# A survey in finding relations in order to calculate natural impulsive and convective mode frequencies in above-ground cylindrical steel storage tanks

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#### SUMMARY:

According to the past earthquakes, cylindrical liquid steel storage tanks have been accounted as a part of vulnerable lifelines and high risk elements. Flexible behavior of tank shell, interaction between liquid and shell, unanchored bottom of tank and large deformations made seismic behavior of this kind of structures more complicated constantly.

Developing throughout the recent decade, API650 (American Petroleum Institute), as a pioneer code for designing storage tanks, has tried to suggest a simple way to define frequencies of the first modes, in appendix E. The modes are divided into two series of convective and impulsive due to different phases in the structure and only the first couple of modes are defined by equations in mentioned code.

In this study, the behavior of tanks has been observed by modeling them including liquid and also elastic base, applying finite element method. To achieve the goal, more than 50 of existing storage tanks that had been designed by API in the past, have been modeled again and their modal analyses have been studied. The first frequencies of modes have been calculated for the same tanks by using appendix E of API-08 11th edition code.

Results show very close similarity in both methods for the first couple of modes that are more accurate in medium ratios of height to diameter for tanks.

Besides, in some cases, the summation of first couple of effective modal masses is not adequate for spectrum analysis suggested by many different codes.

Consequently, by using API guideline parameters, experimental equations have been given for calculating effective impulsive and convective modes to reach sufficient effective modal mass ratio. Furthermore, the range of some parameters has been suggested to be accounted for designing tanks with less vulnerability in accordance with the present frame work.

Keywords: Impulsive mode, convective mode, storage tank, flexible behaviour, frequency

#### **1. INTRODUCTION**

Above ground liquid tanks are high risk and vulnerable elements during earthquake events. They suffer from three most frequent failure modes which are shell buckling, roof failure and uplift and settlement. Past statistical records show the same contribution of aforementioned failure modes in liquid tanks [4].

Over time, the pioneer standards in this field like American Petroleum Institute's (API) [1] and American Water Work Association (AWWA) [2] as the famous ones try to develop their codes in order to provide sufficient details to see all damages.

One of the most important parameter for defining seismic force is natural frequency of tank which are known in two series by the name of impulsive and convective modes and the first couple of these frequencies are defined by equations in appendix E of API-650.

In this paper, by using finite element method, a simulator was built, calibrated and verified by confirmed published document. Moreover, fifty one different existing storage tanks which have been designed by API code have been observed by applying the defined method.

Modal analysis was the next step in order to research on relations between storage tanks parameters and their natural frequencies amounts. Besides, using the API relations for natural frequencies in appendix E, the amount of tank natural frequencies obtained from equations and extracted from analysis have been compared using graphs.

Interestingly enough, for many cases, it was observed that the total effective mass of only two main frequencies do not cover ninety percent of total mass, recommended by many standards. In this regard, the investigation has been extended to reach more than requested effective total mass for each case.

Finally, for every effective frequency, an equation has been defined in order to obtain frequencies with tank parameters.

## 2. RESEARCH METHODOLOGY

Several numerical models were developed by using a finite element program and after verifying the simulator based on confirmed published documents of full scaled tests approved by latest pioneer codes, a system directly was applied for all the tanks as well as a checking schedule was managed to control results.

## 2.1. Simulation theory

A group of typical cylindrical liquid tanks with containing liquid is modeled using FEM program. The steel shell of the tank is simulated using four-node-shell element and the element representing the containing liquid is eight-node-fluid with free surface.

Surveying fourteen oil depot tanks including one hundred and forty eight cylindrical steel storages, dimensions of the fifty one different models were extracted from exiting tanks. The diameters of the modeled tanks are between 7 to 54 meters. Tanks height which consists from four to nine courses of steel plates (maximum and minimum plates' thickness is 38.6 to 6.35 millimetres) are between 6 to 18.5 meters.

#### 2.1.1. Shell element

Shell element in the model has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. A consistent tangent stiffness matrix option is available for use in large deflection (finite rotation) analyses.

#### 2.1.2. Fluid element

The fluid element is used to model fluids contained within vessels having no net flow rate. The fluid element is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. Acceleration effects, such as in sloshing problems, as well as temperature effects, may be included. The fluid element is defined by eight nodes having three degrees of freedom at each node: translation in the nodal x, y, and z directions.

In order to find the stiffness matrix ([K]) of the fluid elements, stress-strain equations which relate bulk strains (using fluid bulk modulus) to internal fluid pressure are studied. Also the damping matrix is developed based on stress and strain differentiation relationships. In time-history analysis, the total damping matrix ([C]) is calculated by adding Rayleigh and Viscosity matrixes according to Eqn. 2.1.

$$[C] = \alpha[M] + \beta[K] + \sum [cf]$$
(2.1)

# 2.1.3. Gap element

The underplaying supporting soil is modeled using compression-only gap conditions. Therefore the non linear geometrical behaviour of uplift and the impact can be calculated by time-history dynamic analysis.

