The value of risk reduction in optimum seismic design

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SUMMARY:
In order to find optimum design values in earthquake engineering, we need to evaluate initial and maintenance costs as well as the losses caused by earthquakes. Intangibles such as human lives may lie among the losses. The paper aims to establish a lower limit to this intangible in the sense of how much a person or society is willing to invest for preserving a life, and to examine the implication in optimum seismic design. Thus the willingness-to-pay approach is adopted, and since it requires explicit use of utility curves, we pay attention to such curves. We postulate simple utility curves which comply with certain conditions referring to the derivative of utility with respect to time in function of income per unit time, as well as utility based on total wealth, including the expected present value of future income. We also review the human capital approach, where an anonymous life is taken as the expected present value of a person's contribution to gross domestic product, during the rest of his/her life. The values obtained with both methods are compared, and the results show that the human capital approach computed as the expected present value of contribution to the gross domestic product underestimates the value that an individual or society is willing to pay to reduce the risk of dying. However, it can be used as a lower limit.

Keywords: Utility, willingness to pay, seismic design, reliability, human life, risk reduction, optimization

1. INTRODUCTION

The structural design problem is one of decision-making tending to optimize the systems. It is basically trying to maximize the utility that is associated with the design of the structure involved, which is dependent upon the benefit resulting from the existence of the structure, the losses implying its possible failure, and its initial cost. Thus, we aim to select a design parameter so that an objective function is maximized (Rosenblueth, 1964; Esteva 1968). Utility is usually treated as if it were a linear function of money. This is acceptable when the maximum variation that is expected in money is small related to the fortune of the person involved. The problem is then reduced to establish monetary equivalents of all possible benefits and losses, and to maximize the corresponding quantities of benefits minus initial cost and consequences of failure, or if direct benefits are insensitive enough to design, to minimize the sum of initial cost and consequences of failure. This economic model is incomplete, since it neither includes the costs of design nor the studies required by this design. It would be advisable to represent the complete model through a decision tree that shows different branches with the decisions that the designer can make as well as the events that can happen. This diagram makes it easier to visualize and systematize optimization, because it is possible to write probabilities and utilities in the right places (Benjamin and Cornell, 1970).

In computing all possible losses, intangibles such as human lives must be taken into account. In this case, it is not a trivial matter to establish monetary equivalents, and this kind of purely economic approach deserves further study, because just considering this loss as an additive term in the formation of an objective function may lead to absurd results. With this in mind and in an effort to establish a lower limit to how much one is willing to invest in order to save a life of a victim for earthquakes, either the question is formulated as an individual or as a group. Here, we study the problem from a decision-making point of view. As long as the willingness to pay approach is used, which explicitly requires of utility curves, special attention is given to them.

2. A BRIEF LITERATURE REVIEW

The principal approaches found in the literature that have been employed to deal with the problem of the value of human life are: human capital, consumption and its variations, considerations of possible legacies or bequests, considerations of possible life insurance, willingness to pay, and quality of life.

The first studies on the value of human life adopt the viewpoint of human capital, which states that the value of human life is the sum of income that a person would have received from his/her job in case of not having died prematurely (Hull, 1899). Nowadays, the human value criterion is based on the premise that on a perfect job market, what a person does is worth to society what that person is paid to do the job. Therefore, it could be argued that when a person dies, society would lose the total income that this person would have received over the rest of his/her life. Here, the value of a person's life is calculated as his/her discounted expected future earnings (Linnerooth, 1979). The most obvious objection to this criterion is that the job market is never perfect, and adopting this perfection seriously distorts the results for individual cases. Moreover, it is necessary to consider the person whose life is involved as well as those who know him/her as members of society. Therefore, each person is willing, and so is society, to contribute to decrease his/her risk of dying, with a larger amount than that resulting from assigning the value given by human capital to his/her life.

