Basis for Acceleration-Based Conceptual Design of Low-Height Base-Isolated Systems



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SUMMARY

The dynamic response of a series of base-isolated systems subjected to ground motions that were recorded in firm soils located inshore of the Mexican Pacific was estimated in order to evaluate the influence of the structural properties of these systems in their floor acceleration demands. While the super-structures were assumed to remain elastic, the energy dissipation capacity of the base-isolation systems was represented through a viscous damping model, and the different damping levels in the super-structures and isolation systems were taken into account through a non-classical damping approach. After identifying in general terms the structural properties of the systems that are able to better control their floor acceleration demands, an equivalent single-degree-of-freedom system that can be used within an acceleration-based format to conceive base-isolation systems, is discussed.

Keywords: Acceleration-based design, Base Isolation, Contents

1. INTRODUCTION

The excessive losses derived from the unsatisfactory seismic performance of buildings designed according to worldwide accepted standard practice has created discomfort in the structural engineering community. This has gained particular importance since the unacceptably high material and socio-economic losses derived from recent worldwide seismic events (Northridge 1994, Kobe 1995, Taiwan 1999, Sichuan 2008, Chile 2010). The level of loss has highlighted the need to: A) Establish design criteria distinct from that specified in current building codes; and B) Develop innovative design approaches that can explicitly control the level of damage and loss suffered by buildings built in high seismicity zones.

The main objective of performance-based formats is to promote the design of earthquake-resistant structures that have the ability to control their dynamic response within thresholds associated to well-defined levels of damage. Currently, the most used response parameter to achieve adequate seismic performance is the maximum lateral displacement/drift demand. Nevertheless, under some circumstances, there are other demands that should be controlled to achieve adequate overall seismic performance. For instance, take into consideration that acceleration-sensitive contents of certain type of buildings, like museums, healthcare and manufacturing facilities, can represent a large percentage of the cost of the building. Recent studies suggest that the estimation of acceleration demands for non-structural components on fixed-based buildings depends on several variables, in such a manner that the formulation of rational design methods rapidly becomes complicate for a practical application. Among other things, acceleration demands depend on the: A) Type of structural system and the variation along height of its mass and structural properties; B) Height of the building and location along height of the non-structural component; and C) Interaction that occurs between the dynamic and mechanic characteristics of the structural system and the frequency and energy content of the ground motion (Villaverde 1997, Medina et al. 2006).

The supports and anchors for non-structural components are usually designed so that they are able to withstand without collapse, toppling and shifting the acceleration demands imposed on them by the design ground motion. Within this black-box approach, no effort is carried out to control the dynamic response of sensitive and valuable contents housed within some non-structural components. In highly refined facilities, such as hospitals and high-tech manufacturing facilities, a black-box design approach may easily result in loss of operation and unacceptable damage on sensitive equipment and contents, in such a manner that there is a need to formulate design methodologies that allow the determination of structural properties for a building that are capable of controlling its acceleration demands.

An option to reduce acceleration demands on non-structural components is through energy dissipation provided by the structural system (Medina et al. 2006, Lavan 2010). An efficient alternative in terms of acceleration reduction is to provide energy dissipation capability directly to the non-structural components (Adam and Fotiu 2000, Villaverde 2006). Nevertheless, in terms of effective and efficient overall acceleration control, there is wide analytical, experimental and field evidence that base-isolation is the best option (Kelly 1982, Dolce and Cardone 2003, Ventura et al. 2003). In spite of their potential for acceleration control, the design of the properties of base-isolation systems and of their super-structures is usually based on strength or displacement-based formats. Accelerations demands, if at all, are estimated as a byproduct of the design procedure, and this may lead to several design iterations when acceleration demands on non-structural components need to be explicitly controlled.

This paper evaluates the influence of the structural properties of some base-isolated systems in their acceleration demands, and through the integration of the results that it presents, discusses the basis for an acceleration-based format for the conception and preliminary design of base-isolation systems. The paper focuses on elastic and viscously damped base isolation systems subjected to intense ground motions recorded at firm soil sites located in the Mexican Pacific Coast.

