteretic characteristic to control performance over broad range of excitation (Fenz DM, 2008a,b,c;Malekzadeh & Taghikhany, 2012). TFPB enables engineers to choose different combination of stiffness and damping in different level of excitation and achieve multiple performance objectives which were not accessible in past. TFPB can be designed in a way which can restrict the base displacement and floor acceleration simultaneously. Finding optimum combination of its design parameters (curvatures radiuses, friction coefficients and displacement capacities) is complicated process and their optimum values depend on input motion

characteristic and seismic performance objective of superstructure.

Herein, after precise numerical modelling of TFPBs, a sensitivity analysis is conducted to recognize effect of variation of design parameters on seismic response of superstructure. The variation of design parameters is determined vs. input motion characteristic in practical ranges for buildings. In next step, design parameters, or their combination which are minimizing superstructure demand such as floor acceleration and base displacement are investigated. In this procedure near-field strong motions with different characteristics in three hazard levels (MCE, DBE, and SLE) are used to achieve above target. Optimum design parameters for nonlinear explicit target functions are computed using genetic algorithm (GA).

2. ADAPTIVE TRIPLE FRICTION PENDULUM BEARINGS

The behaviour of Triple Friction Pendulum Bearings is termed as adaptive because they progressively exhibit different hysteretic properties at different stages of displacement. The stiffness and damping can be changed to predictable values at different controllable amplitudes. These properties let the design of isolation system to be separately optimized in multiple levels of input excitation. As shown in Figure 3.1.a, R_i is the radius of curvature of surface *i*, h_i is the radial distance between the pivot point and surface *i* and μ_i is the coefficient of friction at the sliding interface. The internal construction of these bearings permits sliding on different combinations of surfaces throughout the course of motion, resulting in changes in stiffness and damping (Fenz & Constantinou, 2008a). Different stages of sliding related to adaptive TFPB during different levels of excitation are defined as follow (Figure 3.1.b):

Stage I: Sliding on surface 2 and 3 only, this stage forms one pendulum mechanism, and defines the properties of the isolation system under low levels of excitation (Service Level Earthquake: SLE).

Stage II: Motion stops on surface 2; sliding on surface 1 and 3. This mechanism defines the primary properties of the isolation system under moderate levels of excitation (Design Basis Earthquake: DBE).

Stage III: Motion stopped on surface 2 and 3; sliding on surface 1 and 4. The friction coefficient of upper concave (surface 4) is sufficiently large to prevent sliding until an extreme level of excitation occurs (Maximum Credible Earthquake: MCE).

Stage IV: Slider contacts restrainer on surface 1; motion remains stopped on surface 3; sliding on surface 2 and 4. This mechanism defines properties of isolation bearing beyond MCE.

Stage V: Slider bears on restrainer of surface 1 and 4; sliding on surface 2 and 3(final stage).

3. NUMERICAL MODELLING

Here the hysteretic behaviour of TFPBs is simulated by series model of three independent single friction pendulum bearing (SFPB) according to Fenz and Constantinou (Fenz & Constantinou, 2008). By applying series model, the dynamic equilibriums are derived for an isolated structure and solved numerically in MATLAB. The responses of the isolated structure from numerical model are verified by comparing experimental results reported by Fenz and Constantinou (Fenz & Constantinou, 2008).

3.1. Design parameters for each Single Friction Pendulum (SFPB) in series model

Figure 3.2 is a schematic view of a series model of three SFPB, which have same force as each other but their relative displacements are different and independent. Each element consists of three parallel members: a) a linear elastic spring to create the resisting force of concave surfaces, b) a rigid plastic

friction element with velocity dependence, and c) a gap element to account for the finite displacement capacity of each sliding surface. For each linear elastic spring the stiffness is given by $\frac{1}{\overline{R_{effi}}}$ where

