

The Choice of the Seismic-Load Reduction Coefficient Based on the Analysis of the Plastic Resource of Structure taking into account the Low-Cycle fatigue

Y. L. Rutman & E. Simbort

State University of Architecture and Civ. Eng. (SPSUACE), St. Petersburg, Russia

E. Simbort

Center on EQE&NDR (CENDR), Russia



SUMMARY:

In the present research a nonlinear single degree of freedom system is used for analyzing the behavior of structures under elastoplastic deformation. The paper presents a method for calculating the fatigue damage in steel moment frames and multi-storey buildings with first soft storey. This method is based on The Palmgren-Miner Linear cumulative fatigue damage theory (Miner's Rule). The Manson-Coffin relationship is used to determine the Fatigue life - deformation dependence per cycle. Then an algorithm allowing to relate the value of ductility factor K_μ and the fatigue damage index with the level of deformations of a system is proposed. This research shows how to use obtained results for calculating the maximum values of deformations in the structural elements of moment resisting frames and in the columns of multi-storey buildings with soft storey.

In the paper recommendations for calculating the seismic-load reduction coefficient K_1 are proposed.

Key words: seismic-load reduction coefficient K_1 , ductility factor, low-cycle fatigue, strength design criterion, ductility design criterion

1. INTRODUCTION

These days in the world practice a multi-level design approach (Fardis, 2002), considering several seismic load levels and their corresponding ultimate limit states, is used. Such approach is applied in European normative base (Eurocode 8). Since 2011 this method is also applied in the Russian Federation (SNIP II-7-81*). In Russian norms as a rule, two seismic load levels are used: the maximum design earthquake MDEQ and the design earthquake DEQ.

In accordance with the multi-level design approach a separate serviceability analysis at DEQ level is carried out. At the same time it is also assumed that at the MDEQ level the seismic loads acting on the structures will be higher than the calculated design ones, and consequently much of the earthquake energy will be dissipated by the structure through large plastic deformations (i.e. by the plastic resource of the structure). However this assumption is based on intuitive empirical considerations. During the shaking of strong earthquakes the capacity of structure to undergo plastic deformation reduces as a consequence of the damage accumulation induced by the numerous inelastic cycles, and in quite a number of cases the plastic resource of structure would be insufficient to resist the input earthquake load.

A ductility factor is used to make a quantitative estimate of the ability of a structure to undergo plastic deformations. The ductility factor is the most widely used parameter to estimate the structural behavior beyond the elastic range. This parameter represents the maximum dynamic deflection related to the yield deflection of the system (i.e. the deflection corresponding to the system's transformation into a mechanism) $K_\mu = x_{\max}/x_y$. It has been widely recognized, however, that the level of structural damage due to earthquakes depends not only on maximum displacement but on the cumulative damage resulting from numerous cycles of elasto-plastic deformation.

Therefore in seismic resistant design the low-cycle fatigue has to be taken into account. Research on fatigue in structures dates back to the early 1900's. Low-cycle fatigue has been studied by Krawinkler

and Zohrei (1983), Park et al. (1984), McCabe and Hall (1989), Faijar (1992), Kuwamura and Yamamoto (1997), Campbell et al. (2008) and others.

In accordance with the above-mentioned, in order to estimate the behavior of structure under repeated elasto-plastic deformation conditions and to choose the optimal value of the reduction coefficient this paper presents a method for calculating the fatigue damage in steel moment frames and multi-storey buildings with first soft storey. This method is based on The Palmgren-Miner Linear cumulative fatigue damage theory (Miner's Rule) (Miner, 1945).

2. ANALYSIS METHODOLOGY

2.1. A Single Degree of Freedom Model

In the present research, a non-linear single degree of freedom model is used to analyze the structural behavior under the elasto-plastic deformation conditions. The validity of the application of this model was analyzed by Simbort (2011). The model (see Fig. 2.1a) is described by a differential equation (1):

$$m\ddot{x} + \alpha\dot{x} + F(x, \dot{x}) = -m\ddot{y}_g(t) \quad (2.1)$$

$$\ddot{x} + 2\xi\omega\dot{x} + f(x, \dot{x}) = -\ddot{y}_g(t) \quad (2.2)$$

where $\ddot{y}_g(t)$ is a single degree of freedom system base acceleration.