2. DYNAMICS OF BASE-ISOLATED SYSTEMS

Classical damping approaches are thoroughly used to estimate the dynamic response of elastic systems with structural and non-structural components that exhibit similar damping levels. In case of base-isolated systems, the damping level of the super-structure may be smaller than that of the base-isolation system (in what follows, the words *isolated* and *isolation* will be used, respectively, to denote *base-isolated* and *base-isolation*). Under these circumstances, it is convenient to use a *non-classical damping* formulation to estimate the dynamic response of an elastic isolated system. In terms of non-classical damping, the damping matrix is not orthogonal with respect to the modal shapes of the isolated system, and some components of motion and force are out-of-phase with respect to each other. As a result, the modes of the system can't be uncoupled in terms of damping, and the modal shapes, frequencies and damping levels, depend on the damping matrix of the system. Within this context, a change of variable allows for the determination of the dynamic properties of the non-classical damped system and to establish the modal participation factor of each mode of vibration.

According to the lineal theory for isolated structures, the first mode of vibration of an isolated system is closely related to the lateral displacement of the degree-of-freedom assigned to its isolation system. In this sense, upper modes contribute to the lateral deformation and motion of the super-structure, and thus, damage in the structural and non-structural systems and contents of the isolated structure (Naeim and Kelly 1999). Within this context, it is of interest to study the variation of the modal participation factor associated to the fundamental mode of vibration of various isolated systems in order to understand how to control the effects of upper modes and thus, their consequences to the super-structure.

The isolated systems under consideration in this paper consider an elastic super-structure consisting of a series of masses (one per story), connected through elastic shear springs that condense the lateral stiffness of the different inter-stories. The value of lateral stiffness, reactive mass, and damping level was considered constant along the entire height of the super-structure. In this sense, the stick models

represent low-height structures with a lateral behavior dominated by global shear deformations. An elastic model with viscous damping was considered for the isolation system. This behavior would be typical of normal and high damping rubber isolators, or that of rubber isolators complemented with viscous damping devices.

Table 1 summarizes the properties of the super-structures and isolation systems under consideration. While ξ_B and ξ_S denote the percentage of critical damping in the isolation system and the super-structure, respectively; T_B is the period of vibration the isolation system would have if all the mass in the system was assigned to its degree-of-freedom; and T_S the fundamental period of vibration the super-structure would have if it was supported on a fixed base.

Tuble 1. Structural properties of isolated systems				
Stories	Super-structure		Isolation System	
	T_{S} (sec)	$\xi_{S}(\%)$	$T_B(\text{sec})$	$\xi_B(\%)$
2 4 8	0.05, 0.17, 0.28, 0.40 0.15, 0.30, 0.50, 0.70 0.30, 0.50, 0.70, 1.00	2	1.5 2.0 2.5 3.0	10 15 20 25

Table 1. Structural properties of isolated systems

Two parameters that are relevant to the dynamic response of isolated structures are: A) The ratio between the periods T_B and T_S ; and B) The relation between the mass of the super-structure and that assigned to the isolation system. Within this context, the results that will be presented next will be discussed in terms of the period ratio T_B/T_S , and the mass ratio γ , defined as (Naeim and Kelly 1999):

$$\gamma = \frac{\sum_{i=1}^{N-1} m_i}{m_B + \sum_{i=1}^{N-1} m_i}$$
(1)

where m_i is the mass assigned to the *ith* degree-of-freedom of the super-structure; N the total number of degrees-of-freedom; and m_B the mass assigned to the isolation system.

