# Evaluation of Section and Fiber Integration Points in Fiber Model 

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#### Abstract

SUMMARY: This paper aims to evaluate an adequate number of the section and fiber integration points in fiber model. In order to improve computation efficiency and accuracy, both the section integration points discretized in an element and fiber integration points divided in a section are evaluated by using three nonlinear beam-column elements (DB, FB and PH elements) in finite element programming OpenSees. From the results, it is indicated that for the DB element, six section integration points and $6 \times 6$ fiber integration points are suggested to use; for the FB element, four section integration points and $6 \times 6$ fiber integration points are suggested to use; for the PH element, 0.2 of hinge length ratios and $6 \times 6$ fiber integration points are suggested to use. Finally, the paper presents a verification example to demonstrate the reliable and computationally efficient of the evaluation of integration points.


Keywords: section integration points, fiber integration points, fiber model, DB, FB and PH elements

## 1. INSTRUCTION

Since the elastoplastic damages occur under severe earthquake, reliable prediction of nonlinear behavior of structures in the earthquake is important to assess the seismic safety of the structures. At present, there are mainly four kinds of analytical models, namely fiber model, plastic hinge model, micro-model and hybrid-model. The most advantage of fiber model is that it adopts uniaxial material constitutive and is able to consider the coupling of the axial force and bending. For its concise and clear physical conception and reliable analysis result, researchers pay more and more attention to fiber model.

The formulation of the fiber model is as following: 1)First, the structure is divided to a discrete number of elements which includes beams or columns. 2)Then the elements are discretized into a number of sections. The section deformations are then used to determine the element displacements based on the assumed force and deformation interpolation functions. 3)Finally the sections are divided into a number of fibers. The fiber strains are then used to determine the section deformations based on the plane section assumption. How to determine the appropriate number of section integration points of an element and the appropriate number fiber integration points of a section during the discretization? Theoretically, of course the more the better. But we hope that through the study of reasonable division in section integration points and fiber integration points, computation efficiency and accuracy are both ensured. This paper, using three nonlinear beam-column elements in finite element programming OpenSees: a)Displacement-based element(DB element), b)Force-based element(FB element), c)Plastic hinge element( PH element), evaluates an adequate mumber of both the section integration points discretized in an element and fiber integration points divided in a section.

## 2. EVALUATION OF SECTION INTEGRATION POINTS

A cantilever column structure as shown in Fig. 1(a) is considered to evaluate the accuracies and applicabilities of three nonlinear beam-column elements in OpenSees: a) DB element, b) FB element, c) PH element. This structure is simple and could be modeled by only one element. However, it has the capacity to illustrate responses in different levels: structural level ( which shows structural nodal responses) and sectional level (which shows responses within a section). Three bilinear section moment-curvature curves in Fig. 1(b) are employed to trace the capacity of the elements in different deformation cases, such as section response under hardening ( $\mathrm{r}=0.05$ ), section response under zero post stiffness ( $\mathrm{r}=0.0$, actually a very small value $1 \times 10^{-9}$ is used to instead of 0.0 for improving convergence) and section response under softening ( $\mathrm{r}=-0.05$ ), where r is the post stiffness ratio. For no axial load is applied, the section axial force-deformation curve is set to linearity.

The analyses are performed using the displacement control method. Newton-Raphson strategy is employed to predict the iterative structural nodal displacement increments. At the end of each iteration, the solution obtained is checked by energy convergence criterion with a tolerance of $1 \times 10^{-7}$. In the figures, IPs means the number of section integration points, and ratio means hinge length ratios.


### 2.1. Structural Level Response ( Top Displacement vs. Base Shear)

From the observation in Fig.2-4, we are able to get the following conclusions.
For the DB element, if six integration points or more have been used, a consistent structural level response will be obtained regardless the cases of section under hardening, zero post stiffness and softening. It can be also seen that small number of integration point may also present a consistent structural level response in the case of section under hardening.

For the FB element, if four integration points or more have been used, a consistent structural level response will be obtained in the cases of section under hardening and zero post stiffness. However, the element fails to present a consistent structural level response when section response is under softening because the result varies as the number of integration point varies. In addition, the element is unstable when section response is under softening.

For the PH element, a sensitive correlation between the hinge length and structural level response in the post-yielding region is observed. This means that it must be preset an accuracy hinge length for getting an accuracy structural level response.


