System Identification of the Saraighat Bridge using Ambient Vibration Data: A Case Study

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Summary:
The Saraighat bridge, a 1.3 km long rail-cum-road double-deck steel truss bridge over the mighty river Brahmaputra in Assam, India, was made open to traffic in 1962 and is considered as a lifeline of North-East India. This important bridge structure is targeted for health monitoring due to its age as well as its existence in high seismic zone. Moreover, there exists an interest to observe the modal parameters of this unique bridge structure. Operational modal analysis (OMA) is carried out for modal parameter identification using the ambient vibration data in form of acceleration response. Analysis is carried out using three major techniques both in time and frequency domain. Identified natural frequencies based on all the techniques are observed to be in good agreement.

Keywords: bridge, ambient vibration, operational modal analysis, system identification.

1. INTRODUCTION

Structural system identification helps to construct fairly accurate numerical model of a structural system based on the knowledge of physical input as well as output. This area, generally referred as experimental modal analysis (EMA) (Ewins 2000), considers commonly modal model as the identification model of a structural system. However, in case of large structures it becomes difficult to carry out EMA providing artificial input excitation. Hence, operational modal analysis (OMA) using output-only response data is highly considered for identification of modal model associated with natural frequency, damping ratio and mode shape in case of large structures. Modal parameters identified periodically can be considered for structural health monitoring (SHM). Further, identified modal parameters are required to update the numerical FE model to a more accurate one. Such an updated numerical finite element (FE) model is further required for reasonable prediction of output as well as effective design of control devices.

One technique which is considered as a major contribution for OMA in time domain is natural excitation technique (NExT) (James *et al.* 1995). It explains that the correlation function (CF) between two ambient vibration response measurements has the same analytical form as the impulse response function (IRF). This helps to apply the major non-parametric system identification techniques in OMA using CF in the form of IRF. Eigensystem realization algorithm (ERA) (Juang & Pappa 1985; Juang & Pappa 1986) is such a technique used for multi-input multi output (MIMO) based identification using IRF or CF. ERA is developed based on minimum realization theory in discrete time domain of system engineering. Next, the data-driven stochastic subspace identification (SSI-DATA) technique (Van Overschee & De Moor 1996) identifies the structural system in form of state space matrices directly from the ambient response data. Some of the important inherent steps / measures of SSI-DATA are orthogonal or oblique projection, estimation of Kalman states, least-square, singular value decomposition (SVD) etc. Finally the modal parameters are obtained using the identified state space model. Covariance-driven stochastic subspace identification (SSI-COR) technique (Peeters & De Roeck 1999) is another technique based on the framework of SSI. The block Toeplitz matrix is decomposed using SVD to obtain the observability matrix and the stochastic controllability matrix. Subsequently, the modal parameters are obtained. In many cases, it becomes difficult to make measurement from all the degrees of freedom (DOF) at once while carrying out the ambient testing of large structures. All those DOF are divided into several set-ups using overlapping common reference sensor and data are recorded for different set-ups at different time. A novel approach based on SSI as SSI-reference (Peeters & De Roeck 1999; Peeters & De Roeck 2008) is developed to consider the multiple set-ups in the identification stage itself. Some other important techniques (Maia and Silva 1998) in time domain are referred as: Least-squares complex exponential (LSCE) algorithm, polyreference least-squares complex exponential (PRCE) method, Ibrahim time-domain (ITD) as well as AR and ARMA based techniques. LSCE and PRCE are two techniques which apply least-square for modal parameter identification. LSCE is single input multi output (SIMO) based technique and PRCE is considered as the MIMO extension of LSCE. ITD technique is another SIMO technique which identifies the modal parameters using the free decay responses. Free decay responses can be estimated from the ambient vibration responses using the random decrement technique and this makes possible for the techniques like ITD to identify the modal parameters using ambient vibration data. Further, auto regressive (AR) as well as auto regressive moving average (ARMA) based identification techniques have been implemented for modal parameter identification using ambient responses.