 \overline{R}_{effi} is the effective radius of curvature. \overline{d}_i is the displacement capacity of each gap element and for rigid plastic friction elements, the velocity dependent coefficient of friction for each member is shown by $\overline{\mu}_i$. The dependency of coefficient of friction to velocity is given by following equation (Constantinou, et al., 1990):

$$\mu = f_{\max} - (f_{\max} - f_{\min}) \exp(-a|\dot{u}|)$$
(3.1)



Figure 3.1. a) Section of a TFPB, b) Different stages of sliding related to adaptive TFPB

 f_{max} is the friction coefficient due to high velocities and f_{min} is the friction coefficient in lowest or negligible velocities. The rate parameter "a" control the transition of friction coefficient between f_{max} and f_{min} .



Figure 3.2. Three SFPB connected in series to model a TFPB

In order to substitute adaptive TFPB (Figure 3.1.b) behaviour with three SFPB element in series, design parameters of SFPBs should be define according to Table 3.1.

3.2. Verification of numerical modelling

In order to verify the model, the equations of motion were numerically solved using MATLAB program and their results were compared with analytical outcomes reported by Fenz and Constantinou (Fenz & Constantinou, 2008). The specification of the superstructure in analytical model is shown in Figure 3.3. The 180 degree component of the 1940 El Centro record with PGA=0.31g was used for verification. To induce isolator displacement for having all the sliding regimes in the TFPB, the record was multiplied by a factor of 2.15. The parameters of TFPB in series model for verification are gathered in Table 3.2 that are calculated from Table 3.1. In this study in order to solve "state equation" in MATLAB, "ode15" function was used.

	Coefficient of friction	Radius of curvature	Nominal displacement capacity	Rate parameter
Element 1	$\bar{\mu}_1=\mu_2=\mu_3$	$\overline{R}_{\text{eff1}} = R_{\text{eff2}} + R_{\text{eff3}}$	$\bar{d}_1 = d_{tot} - (\bar{d}_2 + \bar{d}_3)$	$\bar{a}_1 = \frac{1}{2} \frac{a_2 + a_3}{2}$
Element 2	$\bar{\mu}_2=\mu_1$	$\overline{R}_{eff2} = R_{eff1} - R_{eff2}$	$\bar{d}_2 = \frac{R_{\text{eff1}} - R_{\text{eff2}}}{R_{\text{eff1}}} d_1$	$\overline{a}_2 = \frac{R_{eff1}}{R_{eff1} - R_{eff2}} a_1$
Element 3	$\bar{\mu}_3=\mu_4$	$\overline{R}_{eff3} = R_{eff4} - R_{eff3}$	$\bar{d}_3 = \frac{R_{eff4} - R_{eff3}}{R_{eff4}} d_4$	$\overline{a}_3 = \frac{R_{eff4}}{R_{eff4} - R_{eff3}} a_4$

Table 3.1. The parameters of in series model in terms of TFPB parameters in a fully adaptive arrangement (Fenz
& Constantinou, 2008c)



Figure 3.3. The isolated superstructure for verification of model (Fenz & Constantinou, 2008)

Properties of TFPB						
Surface 1	Reff1=435 mm	µ1=0.0204	d1=54 mm	a1=0.1sec/mm		
Surface 2	R _{eff2} =53 mm	μ2=0.01-0.02	d2=19 mm	a1=0.1sec/mm		
Surface 3	R _{eff3} =53 mm	μ3=0.01-0.02	d3=19 mm	a1=0.1sec/mm		
Surface 4	R _{eff4} =435 mm	µ4=0.06-0.13	d4=64 mm	a1=0.1sec/mm		
Properties of series elements						
	_		-			
Element 1	R _{eff1} =106 mm	μ ₁ =0.01-0.02	d ₁ =53.6 mm	a ₁ =0.05 sec/mm		
Element 1	R _{eff2} =382 mm	$\overline{\mu}_1 = 0.02 - 0.04$	d ₂ =56.2 mm	ā ₁ =0.05 sec/mm		
Element 1	R _{eff3} =382 mm	$\overline{\mu}_1 = 0.06 - 0.13$	d ₃ =56.2 mm	ā1=0.05 sec/mm		

Table 3.2. The properties of TFPB and alternative series elements



Figure 3.4. Comparison of results of TFPB force-displacement and roof acceleration time history from MATLAB analysis (red colours) and results governed by Fenz and Constantinou (Fenz & Constantinou, 2008) (blue colours)

Figure 3.4 shows force-displacement of TFPB and acceleration time history of roof respectively.