Figure 1 shows, for isolated two-story super-structures, the modal participation factor corresponding to their first mode of vibration (Γ_1). To provide this factor with a quantitative meaning, it has been normalized in such a manner that a value of one corresponds to a global lateral response fully dominated by the fundamental mode of vibration. While the damping of the isolation system does not have a significant influence on the participation factor, the influence of γ in this factor tends to be moderate. Particularly, as γ increases (the mass in the super-structure increases relative to that of the isolation system), the participation factor tends to increase. Note that as the value of T_B/T_S increases from cero to ten, Γ_1 increases rapidly. Once T_B/T_S reaches the value of ten, Γ_1 tends to increase at a smaller rate; and even at a much smaller rate once it exceeds the value of sixteen. This implies, in congruency with the linear theory discussed by Naeim and Kelly (1999), that an increase in the value of T_B/T_S results, for small and moderate values of this ratio, in a significant decrease of the participation of upper modes to the lateral deformation and dynamic response of the super-structure. Although with some particularities, similar tendencies can be observed for super-structures having different number of stories.

3. FLOOR RESPONSE OF BASE-ISOLATED SYSTEMS

In terms of acceleration control, it is convenient to study the dynamic response of isolated systems subjected to the action of intense ground motions. With this purpose, a set of twenty-two ground motions recorded in the Mexican Pacific Coast was formed. The amplitude of the motions, which

exhibit similar dynamic characteristics, was scaled in such a manner as to exhibit a peak ground acceleration corresponding to a return period of five hundred years for the coast of the state of Guerrero. In general terms, it could be said that the set of motions represents the design earthquake for structures built in firm soil sites located along the coast of Guerrero. Figure 2 shows mean pseudo-acceleration ($S_{a'}/g$) and displacement (S_{d}) spectra, respectively, obtained for the set of motions, elastic behavior, and different percentages of critical damping (ξ). Note the moderate displacement demands for periods larger than two seconds, in such a manner that unlike high intensity epicentral motions with directivity effects recorded in parts of California and Japan, displacement demands at the base of an isolated structure located in the Mexican Pacific would not represent a limitation for its acceleration-based conception.



Figure 1. Participation factor of fundamental mode of vibration of two-story isolated systems: a) $\gamma = 0.80$; b) $\gamma = 0.67$; c) $\gamma = 0.50$; d) $\xi_B = 0.25$



Figure 2. Mean of elastic pseudo-acceleration and pseudo-displacement spectra for ground motions recorded at firm soil sites located at the Mexican Pacific Coast: a) Pseudo-acceleration; b) Displacement

The lateral motion exhibited by the floors of an isolated system can be used to identify the participation of its different modes of vibration. This can be achieved in two manners: A) In the frequency domain through Fourier spectra, and B) In the time domain through floor spectra. The

integration of the information provided by both types of spectra provides useful insights into the disaggregated dynamic response of isolated structures.

3.1. Frequency Domain

A Fourier amplitude spectrum provides direct information about the frequency content of the floor response, and in this sense, allows for an understanding on how the motion is filtered by the isolated structure from the ground level to its floor levels. Figure 3 show Fourier spectra for the time-history of the absolute roof acceleration of four-story isolated systems. Note that while the super-structure analyzed in Figure 3a can be considered flexible ($T_s = 0.7$ seconds for a four-story super-structure on fixed base), the one in Figure 3b can be considered stiff ($T_s = 0.15$ seconds). The spectra included in the figures exhibit a large peak in a frequency band located around the fundamental frequency of vibration of the isolated system. In the particular case of Figure 3a, the spectra exhibit noticeable but smaller response peaks in high frequencies, which are associated to the upper modes of vibration of the isolated system. While the location of the smaller peaks is fairly independent of ξ_B , their amplitude increases moderately in relative terms with an increase in ξ_B . The comparison of the results shown in Figures 3a and 3b also allows for an understanding of the influence of T_B/T_s in the dynamic response of isolated systems. The end result of the larger values of T_B/T_s under consideration in Figure 3b is the disappearance of the minor but well defined peaks located in the higher frequency region.