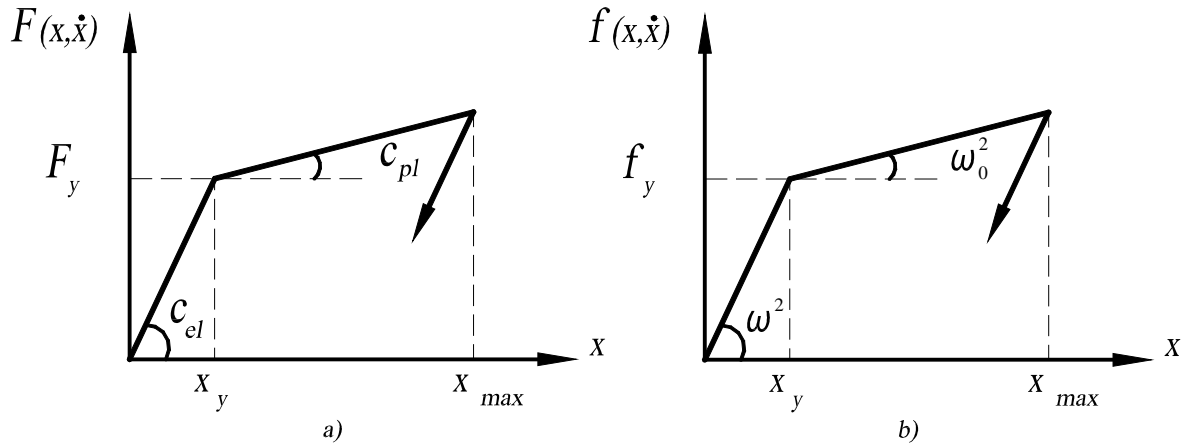


Figure 2.1. Bilinear diagram with elastic unloading path

The unload pattern is described by Masing's kinematic hardening hypothesis (Moskvitin, 1965). Hysteresis loop and cyclic deformation diagram (see fig.1b) are characterized by the following parameters: ω^2 , ω_0^2 , f_y , where ω is a first tone frequency of the system found by solving a linear-elastic problem. Based on empirical considerations, ω_0^2 is assumed to be described by $\omega^2 = (20...50)\omega_0^2$.

$f_y = \frac{F_y}{m}$, where F_y is the single degree of freedom system ultimate load, which can be found by solving a limit equilibrium problem at a horizontal load proportional to the distributed system mass. It is proposed to use the Pseudorrigidity method (PRM) to solve the problem of limit equilibrium of rigid-plastic constructions (Rutman, 1998).

2.2. Analysis Stages

Calculation method of the fatigue damage to moment resisting frames and multi-storey buildings with first soft storey consists of following steps:

1. Determine from the solution of the differential equation (2.2), the non-linear dynamic response of the single degree of freedom system to an earthquake loading represented by accelerogram.
2. Extract the maximum dynamic displacement value x_{\max} .
3. Calculate the ductility factor value $K_{\mu} = x_{\max}/x_y$.
4. Determine the maximum plastic deformations in the elements of the structure. Rutman and Simbort (2011) proposed an algorithm allowing to relate the ductility factor value of the system K_{μ} with the deformation level. In the above mentioned paper a procedure to relate the ductility factor and deformations for a cantilever beam case is described. This procedure is based on the adoption of Bülfinger's exponential law characterizing the stress-strain relationship (Bülfinger, 1729). This exponential law is represented by

$$\sigma = \overline{B}_1 |\varepsilon|^{\mu-1} \varepsilon$$

where $\overline{B}_1 > 0$ and $\mu \leq 1$ are constants.