## 2.1.4. Meshing model into element

By trial and error in several dynamic analyses, the steel shell elements are optimized aiming at higher accuracy and lesser analysis time. Since the liquid elements are dimension-sensitive and their dimension ratio has a great impact on results' accuracy, the optimized shape is very close to a brick. Thus, using manual meshing, the liquid elements were altered to an optimized shape.

#### 2.1.5. Boundary conditions

The boundary conditions in the contact area of liquid and steel shell are only coupled in normal shell direction. Therefore the adjoining nodes are free to move in all directions except for that of normal shell element.

## 2.2. Model verification

After designing a simulator the first deed is to define a verification system that should be based on the experimental published documents. Moreover parametric valid equation for pressure was considered for every analysis as well.

## 2.2.1. Hydrostatic pressure of liquid

Hydrostatic pressure check could be simply primary verification factor. Fig. 2.1 shows the hydrostatic pressure calculated using 3D model which has 14 meter height and contains a liquid by density of 744 kilogram per cubic meter. As can be seen the hydrostatic pressure (steady-state analysis) coincides with nominal "pgh" values calculated manually. As a matter of fact, recent verification was carried out in other various testing models mentioned in the next part.



Figure 2.1. Hydrostatic pressure in model (Average value in element center – Unit : Pa )

#### 2.2.2. Natural frequencies

As the reliable factor, natural frequencies (Impulsive and convective) of a simulated full tank using the

method of simulation is compared with the confirmed values [3]. All mentioned tanks in [3] have been analysed using the research methodology and acceptable results have been obtained consequently. Table 2.1 compares the results between analytic and measured frequencies of the same tank according to the analysis and confirmed published values respectively.

Numbers are in hertz (Hz)	Tank T of Phase I. [3]		Tank No.1 of Phase II. [3]			
The $1^{\text{st}}$ , $2^{\text{nd}}$ and $3^{\text{rd}}$ computed natural impulsive frequencies of effective modes based on [3]	N/A	N/A	N/A	3.01 *	10.38	15.11
The 1 <sup>st</sup> 2 <sup>nd</sup> and 3 <sup>rd</sup> measured natural impulsive						
frequencies of effective modes based on [3]		N/A	N/A	N/A	9.6	14.3
(experimentally)						
The 1 <sup>st</sup> , 2 <sup>nd</sup> and 3 <sup>rd</sup> computed natural impulsive	3 32	10.23	14 54	3 42	10.43	14 79
frequencies of effective modes based on Analysis		10.25	17.57	5.72	10.45	17.77
The 1 <sup>st</sup> Obtained natural convective frequency of	0.25	NI/A	NI/A	NI/A	NI/A	NI/A
effective modes from equation based on [3]	0.23	11/17	11/17	11/71	11/17	1 N/ / 1
The 1 <sup>st</sup> , 2 <sup>nd</sup> and 3 <sup>rd</sup> computed natural convective		0 342	0 390	0.236	0 343	0 3 9 3
frequencies of effective modes based on Analysis	0.230	0.342	0.390	0.230	0.545	0.393

Table 2.1. Com	parison of fre	quencies of n	nodeled tank	s with [3]
	p			

\* 3.81 for rigid foundation

# 2.3. Modal analysis

In this part, an example of modal analysis has been shown. In this regard, main convective and impulsive modes of the tank depicted in section 2.2.1, has been illustrated below.

# 2.3.1. Convective modes

Fig. 2.2 to 2.4 illustrate main natural convective modes with the most effective mass involved in analysis.



Figure 2.2. Natural convective modes



Figure 2.3. Natural convective modes



Figure 2.4. Natural convective modes

# 2.3.2. Impulsive modes

Four impulsive modes of tanks with the most effective mass are shown in Fig. 2.5 and 2.6 and table 2.2 shows several effective modes and their mass percentage.



Figure 2.5. Natural impulsive modes



Figure 2.6. Natural impulsive modes

<b>Table 2.2.</b> Wiodal analysis results					
Frequency	Period	Effective mass percentage			
(Hz)	(s)	(%)			
0.172540	5.7956	33.00			
0.306387	3.2638	0.76			
5.03015	0.1988	53.72			
19.9203	0.0050	0.78			
41.5298	0.0024	1.83			
Total	Mass	90.09			

Table 2.2. Modal analysis results

#### **3. RESULTS**

In appendix E of the eleventh edition of [1], the first convective mode period is calculated by equation E.4.5.2-a and E.4.5.2-b shown in Eqn. 3.1 and 3.2 below, where  $T_c$  is natural period of the convective mode (s), D is nominal tank diameter (m) and H is maximum design product level (m).

