A Simplified Method for Analyzing Structural Hysteretic Energy based on Multiplex Pushover Analysis

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SUMMARY:
A method for analyzing the hysteretic energy of seismic frame structures is proposed. The hysteretic energy of seismic structures is analyzed by using the multiplex model pushover method and time history method with various types of earthquake excitations. Then, the simplified method based on model pushover and the corresponding modify coefficients are proposed by comparing the results of above two methods. It's found that the simplified method is efficient for analyzing hysteretic energy of seismic structures for practical purpose.

Keywords: Seismic structure; Model pushover analysis; Hysteretic energy.

1. INTRODUCTION

The process that strong earthquake ground motions affect structures is actually the process of energy dissipation of inputting seismic energy through the structures' vibration and deformation Former earthquake resistant design mainly emphasized force and displacement response of structures. But recent earthquake resistant design based on the performance, which has significant influence on the earthquake engineering, mainly aims at reliable evaluation of nonlinear deformation and energy dissipation caused by earthquake excitation [1-3].

There have been a lot of methods available to evaluate the deformation or displacement of structures. And the static elastic-plastic analysis method is the most widely used method in which pushover analysis is used for seismic assessment of structures [4-5]. As for energy dissipation property of structures, the nonlinear time history analysis, which is considered as the most accurate method, is often used to calculate the hysteretic energy of structures under ground excitation. But time history analysis depends too much on the properties of ground motions and the analysis models of structures. Meanwhile the method of selecting proper ground motions for time history analysis is also a problem under studying. These difficulties have blocked the application of time history analysis method in practice. Therefore seeking a simplified method to calculate the hysteretic energy of structures has become an issue in the fields of earthquake resistant design of structures.

To address above problem, we propose a modal based pushover method to evaluate the hysteretic energy of structures and hope it to provide the research basis for displacement and energy based seismic design. In the example of this paper, the hysteretic energy of an 8-story frame structure is analyzed using the proposed method and time history analysis method respectively and the results are compared and discussed. It's found that the proposed method is efficient to evaluate the hysteretic energy of structures for practical purpose.

2. MODAL PUSHOVER ANALYSIS FOR EVALUATING THE HYSTERETIC ENERGY OF STRUCTURES

The motion equation for multi-degree of freedom (MDOF) elasto-plastic system is given by

$$\mathbf{M}\ddot{\boldsymbol{v}}(t)+\mathbf{C}\dot{\boldsymbol{v}}(t)+\mathbf{F}[\boldsymbol{v}(t)]=-\mathbf{MI}\ddot{\boldsymbol{v}}_g(t) \tag{1}$$

where M is the mass matrix and C is the damping matrix. $\mathbf{F}[\boldsymbol{v}(t)]$ is the restoring force of the system. In elastic case, $\mathbf{F}[\boldsymbol{v}(t)]=\mathbf{K}\boldsymbol{v}(t)$, where K is the stiffness matrix; $\ddot{\boldsymbol{v}}(t)$, $\dot{\boldsymbol{v}}(t)$ and $\boldsymbol{v}(t)$ are the relative acceleration, velocity and displacement response respectively. $\ddot{\boldsymbol{v}}_g(\mathrm{t})$ is the ground motion (input).

The corresponding energy equation of the system based on the relative displacement response is

$$\int_0^t[\mathrm{d}\boldsymbol{v}(t)]^T\mathbf{M}\ddot{\boldsymbol{v}}(t)+\int_0^t[\mathrm{d}\boldsymbol{v}(t)]^T\mathbf{C}\dot{\boldsymbol{v}}(t)+\int_0^t[\mathrm{d}\boldsymbol{v}(t)]^T\boldsymbol{F}[\boldsymbol{v}(t)]=-\int_0^t[\mathrm{d}\boldsymbol{v}(t)]^T\mathbf{MI}\ddot{\boldsymbol{v}}_g(t) \tag{2}$$

Or $$E_{\mathrm{K}}(\mathrm{t})+E_{\mathrm{D}}(\mathrm{t})+E_{\mathrm{H}}(\mathrm{t})+E_{\mathrm{S}}(\mathrm{t})=E_{\mathrm{I}}(\mathrm{t}) \tag{3}$$

where E_{K} is the kinetic energy, E_D is the damping dissipation energy, E_H is the hysteretic energy, E_S is the elastic strain energy and E_I is the total input energy caused by ground motion. According to Equation (2), the total input energy and the dissipated energy of the system can be yielded by using numerical integration. Once the system is in nonlinear stage, the input energy E_I is mainly dissipated by the plastic deformation of the system, i.e., the hysteretic energy E_H is the main dissipated energy.