The exponential deformation law leads to a relationship:

$$\overline{B}_1 J_m |\chi|^{\mu-1} \chi = -M$$

Hence the relation between the ductility factor and cantilever beam deformations is defined by the Eqn. 2.3.

$$k_{\mu} = \frac{3}{\gamma(m+2)} \frac{\varepsilon_{\max}}{\varepsilon_y} \quad (2.3)$$

In this expression k_{μ} is a cantilever beam ductility factor, which is equal to $k_{\mu} = \frac{\Delta}{\Delta_y}$. Δ is a cantilever plastic deflection. $\Delta_y = \gamma \cdot \Delta_{el}$, where Δ_{el} is cantilever elastic deflection, γ is the structural cross section shape dependent coefficient. $m = \frac{1}{\mu}$, $\mu \leq 1$ are constants. ε_{\max} is the maximum plastic strain deformation of the cantilever beam. ε_y is the strain deformation corresponding to yield point (strength).

The maximum value of K_{μ} factor coefficient is equals to

$$k_{ult} = \frac{3}{\gamma(m+2)} \frac{\varepsilon_{UTS}}{\varepsilon_y} \quad (2.4)$$

5. Select and classify load cycles depending on values and properties of accumulated unilateral deformations.
6. Calculate the fatigue life based on the deformation-kinetic criteria of strength in low-cycle fatigue.

2.3. Low-Cycle Fatigue Calculation

Calculation of damage due to repeated cyclic loads is a well established methodology in some fields of engineering. In the present research in order to calculate the low-cycle fatigue of structural elements deformation-kinetic criteria of low-cycle fatigue resistance are applied. These criteria are based on fatigue and quasi-static damage summation (Kogaev et al., 1985). Fatigue damage is related to cyclic deformations whereas quasi-static damage is related to accumulated unilateral deformations. Therefore the linear summation of damage caused by cyclic and accumulated unilateral deformations is carried out.

Fatigue damage fraction is estimated as

$$d_f = \int_1^{N_f} \frac{dN}{N_{fi}} \quad (2.5)$$

Where N is the number of applied cycles at nominal stress σ_i ; N_f is the number of cycles before failure (crack initiation); N_{fi} is the limit number of cycles to failure at the same stress σ_i and for the same cycle type. N_{fi} can be found using the low-cycle fatigue curve for rigid loading conditions. The Manson-Coffin relationship is used to determine the fatigue life - deformation ε_p dependence per cycle (Okopniy et al., 2001):

$$\varepsilon_p N_f^{m_p} = C_f = \frac{1}{2} \ln \frac{1}{1-\psi} \quad (2.6)$$

where m_p is a plasticity index applied in intervals from 0.4 up to 0.6.

Quasi-static damage fraction is estimated as

$$d_s = \int_0^{e_f} \frac{de}{\varepsilon_f} \quad (2.7)$$

where e is the unilateral plastic deformation accumulated at the static and cyclic loading process; e_f is the unilateral accumulated deformation up to failure (crack initiation); ε_f is the available plasticity (strain capacity) of the material. The ultimate limit state by the terms of the low-cycle fatigue fracture is reached at

$$d_f + d_s = \int_1^{N_f} \frac{dN}{N_{fi}} + \int_0^{e_f} \frac{de}{\varepsilon_f} = 1 \quad (2.8)$$

Then failure is predicted to occur when $d_f + d_s \geq 1$.

3. EXAMPLE

Hereinafter as an example a nine-storey building with first soft-storey is analyzed under earthquake excitation from NEWHALL earthquake data, with peak ground acceleration equal to 5.782 m/s² and length of time equal to 58.98 s. The input data for analysis are the following parameters:

$$\varepsilon_y = 0,002; \quad \varepsilon_{UTS} = 0,15; \quad m = 5; \quad \omega = 7,71 \text{ rad/s}; \quad f_y = 2,1 \text{ m/s}^2; \quad \Delta_y = 0,0175 \text{ m}; \quad \gamma = 1,12$$

3.1. Calculation of the ductility factor and its corresponding deformation level

According to Eqn. (2.4) the maximum value of K_μ factor can be determined by

$$k_{ult} = \frac{3}{\gamma(m+2)} \frac{\varepsilon_{UTS}}{\varepsilon_y} = \frac{3}{1,12(5+2)} \frac{0,15}{0,002} \approx 29$$

After solving the dynamic problem (using Mathcad program) the maximum displacement value of nonlinear SDOF system equals $x_{\max} = 0,312 \text{ m}$ (see Fig. 3.1).