In case of frequency domain, the simplest method is the peak-picking (PP) method (Bishop & Gladwell 1963). It provides reasonably good results under the assumption that the modes are well separated and the damping is lower. Frequency domain decomposition (FDD) method (Brincker *et al.* 2001) is presented based on the SVD of the power spectral density (PSD) matrix at every discrete frequency. Natural frequency and damping ratios can be obtained using the small segment of single degree of freedom (SDOF) density function around the peak of a PSD function. The first singular vector corresponding to a natural frequency is considered as the mode shape of that frequency. FDD method is further improved as frequency-spatial domain decomposition (FSDD) method (Zhang *et al.* 2010) based on similarity establishment with the well-accepted complex mode indicator function (CMIF) method. Modal damping ratios are identified in FSDD by SDOF curve fitting over the density function obtained around the peak of the PSD based on higher MAC.

In the present study, Saraighat Bridge - a large double deck steel truss bridge, is considered for the modal parameter identification. Three important and widely used techniques are employed for identification of modal parameters using recorded ambient acceleration time histories. NExT-ERA and SSI-DATA are used as the time domain techniques while FSDD is considered as a frequency domain technique.

2. MODAL IDENTIFICATION TECHNIQUES

The modal parameters are identified using three important techniques: NExT-ERA, SSI (data driven) and FSDD. Some details regarding these techniques are presented below. At first, the state space equations in discrete time domain are referred as

$$\mathbf{s}(k+1) = \mathbf{A}\mathbf{s}(k) + \mathbf{B}\mathbf{z}(k) \tag{1a}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{s}(k) + \mathbf{E}\mathbf{z}(k) \tag{1b}$$

where, **A**, **B**, **C**, **E** represents the discrete time state space matrices, **s** represents the states, k represents the discrete time, while **z** and **y** represent the input and output respectively. These important modal identification techniques are described in the subsequent sub-sections.

2.1. Natural Excitation Technique with Eigen-system Realization Algorithm (NExT-ERA)

It is a two-step identification where NExT (James *et al.* 1995) is applied to estimate the impulse responses from ambient data in the first phase. Subsequently, ERA (Juang & Pappa 1985) is employed to identify the state space matrices in discrete time domain from the estimated impulse responses. Using the estimated impulse responses the Markov parameters blocks are formed and these Markov parameter blocks are used to form the Hankel matrix. The Markov parameters can be written as

$$\mathbf{M}(k) = \mathbf{C}\mathbf{A}^{k-1}\mathbf{B} \tag{2}$$

Considering the number of input and output as n_1 and n_2 respectively, the size of a Markov parameters becomes $n_2 \times n_1$. Now, the Hankel matrix is represented as

$$\mathbf{H}(k-1) = \begin{bmatrix} \mathbf{M}(k) & \mathbf{M}(k+1) & \cdots & \mathbf{M}(k+j) \\ \mathbf{M}(k+1) & \mathbf{M}(k+2) & \cdots & \mathbf{M}(k+j+1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}(k+i) & \mathbf{M}(k+i+1) & \cdots & \mathbf{M}(k+i+j) \end{bmatrix} \tag{3}$$

where, i=1, 2, ….. r_1-1 and j=1, 2, …. s_1-1, with r_1 and s_1 as integers. Now the size of the Hankel matrix becomes as ($n_2 r \times n_1 s$). Hankel matrix for k = 1, $\mathbf{H}(0)$ is decomposed with SVD as

$$\mathbf{H}(0) = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{T}} \tag{4}$$

where, the sizes of $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}^{\mathrm{T}}$ are ($n_2 r \times n_2 r$), ($n_2 r \times n_1 s$) and ($n_1 s \times n_1 s$) respectively. It is considered that $\mathbf{H}(0)$ has $2N$ non-zero singular values (i.e. rank=$2N$), equivalent to the order of state space system. Therefore $\mathbf{H}(0)$ can be recomputed as

$$\mathbf{H}(0) \approx \mathbf{U}_{2N}\mathbf{\Sigma}_{2N}\mathbf{V}_{2N}^{\mathrm{T}} \tag{5}$$