In an attempt to overcome some objections in the income criterion, it was proposed to use consumption instead. That is, the value of a human life would have been the expected present value of consumption the person would have made had he/she lived. This criterion is consistent with the idea that what gives comfort is what it is consumed, not the unspent income saved or invested. Thus, if well-being is desirable, one must tend to maximize consumption. However, the criterion is wrong by assuming that all that is consumed gives comfort and only what is consumed gives comfort. It is also wrong by assuming that extending the life of a person would bring consumption of everything he/she has not consumed up to the time of his/her death.

On the other hand, in accordance with the equality concept, if it is assumed that all lives are worth the same to society, an anonymous life can be valued as the expected present value of a person's contribution to the gross domestic product (GDP) for the rest of his/her life (Rosenblueth, 1974). This approach circumvents criticism regarding the future income of the person related to market imperfections, as well as objections regarding the assumption that only income gives comfort. Therefore, the approach allows for establishing a lower limit to what society must be willing to pay to reduce the probability of the death of an anonymous life. This limit could be very low, but not less useful, since there are other criteria leading to even lower figures.

Models of legacies or bequests and life insurance are applicable to short-term death or when the person can buy a life insurance, thus they are applied to single events. In the first case, total utility is the expectancy assigned by the person to the condition of surviving risk and to that of dying, the latter being determined by what utility means to the person who dies that his/her heirs enjoy his/her fortune. In the second case, the total fortune is the expectancy of the fortune that he/she would have in case of surviving and the one that his/her heirs would receive in case the person dies.

The main objection to the human capital and its variations is that it looks only at the economic side of the problem. Mishan (1971) suggested that in resource allocation, in order to achieve an improvement in the sense of Pareto, it is required to take into account each person's willingness to reduce his/her risk of dying. A Pareto improvement is said to exist when individuals who gain from a social change are able to compensate those who stand to lose from the change and still leave a net gain. Also, Usher (1971) published a formal treatment to the problem of establishing the amount that a rational person must be willing to invest, in order to reduce such a risk, taking into account his/her utility curve. Although it can be argued that there are no grounds for assuming a relation between results of these two criteria, many empirical data make one think that, even though, there are differences between them, there is indeed a relation (Linnerooth 1979). Rosenblueth (1987) establishes lower bounds of the amount that a person is willing to pay to reduce an originally small risk of losing her life by some amount, including the disutility associated with experiencing death. This author later computes both individual and social values of life by using utility curves per unit time as a function of time, (Rosenblueth 1992).

Nathwani et al (1997) have suggested the maximization of a social indicator of the quality of life as a decision-making criterion, and Ditlevsen (2003, 2004) has defined this index in a slightly more general way. The former authors suggest that the life quality index (LQI) reflects the expected length of good life and, by using it, one can choose between alternative projects or designs. The LQI is a compounded number based on the life expectancy, per capita GDP, and ratio of time spent at work. The estimation of societal willingness-to-pay for safety is studied by Pandey and Nathwani (2004). Moreover, this index has been studied by Rackwitz (2002), Rackwitz and Joanni (2009), for its possible role as a mean to obtain the societal affordable level of safety in buildings.

3. INDIVIDUAL AND SOCIAL VALUES

When human lives are at risk, losses are usually so high that the hypothesis of utility as a linear function of losses is not valid, and it is necessary to define the shapes of the utility functions. In the decision making process it is convenient to associate a qualitative scale to preferences, that is, a measure of intensity of preference. This scalar is called utility. The problem of establishing utilities corresponding to the diverse events that may result from our decisions is an ethic one. One must define for whom these utilities are established or for whom we will optimize (owner, society, etc). The more delicate intangible to associate a utility is the human life. The procedure resorting to the question how much a human life costs in dollars has been used and abused. For moral reasons it is illicit to even think of asking such a question and it must be replaced by, how much a person or society is willing to pay for saving a human life. However, any time that we receive supplementary wage for doing a job at risk, regardless how small it is, or any time we decide how safe a civil structure must be, we are implicitly assigning values to human live, either to one’s or someone else’s life. Therefore, we had better do this in a rational way by selecting the optimum alternative in the frame of decision making theory.