Figure 3. Fourier spectra of the time-history of absolute roof acceleration in four-story isolated systems: a) flexible super-structure, b) rigid super-structure

3.2. Time Domain

Although it is not possible to use floor spectra to assess the acceleration demands on any type of content, this representation can be useful to develop an understanding of what needs to be done from a structural point of view to control the acceleration demands in isolated systems. It should be mentioned that floor spectra cannot be applied to assess the performance of those contents that are capable, due to their mass and dynamic properties, of dynamically interacting with the super-structure to such degree as to modify the global response of the entire isolated system. Within this context, floor response spectra methods have been observed to provide reasonable estimates of acceleration demands for non-structural components with masses that are much smaller than those of the supporting structure, and whose frequencies are not close to one of the natural frequencies of the structural system (Villaverde 1997, 2006). Because the use of these spectra neglect the dynamic interaction between non-structural component and structural system, and do not take into consideration the out-of-phase components of motion that take place in elements with different damping, their use yields overly conservative acceleration demands for "resonant" components.

According to the observations made by Sankaranarayanan (2007), three regions of period can be considered for floor acceleration spectra: A) Short; B) Intermediate; and C) Long. As illustrated in Figure 4a, the first region contemplates non-structural components whose periods are such that the ratio between their period (T_c) and T_B does not exceed 0.5. The period ratios for the second region range from 0.5 to 1.5, and those corresponding to the third region exceed the value of 1.5.

Figures 4 and 5 show the absolute acceleration spectra corresponding to the roof motion of several four-story isolated systems. The spectra, which were obtained for non-structural components with 2% of critical damping, represent the mean spectra of those corresponding to all ground motions under consideration. Note that the figures contemplate different values of ξ_B and T_B/T_S .



Figure 4. Absolute acceleration spectra at roof of flexible four-story isolated systems



Figure 5. Absolute acceleration spectra at roof of stiff four-story isolated systems

Due to the significant participation of upper modes in the dynamic response of flexible superstructures, the spectral acceleration included in Figure 4 shows considerable amplification of accelerations in the short period range. Particularly, there are two noticeable spectral peaks within this range, which are closely related to the second and third modes of vibration of the isolated system. Results from Figure 5 help understanding the consequences of increasing the lateral stiffness of the super-structure relative to the isolation system. In general terms, and in congruency with the linear theory discussed by Naeim and Kelly (1999), it can be said that the contribution of upper modes to the acceleration demands on the contents diminishes considerably. While an increase of ξ_B results in a noticeable reduction of the acceleration in the contents located in the short range of periods, it may be reflected in small increments of acceleration in the control, the ideal situation for an isolated system can be formulated in terms of a flexible isolation system integrated with a very stiff super-structure.

Note that an increase in the damping level of the isolation system reduces the response of nonstructural components that fall in the intermediate region of the floor acceleration spectra, and that this increase is not reflected in smaller acceleration demands in the short period region. This observation is consistent with what has been observed for the acceleration demands on fixed-based systems in terms of the ineffectiveness of energy dissipation in reducing the contribution of upper modes to acceleration demands (Rodriguez et al. 2007). It is also worthy to note that an increase in the lateral stiffness of the super-structure relative to that of its isolation system results for a complex multi-degree-of-freedom system with multi-peaked floor response spectra to exhibit a single-degree-of-freedom behavior with single-peak floor spectra.

4. ACCELERATION RATIOS

One manner of characterizing the amplification of acceleration on an isolated system is through an acceleration ratio:

$$R_n = \frac{\max(ii_n)}{\max(ii_B)} \tag{2}$$

where *n* is the number of stories of the super-structure; \ddot{u}_n represents the time-history of absolute acceleration at the roof level; and \ddot{u}_B represents the time-history of absolute acceleration at the isolation (base) level.