where, the sizes of $\mathbf{U}_{2N}$, $\mathbf{\Sigma}_{2N}$ and $\mathbf{V}_{2N}^{\mathrm{T}}$ are ($n_2 r \times 2N$), ($2N \times 2N$) and ($2N \times n_1 s$) respectively. The estimate of the discrete time state-space are obtained as follows

$$\mathbf{A} = \mathbf{\Sigma}_{2N}^{-1/2}\mathbf{U}_{2N}^{\mathrm{T}}\mathbf{H}(1)\mathbf{V}_{2N}\mathbf{\Sigma}_{2N}^{-1/2} \tag{6a}$$

$$\mathbf{B} = \mathbf{\Sigma}_{2N}^{1/2}\mathbf{V}_{2N}^{\mathrm{T}}\mathbf{E}_2 \tag{6b}$$

$$\mathbf{C} = \mathbf{E}_1^{\mathrm{T}}\mathbf{H}(1)\mathbf{U}_{2N}\mathbf{\Sigma}_{2N}^{1/2} \tag{6c}$$

$\mathbf{E}_1^{\mathrm{T}}$ and $\mathbf{E}_2$, as appeared in the above equations, are defined as follows,

$$\mathbf{E}_1^{\mathrm{T}} = [\mathbf{I} \quad \mathbf{0} \quad \cdots \quad \mathbf{0}] \tag{7a}$$

where, each sub-matrices (identity and zero matrices) is of the size ($n_2 \times n_2$).

$$\mathbf{E}_2 = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \tag{7b}$$

where, each sub-matrices (identity and zero matrices) is of the size ($n_1 \times n_1$).

2.2. Stochastic Subspace Identification (SSI)

Next, SSI-DATA method (Van Overschee & De Moor 1996) is considered for evaluation of the modal parameters. The highlights of this identification technique are mentioned below. The output block Hankel matrix (consisting of $2i$ rows and j columns of output block sub-matrices) is represented in two forms as in Eqns. (8a) and (8b).

$$\mathbf{Y}_{0|2i-1} \stackrel{\text{def}}{=} \left(\begin{array}{cccc} \mathbf{y}_0 & \mathbf{y}_1 & \cdots & \mathbf{y}_{j-1} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{i-2} & \mathbf{y}_{i-1} & \cdots & \mathbf{y}_{i+j-3} \\ \mathbf{y}_{i-1} & \mathbf{y}_i & \cdots & \mathbf{y}_{i+j-2} \\ \hline \mathbf{y}_i & \mathbf{y}_{i+1} & \cdots & \mathbf{y}_{i+j-1} \\ \mathbf{y}_{i+1} & \mathbf{y}_{i+2} & \cdots & \mathbf{y}_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{2i-1} & \mathbf{y}_{2i} & \cdots & \mathbf{y}_{2i+j-2} \end{array}\right) \stackrel{\text{def}}{=} \left(\frac{\mathbf{Y}_{0|i-1}}{\mathbf{Y}_{i|2i-1}}\right) \stackrel{\text{def}}{=} \left(\frac{\mathbf{Y}_1}{\mathbf{Y}_2}\right) \tag{8a}$$

$$\mathbf{Y}_{0|2i-1} \stackrel{\text{def}}{=} \left(\begin{array}{cccc} \mathbf{y}_0 & \mathbf{y}_1 & \cdots & \mathbf{y}_{j-1} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{i-2} & \mathbf{y}_{i-1} & \cdots & \mathbf{y}_{i+j-3} \\ \mathbf{y}_{i-1} & \mathbf{y}_i & \cdots & \mathbf{y}_{i+j-2} \\ \mathbf{y}_i & \mathbf{y}_{i+1} & \cdots & \mathbf{y}_{i+j-1} \\ \hline \mathbf{y}_{i+1} & \mathbf{y}_{i+2} & \cdots & \mathbf{y}_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{2i-1} & \mathbf{y}_{2i} & \cdots & \mathbf{y}_{2i+j-2} \end{array}\right) \stackrel{\text{def}}{=} \left(\frac{\mathbf{Y}_{0|i}}{\mathbf{Y}_{i+1|2i-1}}\right) \stackrel{\text{def}}{=} \left(\frac{\mathbf{Y}_1^+}{\mathbf{Y}_2^-}\right) \tag{8b}$$