There are three types of values that must be taken into account when valuating life, namely, one’s life, someone else’s life, and the value which society assigns to their members. These three types of values differ among them and they pose problems of a different kind. Accordingly, the problems that we will be interested in may be grouped together into two types: individual and social values. In both cases, there is a conflict between the scale of values of the decision maker and that of the person or society. The decision maker is required to maximize the sum of happiness of all human beings, but on the other hand, he carries the responsibility that society imposes on him of serving his/her clients’ interests. In all cases, the criteria applied by the decision maker will be the result of negotiation between the maximization of the sum of happiness of all human beings and the client’s interests. In problems of an individual, the value of human life is inferred from what the person should be willing to pay for reducing the risk of dying or what risk he/she should be willing to assume in exchange for compensation. In social problems, it is an ethics requirement for the decision-maker to take the place of each member of society conceptually. In both cases, whether individual or social, the use of utility curves is required as long as the value of human life is inferred from what a person or society is willing to pay for reducing the risk of dying in exchange for compensation.

4. HUMAN CAPITAL

4.1. Reliability concepts

In the classical theory of reliability, the probability that a structure will fail during a period of time x is given by $P[X \leq x] = F_X(x) = 1 - R_X(x)$, where $F_X(\,)$ is the cumulative distribution function of X. $R_X(\,)$ is the reliability or survival function expressed in terms of time, that is, the probability that the structure is still working at time x, $P[X > x]$. The probability density function (pdf) of the design life is given by $f_X(x) = d[F_X(x)]/dx$, which is known as the unconditional failure rate, since it reflects the probability of failure in the time interval x to $x + dx$ as $dx \to 0$. The hazard function $h_X(x)$, also known as age specific failure rate or conditional failure rate, expresses the likelihood of failure in the time interval x to $x + dx$ as $dx \to 0$, given that failure has not occurred prior to time x, that is: $h_X(x) = f_X(x)/R_X(x)$. The hazard function is not a pdf, though for a fixed time we can define the failure time density (lifetime density) conditioned on survival up to this fixed time. Thus, $h_X(x) \cdot dx$ might be thought of as the instantaneous probability of failure at x, given that it survived up to x. A useful result is that the reliability function $R_X(\,)$ may be expressed in terms of the hazard function $h_X(\,)$ as: $R_X(x) = \exp\left[-\int_0^x h(t)dt\right] = \exp\,[-H(x)]$, where $H(x)$ is the cumulative hazard function.

According to the theory described, the following analogy can be made. We call $F_X(x) = 1 - R_X(x)$ the probability that a person is dead prior to instant x, $R_X(\,)$ the probability that a person is alive at time x, $f_X(x)$ the probability density function of the remaining lifetime, $h_X(x)$ the mortality rate taken as the conditional pdf of the remaining lifetime, given that a person is alive at instant x.

4.2. Contribution to GDP

Now, let $w(x)$ be the person's contribution to GDP at age x and let $u(w(x))$ be the utility derived from the person's contribution to the GDP between ages x and t, thus, the present value of the lifetime utility for a person at age x until she/he reaches age t is

$$U(x,t) = \int_x^t u\big(w(x)\big) e^{-(\gamma-\zeta)(t-x)} dx \tag{4.1}$$

where γ is the discount rate, and ζ is the growth rate of the GDP per capita per unit time.

The expected remaining present value life time utility at age x (conditional on having survived up to x) is then (Yaari, 1965; Arthur, 1981; Shepard and Zeckhauser, 1984)

$$ER(x) = \int_x^\infty \frac{f(t)}{R(x)} U(x,t) dt \tag{4.2}$$

Thus, we can say that the expected present value of the contribution of a person to the GDP is given by

$$W(x) = \frac{1}{R(x)} \int_x^\infty R(t)\, w(t)\, e^{-(\gamma-\zeta)(t-x)}\, dt \tag{4.3}$$

where $w(t)$ is his/her contribution per unit time to the GDP at time t. The social value of the life of a person of age x under impending danger is given by $W(x)$.