Figure 6a shows acceleration ratios for four-story isolated systems having ξ_B of 10% and γ of 0.80. There is a noticeable tendency for the median value and dispersion of R_4 to decrease with respect to an increase in T_B/T_S . It can be said that not only does the level of amplification of motion decreases in the super-structure as its lateral stiffness increases, but that a reduction of the effects of upper modes is reflected in a larger certainty in terms of predicting the levels of acceleration. Figures 6b to 6d show central tendencies for median values of R_4 . As the value of γ increases, the value of R_4 tends to decrease for a given value of T_B/T_S , particularly for small values of this period ratio. An increase in ξ_B results in larger values of R_4 .

5. EQUIVALENT SYSTEM FOR BASE-ISOLATED STRUCTURES

Response spectra are a valuable source of information for the conception of complex earthquakeresistant structures. As long as the parameter that defines the seismic performance of the structure can be estimated reasonably well from its fundamental mode of vibration, a spectrum can be used during the preliminary stages of design to conceive a set of global structural properties that can adequately control the global dynamic response of the structure. Within this context, it is of interest to compare the response of isolated systems with their corresponding spectral ordinates to study the possibility of establishing simple acceleration-based methodologies for the conception of isolated structures.

In terms of this paper, the response parameter of interest is the maximum absolute acceleration. Figure 7 superimposes the maximum absolute roof and base acceleration demands (normalized by the acceleration of gravity) of several four-story isolated systems with γ of 0.8 and ζ_B of 0.10, to elastic pseudo-acceleration spectra obtained for a percentage of critical damping that is equal to the level of damping corresponding to the isolation system. While the abscissa assigned in the plots to the absolute roof and base acceleration demands correspond to the period of vibration of the isolation systems (T_B), the spectral ordinates as well as the absolute acceleration demands correspond to the mean value of the ordinates and demands, respectively, corresponding to all ground motions under consideration.

It is usually considered that a reasonable estimate for the maximum absolute acceleration demand in an earthquake-resistant structure requires from the explicit consideration of at least the first three modes of vibration. It is not surprising then that the acceleration demands in Figure 7 exceed their corresponding spectral ordinates. Nevertheless, note that by adequately designing the stiffness of the super-structure relative to that of the isolation system, it is possible to maximize the contribution of the fundamental mode of vibration to the dynamic response of the isolated systems. Under these circumstances, the absolute roof and base acceleration demands fit well within their corresponding spectra. In these terms, the ideal situation implies a flexible isolation system with a fairly low level of damping, and a stiff super-structure. To illustrate this, note in Figure 7 that the acceleration demands that correspond to isolated systems with T_B/T_S equal or larger than 10 (white triangles), practically fall over their corresponding pseudo-acceleration spectra. Although not illustrated in the figure, it was observed that an increase on the level of damping of the base isolated system resulted, as a consequence of a larger influence of higher modes, in the absolute acceleration demands to slightly move away from their corresponding spectra.



Figure 6. Acceleration ratio for four-story isolated systems: A) Dispersion for $\zeta_B = 0.10$ and $\gamma = 0.80$; B) Central tendencies for $\gamma = 0.67$; C) Central tendencies for $\gamma = 0.80$; D) Central tendencies for $\gamma = 0.89$



Figure 7. Comparison of absolute acceleration demands in isolated structure and corresponding spectral ordinates: a) Roof; b) Base

A reasonable estimate of absolute floor acceleration demands for contents located in isolated systems whose dynamic response is dominated by their fundamental mode of vibration can be obtained from a slightly modified version of the direct method discussed by Yasui et al. (1993):

$$A_{C}(T_{C},\xi_{C}) = \frac{\sqrt{\left[\left(\frac{T_{B}}{T_{C}}\right)^{2}S_{a}(T_{B},\xi_{B})\right]^{2} + S_{a}(T_{C},\xi_{C})^{2}}}{\sqrt{1 - \left(\frac{T_{B}}{T_{C}}\right)^{2} + 4\left(\xi_{B} + \xi_{C}\right)^{2}\left(\frac{T_{B}}{T_{C}}\right)^{2}}}$$
(3)

where A_C denotes the absolute acceleration demand for a low-damped elastic non-structural component; S_a a pseudo-acceleration spectral ordinate; T_C and T_B the period of the non-structural component and that the isolation system would have if all the mass in the structure was assigned to the

degree-of-freedom at the base, respectively; and ξ_c and ξ_B the damping ratios corresponding to the component and the isolation system, respectively.