One of the key steps in SSI-DATA is projection. The projections are computed as in Eqns. (9a) and (9b).

$$\mathbf{O}_i \stackrel{\text{def}}{=} \mathbf{Y}_2 / \mathbf{Y}_1 \tag{9a}$$

$$\mathbf{O}_{i-1} \stackrel{\text{def}}{=} \mathbf{Y}_2^- / \mathbf{Y}_1^+ \tag{9b}$$

The SVD is computed next for the weighted projection, $\mathbf{W}_1\mathbf{O}_i\mathbf{W}_2$ as in Eqn. (10). Three different choices of algorithms are implemented in SSI-DATA based on three choices of weighting matrices for projection matrix: (a) unweighted principal component (UPC) algorithm, (b) principal component (PC) algorithm and (c) canonical variant algorithm (CVA) (Van Overschee & De Moor 1996).

$$\mathbf{W}_1\mathbf{O}_i\mathbf{W}_2 = \mathbf{U\Sigma V}^{\mathrm{T}} \tag{10}$$

The order is determined inspecting $\mathbf{\Sigma}$ (suppose the order is $2N$). Using the significant part of the decomposed matrices as in Eqn. (5), the extended observability matrix now is computed as in Eqn. (11).

$$\mathbf{\Gamma}_i = \mathbf{W}_1^{-1}\mathbf{U}_{2N}\mathbf{\Sigma}_{2N}^{1/2} \tag{11}$$

$\mathbf{\Gamma}_{i-1}$ is found out with stripping the last l (number of outputs) rows from $\mathbf{\Gamma}_i$ and hence denotes the matrix $\mathbf{\Gamma}_i$ without the last l rows. Evaluation of the Kalman filter state sequences is carried out using the Eqns. (12a) and (12b). Here the symbol "†" represents the Moore-Penrose pseudo-inverse of a matrix.

$$\widehat{\mathbf{X}}_i = \mathbf{\Gamma}_i^{\dagger}\mathbf{O}_i \tag{12a}$$

$$\widehat{\mathbf{X}}_{i+1} = \mathbf{\Gamma}_{i-1}^{\dagger}\mathbf{O}_{i-1} \tag{12b}$$

The least-squares solution is carried out using the Eqn. (13) finally to compute an asymptotically unbiased estimate of $\mathbf{A}$ and $\mathbf{C}$.

$$\begin{pmatrix}\mathbf{A}\\ \mathbf{C}\end{pmatrix} = \begin{pmatrix}\widehat{\mathbf{X}}_{i+1}\\ \mathbf{Y}_{i|i}\end{pmatrix}.\widehat{\mathbf{X}}_i^{\dagger} \quad (13)$$

2.3. Frequency Spatial Domain Decomposition (FSDD)

Both the techniques, FDD (Brincker *et al.* 2001) and FSDD (Zhang *et al.* 2010) are quite similar as they evaluate the modal parameters, except the treatment to the SDOF piece of PSD to identify the frequency and damping ratio. FSDD advocates for curve fitting while FDD considers the inverse Fourier transform. In this present work curve fitting is considered for frequency and damping ratio estimation. Frequency domain decomposition is based on the formula, as mentioned in Eqn. (14), relating the output PSD as $\mathbf{G}_{yy}(\omega)$, stochastic input PSD as $\mathbf{G}_{xx}(\omega)$ and FRF matrix as $\mathbf{H}(\omega)$.

$$\mathbf{G}_{yy}(\omega) = \mathbf{H}(\omega)\mathbf{G}_{xx}(\omega)\mathbf{H}(\omega)^{\mathrm{H}} \quad (14)$$