4.3. Discounting function

It is common to turn a future utility into its present value by a discounting function. This could be opposed to an ethical principle stating that the importance of happiness of a person must be independent of the moment experiencing it, now and at any time in the future. On the other hand, individuals tend to overestimate what is nearest and to this generation, rather than to the next. In this study, we use an exponential discount function assuming that the probability of occurrence of an event ending life is finite and increases over time, and that also increases the probability over time that the scale of values of a person differs significantly from what we can assume today. However, some other discount functions have been found in the literature. Pandey and Nathwani (2003) propose a hyperbolic discount function; Lind (2007) fixes the discounting function for risks in the far future of a prospect in relation to its financing.

5. UTILITY THEORY

We will begin by establishing a scale of utilities, of what a person or society invests to preserve or reach an intangible, not as a final answer but just as a guide for the engineer. Some basic concepts of utility theory are reviewed in Appendix A.

5.1. Conditions on utility curves

In order to calculate the value of an intangible such as the human life, it is important to propose utility curves based on the wealth or income of a person. When we give the term *utility* its ordinary meaning, the intensity of desire or an a priori preference, these curves must satisfy certain conditions that are described further ahead (Rosenblueth, 1987). These conditions refer to the derivative of utility with respect to time based on income per unit of time as well as utility based on total wealth including the expected present value of future income. We will express them in these terms. $U = U(W)$ means utility of wealth W, and W_{min} is the minimum value of W to survive. Furthermore, U and W are expected present values of utility and of wealth and income, respectively.

The first obvious condition to be satisfied by utility curves is that the utility of a dead person is nil, provided that W includes all wealth, that is $U(W) = 0 \; if \;\; W < W_{min}$. The second condition is that utility is an increasing function of W; therefore $U'(W) > 0 \; if \;\; W \geq W_{min}$, where the prime denotes a derivative with respect to W. The third condition is related to risk aversion,$U''(W) < 0 \;\; if \;\;\; W \geq W_{min}$, that is, $U(W)$is concave. The presence of aversion to the fact itself of taking risks increases the concavity of the curve, whereas being prone to risks decreases it, but it is presumed in most cases that it is not reversed. The fourth condition is derived from the fact that the coefficient of risk aversion $r(W) =- U''(W)/U'(W)$, (Keeney and Raiffa, 1976), must be a non-increasing function of W. Therefore by making $dr(W)/dW < 0$, we obtain $U''^2(W) < U'(W)U'''(W) \; if \;\; W \geq W_{min}$. The last condition comes from the finiteness of the human being. We are only able to desire with finite intensity, $U(\infty) = U_{max} < \infty$.

5.2. Utility functions

There are some utility functions in the literature that satisfy risk-aversive behavior. These mathematical forms include exponential, logarithmic, and quadratic types. A frequently used utility function is given by $U(W) = (1 - ae^{-bW})U_{max}$, where a and b are constants, but it does not comply with the condition that risk aversion is a decreasing function of W, that is, $(U''^2(W) < U'(W)U'''(W) \; if \;\; W \geq W_{min})$. Instead we can use (Keeney and Raiffa, 1976) $U(W) = [-he^{-c(W-W_{min})} - ke^{-d(W-W_{min})}]U_{max}$, where c, d, h, and k are positive constants. In this study we will use a variation of this function given by:

$$U(W) = (1 - \alpha e^{-m\delta} - \beta e^{-n\delta})U_{max} \tag{5.1}$$

where $\delta = (W - W_{min})/W_{min}$, and U_{max} is the maximum possible utility, assuming that we do not have any economic restriction, and α, β,m, n are constants. Fig. 5.1 shows this function.