Figure 8 compares the absolute acceleration demands derived from time-history analyses with those estimated with Equation 3. As expected, a good approximation of acceleration demands is obtained for systems having a large value of T_B/T_s . The results summarized in Figure 8 confirm the fact that a base isolated structural system behaves as a single-degree-of-freedom system provided the lateral stiffness of the super-structure is large enough with respect to that of the isolation system.



Figure 8. Comparison of absolute roof acceleration demands for contents with 2% damping: a) $T_B = 1.5$ sec, $T_S = 0.15$ sec ($T_B/T_S = 10$), and $\xi_B = 0.20$; b) $T_B = 2.0$ sec, $T_S = 0.15$ sec ($T_B/T_S = 13.33$), and $\xi_B = 0.10$

6. DISCUSSION

Analytical, experimental and field studies have consistently shown the reliability and capability of isolation systems to reduce the dynamic response of earthquake-resistant structures. Damage control in contents is one of the most referred qualities of base-isolated systems. Nevertheless, it is interesting to note that currently, there is a lack of performance-based formats for the preliminary quantification of a set of global structural properties for isolated systems that can explicitly control their acceleration demands within well-defined thresholds.

In terms of *acceleration supply*, several studies are currently under way to understand the capacity of different types of contents in terms of lateral acceleration. The evolution and maturity of these studies should allow the definition of acceleration thresholds in terms of the type of contents and their dynamic characteristics. In the meantime, it is important for the community of structural engineers to work on the *acceleration demand* part of the equation. In these terms, the results summarized in this paper indicate that if no other consideration restricts the design procedure, the structural properties of the isolation system and its super-structure could be established in such a way as to control their global and local lateral acceleration demands. Particularly, if the lateral stiffness of the super-structure is properly designed with respect to that of the isolation system, the global structural properties of an isolation system and its superstructure (T_B , ξ_B and T_S) can be determined within an acceleration-based format with the aid of: A) Acceleration ratios such as those summarized in Figure 6; B) Pseudo-acceleration spectra corresponding to the design ground motion; and C) Equation 3.

Although an increase in the level of damping in the isolation system tends to increment the contribution of upper modes, the use of stiff structural systems (characterized by values of T_B/T_S ranging from eight to ten) results in adequate control of upper mode contribution. The use of percentages of critical damping in the isolation system that are equal or smaller than 10% reduce the lateral stiffness requirements for the super-structure, in such a manner that values of T_B/T_S of at least six seem to be sufficient in terms of upper mode control. In very simple terms, it can be said that flexible isolation systems with low levels of damping coupled with stiff super-structures result in

substantial reductions of the participation of upper modes to the global response of isolated structures. Within this context, it is possible to formulate a simple model for base-isolated systems that would allow for the acceleration-based conception of its global structural properties. Once these properties have been determined, they can be used to achieve an adequate design at the local level of the isolators and the structural elements of the super-structure.

In many cases, the displacement demand at the base of an isolation system does not define its seismic design. Take the case of the Mexican Pacific Coast; studies carried out so far suggest that the largest displacement demands at the base of isolated structures should not exceed 25 to 30 cm (even for low levels of damping, as illustrated in Figure 2b). Nevertheless, under certain circumstances, the lateral displacement in the isolators may be the defining aspect during their seismic design and in this case, a compromise should be taken between displacement control at the base and acceleration control in the super-structure.

Another issue to emphasize is the fact that a reduction in upper mode effects results in predictable behavior of the super-structure in terms of its acceleration demands. Thus, not only are the acceleration demands smaller, but the large uncertainty and complexity observed in these demands on fixed-based systems are practically reduced to estimating accurately the acceleration demands at the base of the isolated structure.

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