Back in the 1950s, Housner [6] stated that pseudo-velocity spectra could be equivalent to breakage spectrum of elastic-plastic system. However, for just basing on a few earthquake ground motions, this equivalent format often underestimated the breakage energy of structures. Akiyama [7] integrated energy analyzing concept and method on the basis of predecessors' researches and concluded that the total earthquake input energy is only related to the natural period and gravity of the system, i.e., the total input energy of an MDOF system can be approximated by the total input energy of an SDOF system with the same natural period. This work was physically meaningful for evaluating the total input energy and hysteretic energy of multi-story or high-rise building structures.

To distinguish the relationship between the hysteretic energy and total input energy of structures and find a simplified method for calculating hysteretic energy, 8 different ground motions (including short, mid and long duration types) are selected as the inputs, and the ratio of hysteretic energy to the total input energy for a SODF system with nature period changed from 0.1 to 6.0 second are calculated using the time history analysis method. The results corresponding to various ground motions are illustrated in Figure 1. It is shown that the ratio of hysteretic energy to the total input energy is somewhat stable to different ground motions. Combined with Akiyama's work, it is conclude that the hysteretic energy mainly depends on the response of the first mode. Because pushover analysis method which assumes that dynamic response of structures mainly depends on the first modal response is a simple and feasible way to evaluate the nonlinear deformation of structures, we use this method to calculate the hysteretic energy of structures, and by comparing the results of pushover analysis and

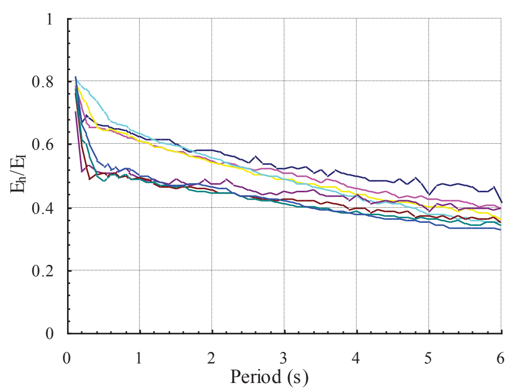


Figure 1. The ratio of hysteretic energy to total input energy for the SODF system.

nonlinear time history analysis method we hope to find a simplified way to calculate the hysteretic energy.

The main idea of pushover analysis is to use gradually static loading pseudo-dynamic method to approximate the dynamic response of structures. The main deficiencies for conventional pushover analysis method are structural displacement response is dominated by the first modal vector, and that the loading pattern is considered to be unchanged in the whole process of dynamic response. But the real structural displacement response is codetermined by all modes which change with the stiffness of structures. In 2002 Chopra and Goel [8] combined the modal decomposition method with pushover analysis method to consider effects of higher modes and ser forth the modal pushover analysis method. The basic assumptions of modal pushover analysis method are:

(1) The coupling between each modal deformation after structure yields is neglected.

(2) The total structural dynamic response is got by using SRSS method to every modal response.

Strictly speaking, above assumptions only hold for elastic system, but according to Chopra's work the interaction between various coupling modes can be neglected.

In this paper, the modal pushover analysis method is employed to analyze the hysteretic energy. Multi-modal is considered for analysis (generally 2 or 3 orders). For each mode, pushover analysis is used to gain the total dissipated energy of each structural member until the structure reaches the performance target. The dissipated energy caused by deformation of beams and columns includes bending strain energy and shear strain energy.

For the pushover analysis, a appropriate loading step is chosen. Gradually loading to the structure, and make records step by step, till complete the whole pushover process. Then the results of $F-\Delta$ and $M-\theta$ of columns, together with the beam $M-\theta$ relationship are receives, which can be used for further calculating of hysteretic energy. For each loading step, the increment dissipated energy are given as follows:

Before yield: $$E'_{\mathrm{F}\Delta}=\frac{1}{2}F\Delta \quad \text{and} \quad E'_{\mathrm{M}\theta}=\frac{1}{2}M\theta \tag{4}$$

After yield: $$\Delta E_{\mathrm{F}\Delta}=\frac{(F_{i+1}+F_i)}{2}(\Delta_{i+1}-\Delta_i) \quad \text{and} \quad \Delta E_{\mathrm{M}\theta}=\frac{(M_{i+1}+M_i)}{2}(\theta_{i+1}-\theta_i) \tag{5}$$

The dissipated energy for each beams and columns are given by

$$E_{\mathrm{beam}}=E'_{\mathrm{M}\theta}+\sum\Delta E_{\mathrm{M}\theta} \quad \text{and} \quad E_{\mathrm{column}}=E'_{\mathrm{F}\Delta}+\sum\Delta E_{\mathrm{F}\Delta}+E'_{\mathrm{M}\theta}+\sum\Delta E_{\mathrm{M}\theta} \tag{6}$$