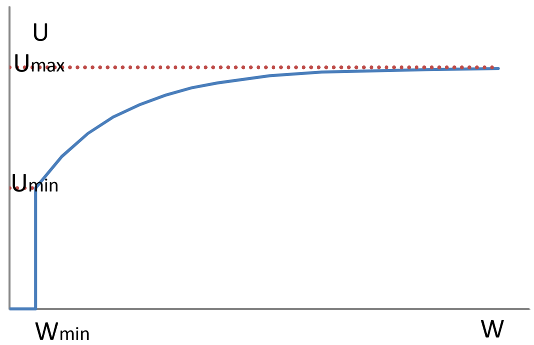

Figure 5.1. Utility function

6. WILLINGNESS TO PAY

The theoretical foundation for the willingness-to-pay criterion is that individuals will maximize their own utility by trading-off wealth or income for reducing the probability of death (Linnerooth, 1979). This method is a function of age, income and the perception of risk, and it assumes that individuals are rational, maximize utility, and correctly perceive the wealth-risk trade-off. As long as the method uses individual utility curves it does not say anything about the value that society assigns to a human life.

Therefore, let us consider a person whose utility curve is known, and she/he does not have life insurance, legacies or bequests. Let us further ask her/him how much she/he should be willing to pay to assume a risk of losing her/his life. Now, let $U(W(x))$ be the utility associated to the expected present value of her/his future income, and C the quantity that she/he will be willing to pay for not taking an activity with probability p of dying. Then C must be such that: $(1-p)U\big(W(x)\big) = U(W(x)-C)$. For any shape of the utility curve one can assign values to C, and obtain the corresponding p. Once C and p are known the human value $L_0(x)$ ruling under these circumstances is given by the ratio between the change in income to the change in risk, C/p. The limit of $L_0(x)$ when p tends to zero is given by the following approximation (Rosenblueth 1987):

$$L_0(x) = U/U' \qquad (6.1)$$

7. APPLICATIONS

7.1. Contribution to GDP

Mortality rates and contribution to GDP were inferred from data taken from Mexico's Census Bureau (INEGI-2008), as shown in Figs. 7.1 and 7.2 respectively. Eqn. 4.3 is used to compute the expected present value of the contribution of a person aged x to the gross domestic product. We adopt the discount rate $\gamma = 0.05/yr$, and the growth rate of the GDP $\zeta = 0.02/yr$. Fig. 7.3 depicts the results.