Ambient vibration is commonly modelled as white noise process and based on this assumption the PSD matrix becomes a constant matrix. Substituting the FRF matrix with the pole/residue form into the output PSD matrix, it also reduces to pole/residue form. Ultimately the output PSD matrix can be written as

$$\mathbf{G}_{yy}^{\mathrm{T}}(\omega) \approx \boldsymbol{\varphi}_k \left[\mathrm{diag}(2\mathrm{Re}\left(\frac{c_k}{(\sqrt{-1})\omega - \lambda_k}\right))\right] \boldsymbol{\gamma}_k^{\mathrm{H}} \quad (15)$$

where, k is the k^{th} pole, c_k is a scalar constant associated with the shape vectors as $\boldsymbol{\varphi}_k$ along with $\boldsymbol{\gamma}_k$. Further by taking the SVD on the estimated output PSD at discrete frequencies $\omega = \omega_j$, following expression can be obtained,

$$\widehat{\mathbf{G}}_{\mathrm{yy}}^{\mathrm{T}}(\omega_j) = \mathbf{U}_j \boldsymbol{\Sigma}_j \mathbf{V}_j^{\mathrm{H}} \quad (16)$$

where, $\mathbf{U}_j$ and $\mathbf{V}_j$ are the unitary matrices, while $\boldsymbol{\Sigma}_j$ is the diagonal matrix consisting of scalar singular values. Observing Eqns. (15) and (16), it is observed that when the frequency approaches to a modal frequency k the k^{th} mode shape dominates there and the first singular vector becomes an estimation of the k^{th} mode shape.

3. THE SARAIGHAT BRIDGE

The Saraighat bridge is a simply supported rail-cum-road bridge which have 10 main spans and 2 approach spans. The length of each of the main spans and approach spans are 118.72 meters and 31.4 meters respectively. This is a steel truss double-deck bridge carrying the rail and road traffic at lower and upper deck respectively. A photographic view of Saraighat bridge is shown in the Fig. 3.1.

Figure 3.1. Photographic view of the Saraighat bridge.

All the spans are structurally uncoupled; hence modal identification is indeed required for each of the spans to complete the modal identification of the whole bridge. However the design of each of the main spans is similar and presently one main span is considered for modal identification.

4. AMBIENT VIBRATION MEASUREMENT

Data is recorded with uni-axial force balance accelerometers (EpiSensor ES-U2, Kinematrics Inc., USA) and 48 channel dynamic data acquisition system (HBM GmbH Germany). Limited numbers of sensors are employed for ambient data recording. The locations are represented using the Fig. 4.1. where, a location associated to a number is chosen considering the level (top, middle or bottom) and left / right positioning following the direction 1 to 17. Letter 'T', 'M', 'B', 'L' and 'R' represent the top, middle, bottom, left and right respectively. A location associated to e.g. 8 at middle level and left side is represented as 8ML.

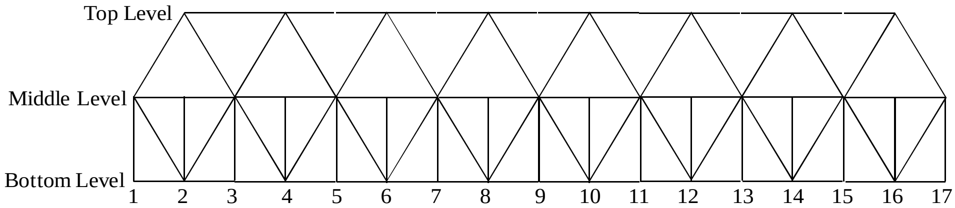

Figure 4.1. Representation of the possible locations of sensors.

Presently, 7 numbers of sensors are used at the locations: 9BL (transverse, vertical and longitudinal), 9BR (transverse and vertical) and 8BL (transverse and vertical). Sensors at the location 8BL are considered as the reference channels.