Derived results from numerical modelling were compared with analytical outcomes reported by Fenz and Constantinou (Fenz & Constantinou, 2008). It is concluded that with tolerable error the results of numerical modelling can be used to predict real responses of superstructures and isolator.

4. INPUT GROUND MOTIONS

Table 4.1 shows list of near-field ground motions which are selected for this study in order to optimum design and sensitivity analysis of TFPBs under close distance earthquakes. In this Table, seven records with pulse periods between 1 to 7 seconds have been adopted to cover wide range of first mode vibration period of isolated structures. These pulse-like near-field motions were picked out from PEER NGA database in according to Baker research in 2007 (Baker, 2007).

To have engineering judgment regarding to sensitivity analysis and to observe all stages of nonlinear behaviour of adaptive TFPBs, input accelerograms have been normalized to three levels of MCE, DBE, and SLE. The design peak ground acceleration for these levels were selected as 0.759, 0.517 and 0.291 times of gravity acceleration respectively and in according to PSHA analysis for a case study region in Iran (Qazvin City).

Record No.	Name of Earthquake	Year	Record No. in Baker classification	Station	Pulse period T _p (sec)	Magnitude (M _w)
1	Morgan Hill	1984	24	Coyote Lake Dam (SW Abut)	1	6.2
2	Loma Prieta	1989	33	Alameda Naval Air Stn Hanger	2	6.9
3	Cape Mendocino	1992	38	Petrolia	3	7
4	Imperial Valley-06	1979	13	El Centro Array #6	3.8	6.5
5	Imperial Valley-06	1979	14	El Centro Array #7	4.2	6.5
6	Landers	1992	40	Lucerne	5.1	7.3
7	Chi-Chi, Taiwan	1999	64	TCU038	7	7.6

 Table 4.1. Specifications of Input ground motions

5. SENSITIVITY ANALYSIS

Sensitivity analysis is a technique to determine how different values of an independent design parameter of TFPB will impact to other dependent parameter under seismic strong motion. This technique is used within specific boundaries for each design parameters in agreement with practical ranges for buildings and current products by manufactures. Sensitivity analysis is a way to predict the outcome of a decision on selecting design parameters which have key role on optimum design of TFPBs. It can be determined how changes in design parameters of TFPB will impact the target variables like base displacement or roof acceleration of isolated structures.

The superstructure in sensitivity analyses was the same as modelled structure used for verification. The researches on behaviour of short and medium stories isolated building with low damping bearings demonstrate the dynamic characteristics of superstructure has negligible effect on seismic performance of isolated system (Naeim & Kelly, 1999). In sensitivity analysis, one parameter was considered as variable while others stay constant. Sensitivity analysis was done under seven pulse like ground motion in three earthquake design levels (MCE, DBE and SLE) observing the response of structure. The response of the structure was characterized in three different object criteria: 1-Maximum Relative Story Displacement (MRSD), 2-Maximum Horizontal Floor Acceleration (MHFA) and 3-Maximum Displacement in Isolation Level (MDIL). Due to the fact that the isolated superstructures vibrate in rigid mode, the floor acceleration of stories is a linear function of relative story displacement of superstructure so the analysis shown a same trend of MHFA and MRSD versus variation of design parameters. Here after sensitivity analysis, seven independent design parameters of TFPB were selected. The chosen parameters are:

- 1. Effective radius of curvature of sliding surfaces 1 and 4, R_{eff1}
- 2. Effective radius of curvature of sliding surfaces 2 and 3, R_{eff2}
- 3. Coefficient of friction of sliding surface 1, μ_1
- 4. Coefficient of friction of sliding surface 2 and 3, μ_2
- 5. Coefficient of friction of sliding surface 4, μ_4
- 6. Displacement capacity of sliding surface 1 and 4, d_1
- 7. Displacement capacity of sliding surface 2 and 3, d_2

The constant values and range of variation interval for seven selected design parameter are shown in Table 5.1. All parameters were specified in agreement with range of values reported by manufacturers (Fenz & Constantinou, 2008). The constant values were chosen equal to the values reported for verification model (Fenz & Constantinou, 2008c).

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	R _{effl} (m)	$R_{eff2}(m)$	μ	μ_2	μ3	d ₁ (m)	d ₂ (m)	
Variation interval of each parameter	0.44-1	0.044-0.1	0.06-0.018	0.01-0.02	0.05-0.15	0.054-0.11	0.02205	
Constant value of each parameter	0.435	0.053	0.02	0.015	0.06	0.064	0.019	

Table 5.1. The values of design parameters for sensitivity analysis

6. RESULTS OF OPTIMIZATION APPLYING GENETIC ALGORITHM (GA)

Genetic Algorithm (GA) is a search meta-heuristic optimization tool that mimics the process of natural evolution. It is based on the idea that firstly the productions of natural processes are optimum and secondly the method of reproduction in natural evolution is optimum itself. In GA a collection of possible solutions (population or candidate solutions) is considered first and then using the search method the better answers will be chosen. Applying the processes governing the natural systems and using chosen answers, new collection of solutions will be generated in next step as a new generation. These steps will be iterated to reach more progressed generations. At the end, one of the generations with desirable adaptive response to the solution of the problem will be chosen as the optimum solution (Goldberg, 1989).

The purpose of this research is to find the optimum design parameters of TFPBs for near-field motions which is an optimization problem for minimizing specific implicit functions of design parameters of a TFPB. Hence in this section, target response functions of design parameters are defined for the GA optimization tool. There will be some assumptions for GA optimization analysis in the following also. The results of optimization process using GA for 3 levels of hazard are summarized using diagrams afterwards.

6.1. Defining problem and assumptions

In short and medium height isolated structures, the dynamic response of system is not hugely impressed by the superstructure specifications. The most effective parameter of superstructures that its value will directly change the vibration period of first mode is total mass. But as we know, one of the most important properties of pendulum bearings is that the vibration period of first mode is a function of radius of sliding surface. This trait makes them different from other types of isolation systems such as elastomeric ones which the vibration period of the structure is a function of total mass of superstructure and stiffness of isolation system. According above discussion the derived results from optimum solution analysis of TFPB can be generalized for short and medium height isolated structures.

To optimally design a TFPB in a way that minimize MRSD and MDIL, the first step in applying GA

method is to define the fitness functions (Equations 6.1 and 6.2 for f1 and f2) which are an implicit function of TFPB's seven design parameters. For iterations of optimization process, one of these target functions will be evaluated.

$$f_1(R_{eff 1}, R_{eff 2}, \mu_1, \mu_2, \mu_3, d_1, d_2) = (\text{Re lative Story Drift})_{max} = MRSD$$
(6.1)

$$f_2(R_{eff_1}, R_{eff_2}, \mu_1, \mu_2, \mu_3, d_1, d_2) = (Displacement of Isolator Level.)_{max} = MDIL$$
(6.2)

The intervals of variation for each parameter are selected as same values of sensitivity analysis in Table 5.1. To satisfy the fully adaptive assumption and modelling assumptions, a nonlinear constraint was used together with the interval limitation.