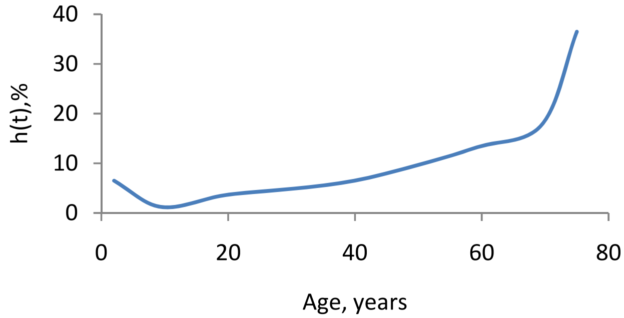

Figure 7.1. Mortality rates in terms of age

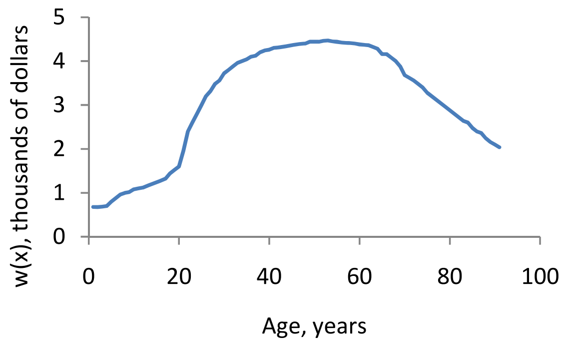

Figure 7.2. Yearly contribution to GDP

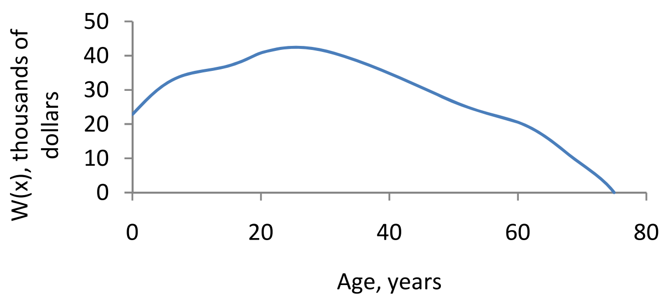

Figure 7.3. Expected present value of the contribution to GDP

7.2. Willingness to pay

Using the utility curve given in Eqn. 5.1 with $\alpha = 0.1$, $\beta = 0.4$, $m = 0.01$, and $n = 0.18$, we calculate L from Eqn. 6.1. In Fig. 7.4 we present the factor $f = L/W$ for different values of W/W_{min}. If the human capital criterion assigns the value of W to life, f would be the factor by which we must multiply to obtain the value that the person would assign to her/his life. These factors are always greater than number one and could be much greater.

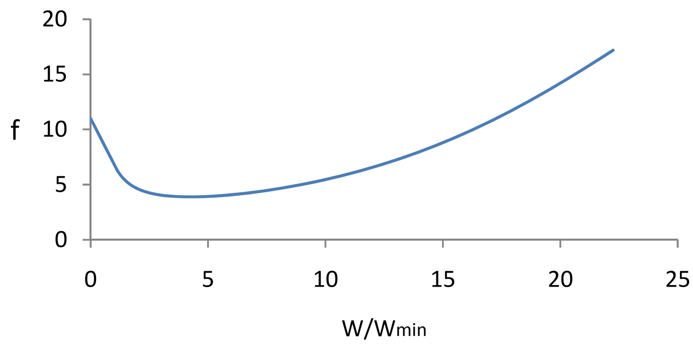

Figure 7.4. Factor for small risks, without life insurance, legacies or bequests

8. CLOSING REMARKS

The human capital approach taken as the expected present value of contribution to the GDP gives a lower limit in the estimation of how much a person or society is willing to invest for preserving a life. In order not to look just at the economic side of the problem, each person's willingness to reduce her/his risk of dying is taken into account by means of her/his utility curve. Thus, this curve is used to compute the factor by which one has to multiply the value given by the human capital approach in order to obtain the value that the person would assign to her/his life. Research has to be done especially regarding the choice of utility curves. These curves should be time evolutionary, that is, a utility curve corresponding to each instant, thus a relation can be established between the individual's income and age.

APPENDIX A. BASIC CONCEPTS OF UTILITY

Utility is the logical measure of the intensity of preference that satisfies the axiom of von Neumann and Morgenstern (1943) that makes utility synonymous with expected utility. We will refer first to preferences. Let > denote the preference relation, so that if A and B are two possible states, then A>B means that A is preferred over B. Likewise, let < and ~ denote relations of no preference and indifference, respectively. Now, let us consider a set of possible states of M, $\{s_i\}$, $i = 1, \ldots, m$. Let us also assume that they constitute a completely ordered set with respect to the relation of preference, which means that for all i and all j it is necessarily true that $s_i > s_j$ or $s_i < s_j$, and if $s_i > s_j$ and $s_j > s_k$ then transitivity relation $s_i > s_k$ is met. Under these circumstances, U_i is the utility of state s_i and U_j of s_j if and only if $s_i > s_j$ means that $U_i > U_j$, and consequently $s_i < s_j$ that $U_i < U_j$, and $s_i \sim s_j$ that $U_i = U_j$.

In order to assign numerical values to utilities, we can proceed as follows: First we order the states s_i so that the intensity of preference is non-decreasing with i, thus $s_1 < s_2 \ldots < s_m$. If $s_i \sim s_m$ we assign any value to all U_i. If $s_m > s_1$ we assign arbitrary values to $U_1$and U_m subject to $U_1 < U_m$. Then we present the engineer with the possibility of participating in a lottery A in which, with probability p, will be in state s_1 and with probability $1 - p$ in s_m. Next, we ask him for his preference between s_i and the lottery. We vary p up to $s_i \sim A$. Now let p_i be the value of p for which the engineer is indifferent to between s_i and the lottery. Thus $U_i = p_i U_1 + (1 - p_i) U_m$, since the utility of A is the second member of this equation due to the von Neumann and Morgenstern axiom.

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