5. IDENTIFICATION OF MODAL PARAMETERS

All three techniques are implemented using the same ambient vibration data. At first, NExT-ERA is implemented. In the implementation of NExT, first the PSD are computed and inverse Fourier transform is employed subsequently to evaluate the auto and cross correlation function. PSD are computed based on Welch's method using hanning window function. ERA is then employed using the cross correlations to obtain the modal parameters. A stabilization diagram, which helps to identify the physical modes using various model orders, is generated and shown in Fig. 5.1. Range of model order in state space is considered from 20 to 100. The stabilization criteria are taken as: 1% for frequency, 5% for damping, and 2.5% for mode shape. Meaning of the used symbols (plotted at left horizontally and vertically at centre) are: 'f' as pole with stable frequency, 'd' as pole with stable frequency as well as damping, 'v' as pole with stable frequency as well as mode shape, 's' as pole with stable frequency as well as mode shape as well as damping and '+' as new pole. The background curve is considered as the maximum of all the PSD curves for better displaying of the peaks.

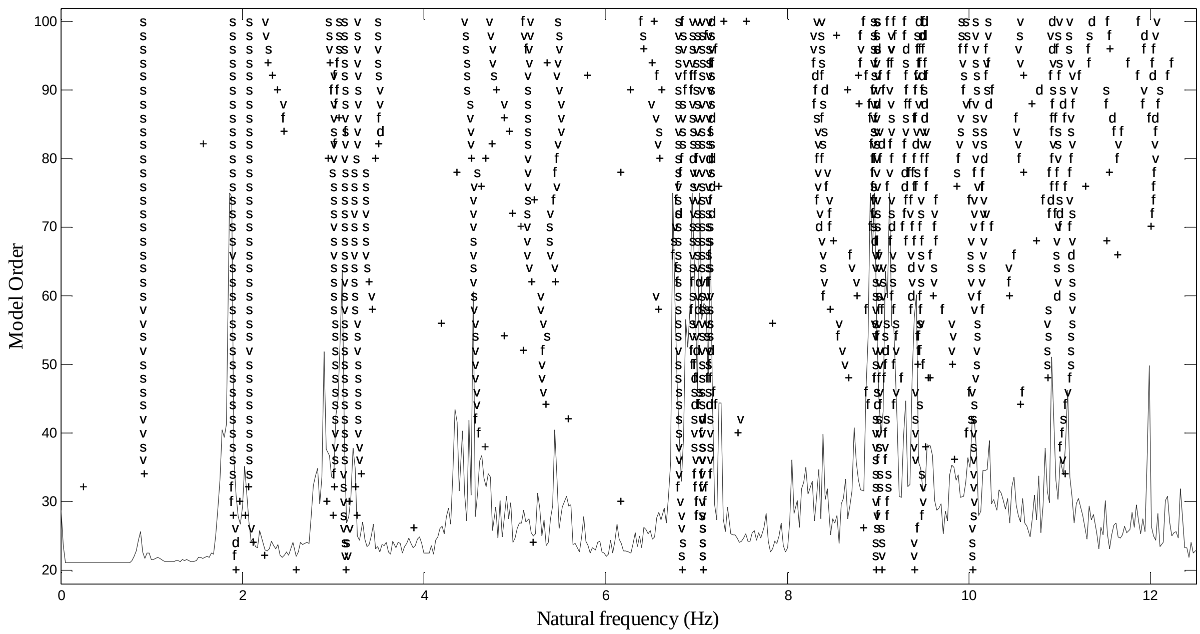

Figure 5.1. Stabilization diagram based on NExT-ERA

SSI (data driven) is implemented next. Out of the three available weighting schemes CVA is finally chosen since, it provides the better stabilization diagram for the considered set of data and considered signal processing parameters. Stabilization diagram based on SSI-DATA using CVA weighting scheme is shown in the Fig. 5.2. Physical modes are well observed from this stabilization diagram in the frequency range of 0–10 Hz.