The population type was considered as a numerical vector and its size was chosen to be 20 for first try and error. The creation function was considered as a Constraint Dependant Function and the Selection Function which is responsible for choosing the next productive generation was assumed as Stochastic Uniform. To produce new generation (reproduction), Elite Count assumed to be 2 and the Crossover Fraction assumed to be 0.8. Mutation Function considered as constraint dependant and Crossover Function as Scattered type. Initial Penalty and the Penalty Factor assumed to be 10 and 100 respectively. Stopping Criteria was chosen: 1- the number of generation passes 100 and 2-the tolerance of Fitness Function becomes less than 10^{-6}

6.2. Variation of optimum design parameters

As an example, the results of optimization process for R_{eff1} to minimize the fitness function f_1 (MRSD) are shown in Figure 6.1 for three seismic hazard levels (MCE, DBE and SLE). As it is shown, the ground motion records in Table 4.1 have been arranged by their pulse periods order in X axis from 1 to 7 seconds. It can be concluded that for different hazard levels, the optimum values for R_{eff1} to minimize MRSD are in a close range and have a similar variation regarding to pulse periods. The optimum values of R_{eff1} for wide range of pulse periods (1-6 sec) are limited between 0.8 and 1 meter. Hence choosing a radius of curvature between 0.8 to 1 meter for sliding surface 1 and 4 can give minimum relative displacement of story.



Figure 6.1. Optimum values of Reff1 that minimize MRSD in three levels of MCE, DBE and SLE

Figure 6.2 also displays the optimum values of R_{eff1} that minimize fitness function f_2 (MDIL). In this figure the variation of optimum R_{eff1} has been shown for ground motions with different pulse periods in three levels of MCE, DBE, and SLE. In a wide range of periods (2 to 7 sec) the optimum values of R_{eff1} vary between 0.4 and 0.6 meter. In other words, choosing a value between 0.4 and 0.6 meter for R_{eff1} can minimize the maximum displacement in isolation level of structures that is imposed by near field ground earthquakes with 2 to 7 periods of pulse. It is concluded from the diagram that for larger pulse periods of earthquake, the optimum design values of R_{eff1} will be larger.



Figure 6.2. Optimum values of Reff1 that minimize MDIL in three levels of MCE, DBE and SLE

The same analyses have been done for other parameters. According to the optimization analysis for other parameters, Table 6.1 proposes ranges of optimum values that minimize maximum relative story displacement (MRSD) and maximum story acceleration (MDIL).

Design parameter	Proposed range for minimizing MRSD (or MSA)	Proposed range for minimizing MDIL
Effective radius of curvature of sliding surfaces 1 and 4, R _{eff1}	0.8-1 meter	0.4-0.6 meter
Effective radius of curvature of sliding surfaces 2 and 3, R _{eff2}	0.06-0.1 meter	0.05-0.07 meter
Coefficient of friction of sliding surface 1, µ ₁	0.02-0.06	0.04-0.06
Coefficient of friction of sliding surface 2 and 3, μ_2	0.01-0.018	0.015-0.018
Coefficient of friction of sliding surface 4, µ4	0.06-0.14	0.08-0.14
Displacement capacity of sliding surface 1 and 4, d1	0.08-0.11 meter	0.06-0.08 meter
Displacement capacity of sliding surface 2 and 3, d ₂	0.022-0.035 meter	0.04-0.05 meter

Table 6.1. Proposed range for design parameters to minimize fitness functions

Figure 6.3 shows the minimums of MRSD (f_1) and MDIL (f_2) due to optimum values of design parameters using GA.



Figure 6.3. a) Minimum values of f_1 (MRSD) according to optimum design parameters, b) Minimum values of f_2 (MDIL) according to optimum design parameters

6.3. Optimization for minimizing fitness functions simultaneously

In previous section, the optimization have been done independently for each of fitness functions f_1 and f_2 . However the ideal is to simultaneously minimize all of fitness functions to limit MDIL and MRSD.