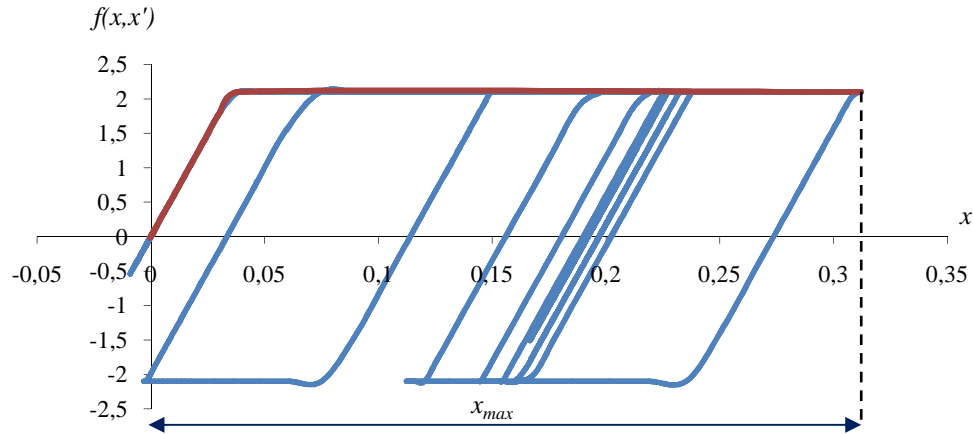


Figure 3.1. Cyclic force-deformation curve for the equivalent SDOF system

Using the equivalent cantilever beam concept (Rutman and Simbort, 2011) the displacement values of soft-storey columns were calculated. These are equal to $\Delta = 0,156 \text{ m}$. Consequently the value of the ductility factor for this system is equal to

$$k_{\mu} = \frac{\Delta}{\gamma \Delta_{el}} = \frac{0,156}{1,12 \cdot 0,0175} = 8 < k_{ult}$$

Deformation level corresponding to given ductility factor is equal to

$$\varepsilon_{\max} = \frac{\gamma(m+2)k_{\mu}\varepsilon_y}{3} = \frac{1,12 \cdot 7 \cdot 8 \cdot 0,002}{3} = 0,042 < \varepsilon_{UTS}.$$

3.2. Low-cycle fatigue analysis

The input data for the low-cycle fatigue analysis are the following parameters:

$$\varepsilon_p = \varepsilon_{UTS} = 0,15; \quad m_p = 0,6$$

Using the formula (2.6) we will define the relationship between low-cycle fatigue resistance properties and the material plasticity under monotonic loading. Hence at $N_f = \frac{1}{2}$, $\varepsilon_p N_f^{m_p} = C_f = 0,099$.

Having selected and classified the loading cycles using Eqns. (2.5) and (2.7) we define the fatigue and quasi-static damage fractions. Thereafter using Eqn. (2.8) the linear damage summation is carried out. The results are presented in table 3.1.

Table 3.1. Fatigue and quasi-static damage index calculation

Cyclic deformations, e_c	Accumulated unilateral deformations, e_u	N	N_{fi}	N/N_{fi}	e	e_f	e/e_f	$d_f + d_s$
1.60%	2.60%	1	21	0.048	0.026	0.150	0.173	
1.10%		1	39	0.026				
0.22%		1	565	0.002				
0.17%		1	925	0.001				
			d_f	0.076		d_s	0.173	0.249
Remaining Fatigue Life								75%

From the table data (3.1) the fatigue and quasi-static damage fractions are 8% and 17% respectively. Therefore the accumulated damage is equal to 25%. The remaining plastic resource equals 75%.