$$T_c = 1.8K_s \sqrt{D} \tag{3.1}$$

$$K_s = \frac{0.578}{\sqrt{\tan\left(\frac{3.68H}{D}\right)}} \tag{3.2}$$

Using API parameters, Graph 3.1 draws the three most effective periods obtained by analysis for every tank (Illustrated in Fig. 2.2.left, right and 2.3.left—from high to low effective total mass percentage—as an example) and also it draws the convective period calculated by API equation for every tank against X parameter (Eqn. 3.3).

$$X = K_s \sqrt{D} \tag{3.3}$$

As it is shown, Analysis parameters have linear relations with X as API suggested. Therefore based on the results, the three first effective convective periods could be calculated by Eqn. 3.4, 3.5 and 3.6.

$$T_c = 1.9111X + 0.0272 \tag{3.4}$$

$$T_{c-2} = 0.8517X + 0.5948 \tag{3.5}$$

$$T_{c-3} = 0.6846X + 0.502 \tag{3.6}$$



Graph 3.1. Natural convective periods against X

Eqn. 3.7 is suggested for calculating impulsive period by [1] in appendix E of eleventh edition (Eq.E.4.5.1-1a) whereas H and D are the same as before and the rest of the parameters are defined below:

- T<sub>i</sub>: Natural period of vibration for impulsive mode of behaviour (s)
- C<sub>i</sub>: Coefficient for determining impulsive period of tank system
- t<sub>u</sub>: Equivalent uniform thickness of tank shell (mm)
- $\rho$ : Density of fluid (kg/m<sup>3</sup>)
- E: Elastic modulus of tank material (MPa)

$$T_{i} = \left(\frac{1}{\sqrt{2000}}\right) \left(\frac{C_{i}H}{\sqrt{\frac{t_{u}}{D}}}\right) \left(\frac{\sqrt{\rho}}{\sqrt{E}}\right)$$
(3.7)



Graph 3.2. Natural impulsive periods against H/D

In Graph 3.2, Main impulsive mode period of every tank- the blue points are calculated by API

equation and the red ones are obtained from analysis—is drawn against H/D.

According to the results, as it is shown in graph 3.2, for the H/D ratio around 0.5, analysed amounts coincides with the calculated ones. Evidently, this ratio is a turning point after which the relative positions of the blue and red points are switched around.

By using Eqn. 3.7, the Y parameter is defined (Eqn. 3.8) which is used as the horizontal axis in Graph 3.3 and the two mentioned periods along with three more impulsive periods resulted from analysis that involve the most effective mass, was sorted against Y on vertical axis. The positioning of the points suggests that a linear relation can be proposed for the periods by Y parameter with an acceptable precision. The trend-line for each period is shown on the graph.

E: Elastic modulus of tank material (MPa).

$$Y = \left(\frac{1}{\sqrt{2000}}\right) \left(\frac{H}{\sqrt{\frac{t_u}{D}}}\right) \left(\frac{\sqrt{\rho}}{\sqrt{E}}\right)$$
(3.8)



Graph 3.3. Natural impulsive periods against Y

[5] referred by [1] in appendix EC (commentary in appendix E), suggested that  $t_u$  may be calculated by taking a weighted average over the wetted height of the tank wall, assigning the highest weight near the base of the tank where the strain is maximal whereas a simple average of tank shell thickness was assumed for the  $t_u$  in the two previously mentioned graphs.

By using the same way of drawing graph 3.2 and 3.3 but with this different assumption for  $t_u$ , graph 3.4 and 3.5 were drawn respectively.

Apparently, in graph.3.4 it can be observed that analysed and calculated periods are become closer to each other especially for the tanks with H/D ratios under 0.55. This modification is also supported by the two related trend-lines in graph.3.5.

Nevertheless, all trend-lines in graph 3.3 for calculating the impulsive periods have much adequate correlation in comparison with the same lines in graph 3.5—excluding the last one—even with simpler calculations (compare graph 3.3 and 3.5).



Graph 3.4. Natural impulsive periods against H/D

Finally, by using graphs 3.3, 3.5 and the equations the impulsive periods of tank can be calculated. Moreover, because of using actual and existing typical tank data, the designers can estimate tank natural frequencies. For instance, according to graph 3.4, the designer could observe that the tanks with H/D ratio around 7.5 can have the minimum possible period while lower values for the tank impulsive period result in lower acceleration.



Graph 3.5. Natural impulsive periods against Y (second assumption for t<sub>u</sub>)

# 4. CONCLUSION

In this paper by using of FEM modeling and confirmed documents a simulator method was proposed and verified. Moreover, with collecting one hundred and forty eight existing storage tanks data, fifty one different types have been applied in the verified model.

Static and modal analysis were conducted for every tanks in order to research on the natural frequencies of typical tanks and the analysis followed to extract adequate modes aiming to reach more than ninety percent of mass involvement.

Using API suggestion, a parameter was selected and equations with the proper result are suggested for calculating natural impulsive and convective frequencies.

Finally, use of real data of the typical existing tank in drawing graphs and building equations could be considered as a strong point of the current research.

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