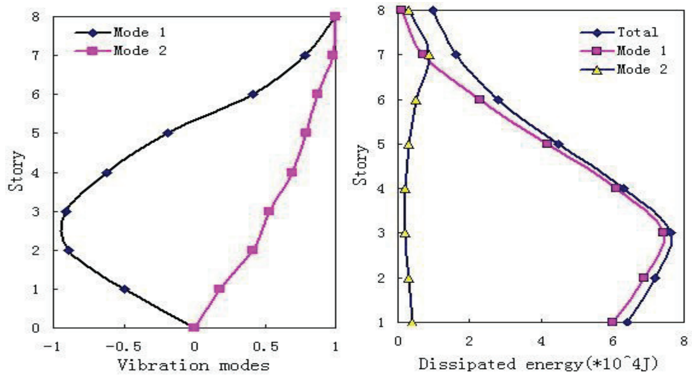
Where, Δ_1, Δ_2, … , Δ_n is the drift for each loading step; F_1, F_2, …, F_n means the corresponding column shear force; M_1, M_2, … , M_n stands for the ending moment of columns (or beams) and θ_1, θ_2, … , θ_n is the corresponding rotation angle.

The story dissipated energy can be obtained by summing up the corresponding dissipated energy of columns and beams of each story [9]. Repeating above process for each mode and combining the corresponding dissipated energy using SRSS method will give the total dissipated energy of each story.

3. ANALYSIS OF A FRAME STRUCTURE

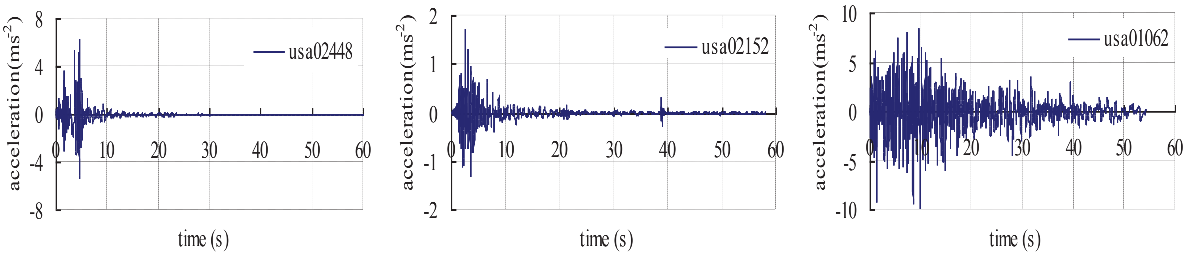
In this example, a 3×3 bay, 8-story reinforced concrete frame used for analysis. The column spacing is 7.2 m for both directions. The height for first floor is 4.5 m and 3.6m for the second and upper floors. In the floors from the first to the fifth, the dimension of columns is 550×650 mm^2 at two ends and 600×650 mm^2 in the middle of building. In the floors from the sixth to the roof floor, the dimension of all the columns is $500 \times 500\,\text{mm}^2$. The dimension of all the beams is $300 \times 600\,\text{mm}^2$. The weight for first floor is 745.1 kN, 737.4 kN for floor 2 to7 and 678.9 kN for the roof floor. The dead load is 4.0 kNm^{-2} and live load is 3.5 kNm^{-2} in the floor 1 to 7. For the roof floor, the dead load is 6.0 kNm^{-2} and live load is 2.0 kNm^{-2}.

The first two mode periods of the frame are 1.26 second and 0.39 second respectively and the corresponding vibration modes are shown in Figure 2(a). The story dissipated energy for the first two vibration modes and the total dissipated energy for each story obtained using the SRSS method are shown in Figure 2(b). It is clear that the dissipated energy for the first vibration mode mainly contributes to the total dissipated energy. The majority of the input energy is dissipated in floors 1 to 6 and the dissipated energy in upper floors (floors 7 and 8) is quite small.



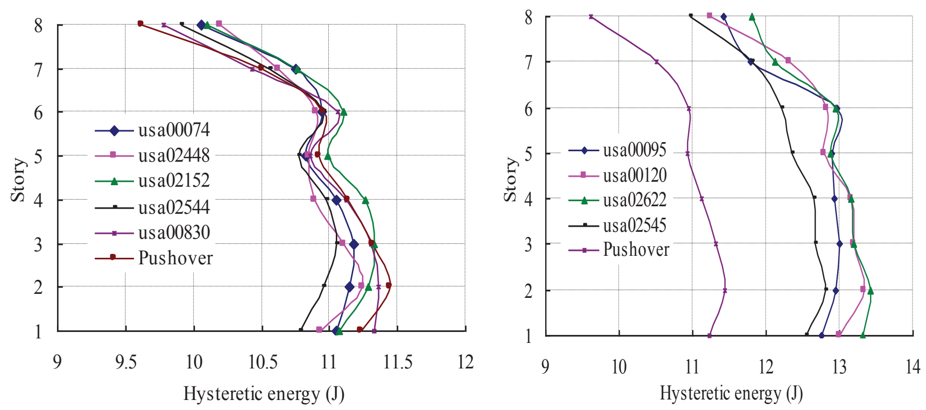
(a) (b)
Figure 2. mode and dissipated energy Analysis results