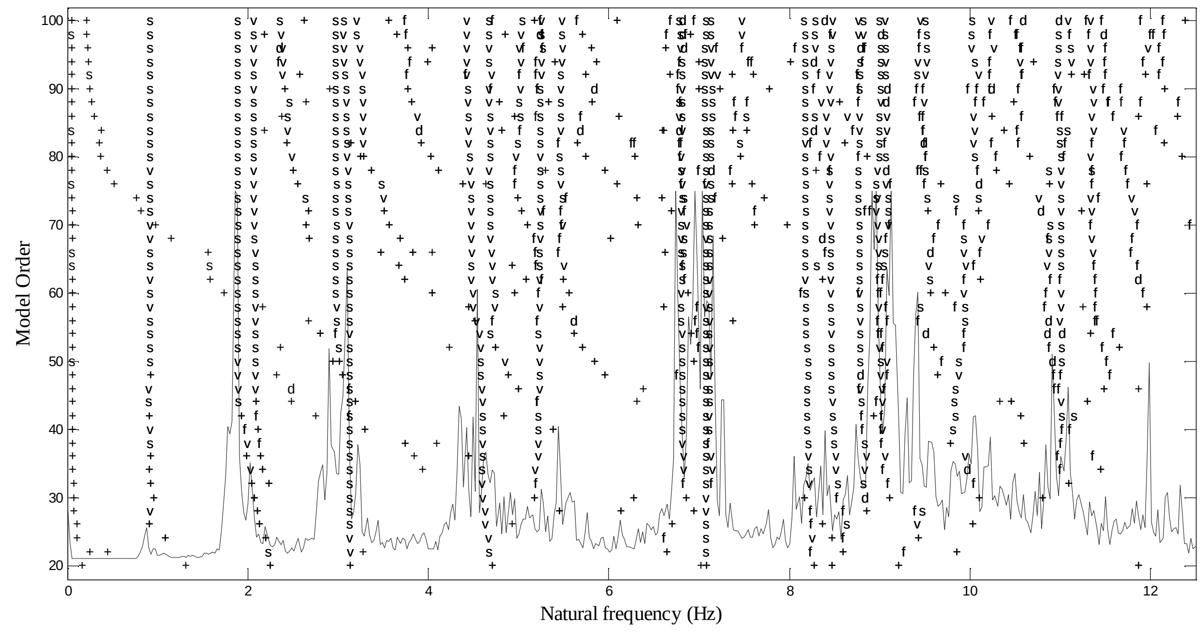

Figure 5.2. Stabilization diagram based on SSI-DATA.

Finally, the FSDD is implemented for the modal parameter identification. The first three singular values of the PSD matrices at different discrete frequencies along with the identified damped frequency peaks are shown in Fig. 5.3.