4. CONCLUSIONS

The present procedure not only provides a notion about the integrity or failure of the structures but also allows to estimate with an appropriate degree of accuracy the remaining plastic resource of the element of study. In addition this method is valid for both monotonic and cyclic load cases.

It should be noticed that fatigue d_f and quasi-static d_s damage indices do not substitute the existing parameters applied in seismic-resistant design to quantify the plastic deformation capacity of a structure. They serve as an additional tool that can be used to better understand the performance of structures under earthquake excitations.

On the basis of the obtained data it can be concluded that K_1 coefficient should be adopted not only depending on the type of seismic force-resisting systems but also taking into account the ground excitation characteristics, dynamic characteristic of the structure (Rutman and Simbort, 2011) and the low-cycle fatigue.

REFERENCES

- Fardis, M. N. (2002). Code Developments in Earthquake Engineering. *12th European Conference on Earthquake*. Paper reference 845.
- Eurocode 8 (2004). Design of Structures for Earthquake Resistance.
- SNiP II-7-81*(2011). Russian Seismic Building Code.
- Krawinkler, H. and Zohrei, M. (1983). Cumulative Damage in Steel Structures Subjected to Earthquake Ground Motion. *Computer & Structures*. **16:1-4**, 531-541.
- Park, Y., Ang, A., Wen, Y. (1984). Stochastic model for seismic damage assessment. *ASCE Proceedings of the 5th Engineering Mechanics Division Specialty Conference*. 1168-1171.
- McCabe, S. L. and Hall, W. J. (1989). Assessment of Seismic Structural Damage. *Journal of Structural Engineering*. **115:9**, 2166-2183.
- Kuwamura, H. and Yamamoto, K. (1997). Ductile crack as a trigger of brittle fracture in steel. *Journal of structural Engineering* **126:3**, ASCE.
- Campbell, S. D., Richard, R. M. and Partridge, J. E. (2008). Steel moment frame damage predictions using low-cycle fatigue. *14th World Conference on Earthquake Engineering*. **Vol 5:1**. 225-233
- Faijar, P. (1992). Equivalent ductility factors, taking into account Low-Cycle Fatigue. *Earthquake Engineering and Structural Dynamics*. **Vol. 21**, 837-848.
- Miner, M. A. (1945). Cumulative damage in fatigue. *Journal of applied mechanics*. **12:3**, A159-A164.
- Rutman, Y. L. and Simbort, E. G. (2011). The Choice of the Seismic-Load Reduction Coefficient K_1 based on the Analysis of Plastic Resource of Structure [in Russian]. *Bulletin of Civil Engineers (SPSUACE), St. Petersburg, Russia*. **№(2)27**, 78-81.
- Simbort, E. G. (2011). A Comparison of Nonlinear Dynamic Analyses performed by both Single and Multi Degree of Freedom Systems [in Russian]. *Magazine of Civil Engineering*. **№6(24)**, 23-27.
- Moskvitin, V. V. (1965). Plasticity under variable loads [in Russian]. MGU, Moscow.

- Rutman, Y. L. (1998). Pseudorigidity method for resolving problems of limit equilibrium of rigid-plastic constructions [in Russian]. St. Petersburg, Russia.
- Bülfinger, G. B. (1729). Comm. Acad. Petrop. 4, 164
- Kogaev, V. P. (1985). Calculations of Machine Elements and Structures for Strength and Durability [in Russian], Mashinostroenie, Moscow.
- Okopniy, Y. A., Radin, V. P. and Chirkov, V. P. (2001). Mechanics of Materials and Structures [in Russian], Mashinostroenie, Moscow.
- Rutman, Y. L. and Simbort, E. G. (2011). An Analysis of Ductility Factor for a Reasonable Choice of the Seismic-Load Reduction Coefficient K_1 [in Russian]. Earthquake Engineering. Safety of Structures. № 4, 21-25.