To find a simplified method based on the modal pushover analysis to evaluate the hysteretic energy [10-13] of the structure, time history analysis method is applied to calculate the hysteretic energy of each floor. In time history analysis, the selection of the ground motions is a concerned issue. In this paper, the method based on spectral compatible criterion and specified in Refs. is firstly used to select proper ground motions. However, this method only considers the spectral characteristics and does not consider the effect of duration and energy distribution. As we know, the duration of ground motions has significant influence on the nonlinear response of structures. In Ref. [14], according to duration and peak index of energy distribution, ground motions are divided into three types which consist of short (S), mid (M) and long (L) duration types. Thus we also use this method to classify the selected ground motions. To demonstrate the duration property of different type of ground motions, the accelerograms of three ground motions, one for each type, are plotted in Figure 3.



(a) (b) (c)
Figure 3. The accelerate grams of various types of ground motions: (a) S type; (b) M type; (c) L type.

Using step by step integration method to item 3 on left hand side of equation(2). In this paper hysteretic energy is first analyzed, and hysteretic energy for the structure component is the sum of works which every structure component in every moment does on the corresponding displacement. Therefore, hysteretic energy for column can be calculated by using area integral calculus to restoring force-relative inter-story displacement F-Δ curve and moment-rotating angle $M-\theta$ curve. The hysteretic energy for beams can be calculated by using area integral calculus to moment –rotating angle $M-\theta$ curve for beams, because shear energy for beams is very small, the effect of shear energy could be neglected. After hysteretic energy for every component, dissipating energy for every layer of structure could be calculated by add beam dissipating energy and column dissipating energy in the same layer. Energy distribution for every floor of frame structure is determined by adopting Pushover analysis and time-history method, it is shown as Figure 4.



(a) (b)

Figure 4. The hysteretic energy obtained using modal pushover analysis and time history analysis corresponding to various types of ground motions in natural log form: (a) S and M types; (b) L type.

Figure 4(a) shows that comparison of hysteretic energy distributions of every floor of structure under different methods which is pushover analysis and time-history analysis for S and M types ground motions. From Figure 4(a), the difference between elastic-plastic dissipating energy which is calculated through push over analysis for fame structure and the one which is calculated through inputting earthquake waves of model S and model M is not large, less than thirty percent, whatever inter-storey dissipating energy and the total dissipating energy. Therefore, hysteretic energy for structure in the effect of S and M types is completely calculated by using pushover analysis method which is simple and accords with the engineering precision.

Figure 4(b) shows the comparison between hysteretic energy distribution for very layer of frame structure, which is calculated by using pushover analysis, and the one which is calculated by using time-history analysis in inputting L type earthquake waves. However, from Figure 4(b), the difference between elastic-plastic dissipating energy which is calculated through push over analysis for fame structure and the one which is calculated through inputting earthquake waves of L type is large, reach to 6 to 7 times, whatever inter-storey dissipating energy and the total dissipating energy.

From the comparison for these analyses, it is simple and easy to compute by adopting Pushover push-over analysis method, but to seek the method to calculate hysteretic energy for structures under the action of L type earthquake waves, in this paper the ratio of hysteretic energy for every storey of frame structure which is calculated under every type of earthquake waves to the one which is calculated by using pushover analysis. Let E_{Hp} show hysteretic energy for every storey of frame structure which is calculated by adopting pushover analysis, EHS show hysteretic energy for every storey of frame structure which is calculated by using time-history analysis, thus, for the short duration or mid duration $\ln E_{Hs} = (0.96 \sim 1.07) \ln E_{Hp}$ (it is 0.96 on the first storey and 1.07 on the top storey with the variation of storey.) and for the long duration $\ln E_{Hs} = (1.15 \sim 1.22) \ln E_{Hp}$ (it is 1.15 on the first storey and 1.22 on the top storey with the variation of storey.)

4. DISCUSSION AND CONCLUSION

To simplify the computation, pushover analysis is considered, adopting analysis, which is using monotonic loading to calculate the dissipating energy, to make approximate analysis to energy response for structure on the earthquake action [15]. The specific method is: firstly, achieve the first of several modals, obtain hysteretic energy for every structure component under the analysis by using monotonic loading analysis, then add these hysteretic energy for every component of each storey to obtain hysteretic energy for every storey E_{Hp}, lastly, according to earthquake wave acting on the structure, conform to method in literature to classify the selected earthquake waves by duration. E_{Hs} could be approximately obtained by individually multiply corresponding coefficient to ln E_{Hp}. So it is valuated that the performance of dissipating energy for frame structure acted by selected earthquake waves.

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