Figure 2. Top displacement vs. base shear responses with r=0.05


Figure 3. Top displacement vs. base shear responses with $\mathrm{r}=0.0$


Figure 4. Top displacement vs. base shear responses with $\mathrm{r}=-0.05$

### 2.2. Sectional level Response ( Section Curvature vs. Section Moment)



Figure 5. Section curvature vs. section moment responses with $r=0.05$


Figure 6. Section curvature vs. section moment responses with $r=0.0$


Figure 7. Section curvature vs. section moment responses with $r=-0.05$

From the observation in Fig.5-7, we are able to get several conclusions.
For the DB element, the use of only three integration points is able to get a consistent sectional level response regardless the cases of section under hardening, zero post stiffness and softening. However, from above results of structural level response we can conclude that six integration points or more should be used to get both consistent structural and sectional level responses.

For the FB element, if three integration points or more have been used, a consistent sectional level response will be obtained in the case of section under hardening. If larger analysis step and more integration points have been used in the case of section under zero post stiffness and softening, inaccurate yielding curvature maybe obtained. The use of more integration points makes the FB element more unstable in the case of section response under softening.

For the PH element, it presents a consistent sectional level response regardless the hinge lengths. However, as above illustrated, the hinge length influences structural level responses, and one can conclude that structural level response controls the consistent result of this element.

## 3. EVALUATION OF FIBER INTEGRATION POINTS

Based on the results of evaluation to the section integration points in the preceding section, fiber integration points is evaluated in this section. A cantilever column structure as shown in Fig. 8(a) is considered to evaluate the accuracies and applicabilities of three nonlinear beam-column elements in OpenSees: a) DB element, b) FB element, c)PH element. It also illustrates responses in different levels: structural level ( which shows structural nodal responses) and sectional level (which shows responses within a section). Three bilinear material stress-strain curves in Fig. 8(b) are employed to trace the capacity of the elements in different deformation cases, such as material response under hardening ( $\mathrm{r}=$ 0.05 ), material response under zero post stiffness ( $\mathrm{r}=0.0$, actually a very small value $1 \times 10^{-9}$ is used to instead of 0.0 for improving convergence) and material response under softening ( $r=-0.05$ ), where $r$ is the post stiffness ratio.

The analyses are performed using the displacement control method. Newton-Raphson strategy is employed to predict the iterative structural nodal displacement increments. At the end of each iteration, the solution obtained is checked by energy convergence criterion with a tolerance of $1 \times 10^{-7}$.

For the DB element, six section integration points are used. For the FB element, four section integration points are used. For the PH element, 0.2 of hinge length ratios are used. In the figures, NUM means the number of fiber integration points.


Figure 8. Cantilever column structure

### 3.1. Structural Level Response ( Top Displacement vs. Base Shear)

From the observation in Fig.9-11, several conclusions were reached.
For the DB element, the FB element and the PH element, if $6 \times 6$ points or more have been used, a
consistent structural level response will be obtained regardless the cases of material under hardening and zero post stiffness. However, all of them fail to present a consistent structural level response when material is under softening because the results were in little variation with the different number of integration point.


Figure 9. Top displacement vs. base shear responses with $\mathrm{r}=0.05$


Figure 10. Top displacement vs. base shear responses with $\mathrm{r}=0.0$


### 3.2. Sectional level Response (Section Curvature vs. Section Moment)



Figure 12. Section curvature vs. section moment responses with $r=0.05$


Figure 13. Section curvature vs. section moment responses with $r=0.0$


Figure 14. Section curvature vs. section moment responses with $r=-0.05$
From the observation in Fig.12-14, the following conclusions could be reached.
For the DB element, the FB element and the PH element, if $6 \times 6$ points or more have been used, a consistent structural level response will be obtained regardless the cases of material under hardening and zero post stiffness. However, all of them fail to present a consistent structural level response when material is under softening because the result a little varies as the number of integration point varies.

## 4. VERIFICATION

Based on the results of the above discussion, the author took part in a blind prediction contest for a pseudo-static collapse experiment of a three-story concrete frame structure (Fig. 15)which is sponsored by AERDP, ASC(Association of Earthquake Resistance and Disaster Prevention, Architectural Society of China), and honored excellent prize. The compare of prediction result and test result is shown in Fig. 16.


## 5. CONCLUSIONS

The objective of this study is to evaluate an adequate mumber of the division of section integration
points and fiber integration points using three nonlinear beam-column elements (DB, FB and PH elements) in OpenSees. For all of them, both section integration points and fiber integration points are evaluated in structural level response and sectional level response. For the DB element, six section integration points and $6 \times 6$ fiber integration points are suggested to use to get both consistent structural and sectional level responses. For the FB element, four section integration points and $6 \times 6$ fiber integration points are suggested to use to get both consistent structural and sectional level responses. For the PH element, 0.2 of hinge length ratios and $6 \times 6$ fiber integration points are suggested to use to get both consistent structural and sectional level responses. Based on the results of the evaluation, the author took part in a blind prediction contest, and the results proved that the evaluation of integration points was reliable.

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