So a new fitness function was defined according to equation 6.3.

$$f\left(R_{eff\,1}, R_{eff\,2}, \mu_1, \mu_2, \mu_3, d_1, d_2\right) = \sum_{i=1}^n a_i \, \frac{f_i}{\min(f_i)}$$
(6.3)

According to equation 6.3., f_i are the fitness functions that identify only one (maximum) response of structure (for example f_1 and f_2 which was defined by equation 6.1 and 6.2). These functions are known as Single Objective functions. Hence "n" is the number of single objective functions. $Min(f_i)$ is obtained from optimization process of each single objective independently (for example the $Min(f_1)$ and $Min(f_2)$ are the minimum of f_1 and f_2 that are obtained in previous section). a_i is the weight of each single objective functions that shows the importance of each single function. The summation of values

of a_i is usually equal to one $(\sum_{i=1}^{n} a_i = 1)$. Hence function f is a summation of single objective

functions, it is known as a Multi Objective function. Applying the optimization process done in last section, the results will minimize the structural response simultaneously. Due to this fact that the optimization was done for f_1 and f_2 , considering n=2 the multi objective function f can be defined. The importance of each single objective function considered to be equal so $a_1 = a_2 = \frac{1}{n} = 0.5$. The optimum

values of design parameters have been plotted for different earthquakes in 3 levels of hazards. Table 6.2 summarizes those optimum values suggesting interval for each parameter.

Table 6.2.	Proposed	range for	design	parameters to	o minimize	multi ob	jective	function
		0	0	1				

Design parameter	Proposed range for minimizing f
Effective radius of curvature of sliding surfaces 1 and 4, R_{eff1}	0.7-1 meter
Effective radius of curvature of sliding surfaces 2 and 3, R_{eff2}	0.05-0.07 meter
Coefficient of friction of sliding surface 1, μ_1	0.045-0.06
Coefficient of friction of sliding surface 2 and 3, μ_2	0.013-0.019
Coefficient of friction of sliding surface 4, μ_4	0.1-0.15
Displacement capacity of sliding surface 1 and 4, d_1	0.08-0.11 meter
Displacement capacity of sliding surface 2 and 3, d ₂	0.03-0.05 meter

Figure 6.4 shows the minimum values of function "f" in different hazard levels due to optimum design parameters. As it is shown for most of earthquakes the value of "f" is close to one. According to definition of function f, this means that both of single objective functions f_1 and f_2 are at their minimum values. So the design parameters can be arranged in a way that minimizes both of structural responses simultaneously. This shows the exclusive specification of TFPBs.



Figure 6.4. The minimum values of multi objective function *f* due to optimum values of design parameters in 3 hazard levels

7. CONCLUSION

The goal of this paper was to introduce a method to design the parameters of TFPBs used in isolated structures imposed by near field motions. In this process the mechanical behavior of the TFPBs was modeled by 3 series single FP elements. As the result the parameters of series model was obtained based on TFPB design parameters. Afterwards the state equations for an isolated structure was governed parametrically to be solved using MATLAB. The equations were solved analytically using response history analysis and the results were verified. The response of structure to fluctuations of design parameters was evaluated by sensitivity analysis. Seven different records of near field motions with pulse periods between 1 and 7 seconds in three hazard levels were considered for analysis. The procedure continued applying GA to optimize the design parameters for two single objective and one multi objective functions. At the end a range for optimum values of design parameters were proposed to minimize the structural responses. The results conclusion can be summarized as following:

- Variation of seven considered design parameters has not same effect on structural response in optimization process.

- Due to this fact that the base isolation systems are mostly designed for low and medium height buildings and the serving superstructures have a rigid dynamic behavior, and also considering the fact that the mass of superstructures does not affect the vibration periods of structures isolated via friction pendulum bearings unlike the ones isolated with elastomeric isolators, therefore the results of optimization process in this research can be generalized for different structures.

- Except some occasions the optimum design parameters were closed to each other in 3 levels of MCE, DBE, and SLE so the influence of hazard levels could be negligible.

- For some parameters such as effective radius of curvatures of surface 1 and 4, a close range could be considered for optimum values.

- For most of the earthquakes, the minimum values of multi objective function "f" show that the maximum responses of structure can be minimized simultaneously.

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