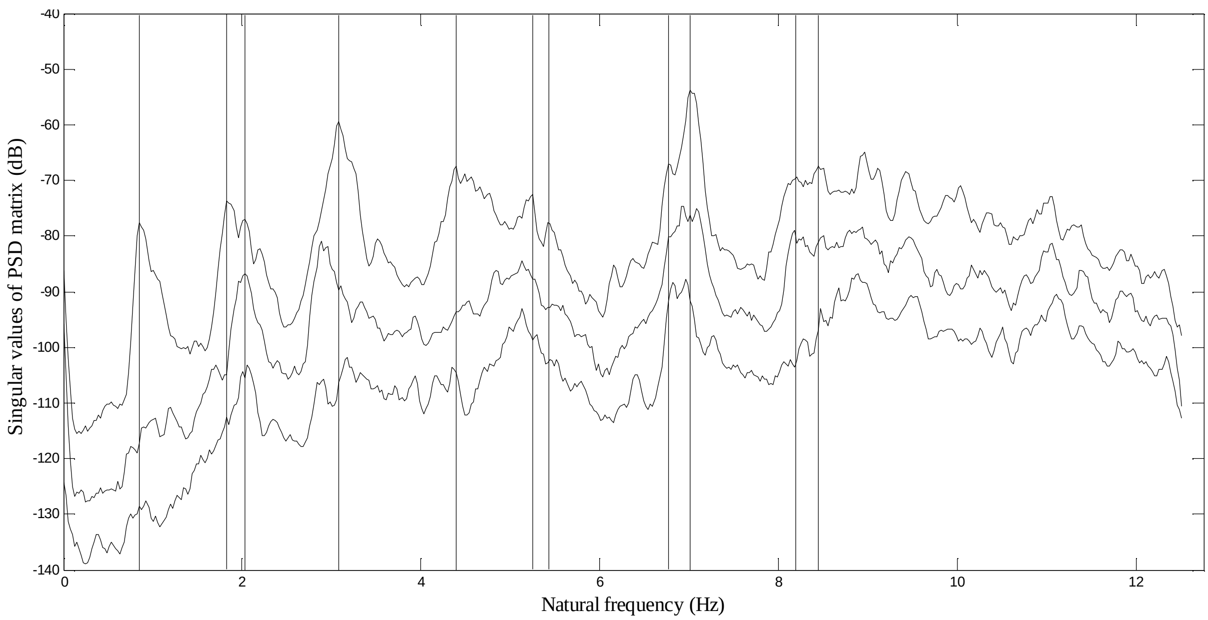

Figure 5.3. First three singular values of PSD matrix along discrete frequencies.

First ten numbers of frequencies are chosen considering all three techniques. The frequencies (Hz) and damping ratios (%) are shown in the table 5.1. It may be observed that the 4th frequency could not identified using the singular value plot as in the Fig. 5.3. Further, the 7th frequency identified using the SSI is not identified using NExT-ERA as well as FSDD.

Table 5.1. Identified frequencies and damping ratios

	NExT-ERA		SSI		FSDD	
	Frequency(Hz)	Damping Ratio (%)	Frequency(Hz)	Damping Ratio (%)	Frequency(Hz)	Damping Ratio (%)
1.	0.8684	0.0500	0.8802	0.0501	0.8944	0.0070
2.	1.8608	0.0219	1.8490	0.0297	1.8650	0.0048
3.	2.0380	0.0130	2.0262	0.0086	2.0506	0.0074
4.	2.9832	0.0286	2.9360	0.0255	-	-
5.	3.0896	0.0069	3.0777	0.0313	3.0646	0.0049
6.	4.4515	0.0122	4.4246	0.0076	4.3874	0.0037
7.	-	-	4.6491	0.0248	-	-
8.	5.1099	0.0060	5.2044	0.0085	5.2356	0.0030
9.	6.7639	0.0033	6.7758	0.0068	6.7477	0.0012
10.	7.0357	0.0029	7.0357	0.0038	7.0092	0.0014

Observing the mode shape behaviours along transverse and vertical directions at the locations 9BL and 9BR, it is reasonably concluded that 1st frequency (0.8684 Hz) and 2nd frequency (1.8608 Hz) represent the 1st transverse and 1st vertical modes. Subsequently, the MAC is evaluated between the pairs of the three different techniques corresponding to the identified frequencies. Evaluated MAC values are shown in table 5.2.

Table 5.2. MAC values between identified mode shapes using three techniques.

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
NExT-ERA/SSI	0.9970	0.9768	0.8936	0.6318	0.9787	0.6833	-	0.6732	0.8255	0.9790
NExT-ERA/FSDD	0.9949	0.9971	0.9336	-	0.9240	0.6640	-	0.7092	0.9489	0.9129
SSI/FSDD	0.9977	0.9613	0.7243	-	0.9778	0.9426	-	0.8349	0.8002	0.9405

From the table 5.2, it can be roughly stated that MAC values show reasonable agreement at different frequencies.

6. CONCLUDING REMARKS

Both the time and frequency domain techniques have been employed for identification of the modal parameters of the Saraighat Bridge. Ambient vibration data in form of acceleration response is used for the modal identification. Following conclusions are made based on the observations of the identified modal parameters of this Saraighat Bridge.

(i) Identification of all the modes may not be possible using a single identification technique. Hence, employing multiple techniques could be considered for better modal parameter identification.

(ii) The frequencies identified using three different techniques show less dispersion.

(iii) On the other hand, the identified damping ratios show higher dispersion. Damping ratios found using NExT-ERA, are matching reasonably with those obtained using SSI. However damping ratios found using FSDD are not in good agreement with those obtained using previous two methods.

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