# Estimation Method of Seismic Response Based on Momentary Input Energy Considering Hysteresis Shapes of a Building Structure

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#### **SUMMARY:**

A simple method of estimating seismic maximum response without non-linear time history analysis using by relationships between maximum momentary input energy and maximum displacement is presented. On the basis of a hysteresis shape of a structure, maximum displacement can be calculated using the maximum momentary input energy. We proposed a relationship between maximum response and momentary input energy as a function of plasticity rate and amplitude ratio. By using 10 observed or simulated ground motions, effectiveness of the evaluation precision is investigated. As a result, it is found that momentary input energy spectrum of ground motion can be estimated utilizing elastic response spectrum of pseudo velocity with damping factor of 10%. Hysteretic energy dissipation can be expressed by a function of plasticity rate and amplitude ratio. The proposed method can obtain the maximum response displacement of an inelastic single degree freedom system without non-linear time history analysis.

Keywords: Momentary input energy, Hysteresis shapes, Pseudo-velocity response spectrum

## **1. INTRODUCTION**

Simple estimation methods of seismic maximum response without non-linear time history analysis are helpful in designing building structures. Especially, maximum drift angle is one of the important factors in the sense to express the degree of damage induced by earthquake. It is necessary to estimate inelastic response with high accuracy using by provided elastic response spectrum. Equivalent linearization is the well known method, which is taken into account in many aseismic guidelines, for example, EC8, ASCE/SEI 41-06 and Japanese Limit Strength Method.

On the other hand, some simple methods utilizing seismic energy response have been widely studied. Hagiwara *et al.* (2009) have studied on seismic response for combination systems, consisting with multiple inelastic springs, according to relationship between total input energy and natural period of the system. Inoue *et al.* (1998) have defined a momentary input energy as a seismic energy inputted during a half cycle of hysteresis. And an estimation method of maximum displacement (as ductility factor) utilizing maximum momentary input energy corresponding to hysteretic energy dissipation have been proposed. Previous research has suggested that maximum momentary input energy spectrums of observed earthquakes are independent of damage level *i.e.*, elastic or inelastic level. Assuming response type of hysteritic shape, that is a partial response type, impartial response type and their intermediate response type, maximum ductility factor can be estimated. However this method has some limitations, such as the need for assumption of response type.

In this research, we present a simple estimation method of maximum seismic response for R/C structures utilizing momentary input energy. Because of elasto-plastic behavior of R/C structure during earthquake, maximum response has a higher correlation with momentary input energy than amount of total input energy.

#### 2. EXAMINED SDF SYSTEM AND INPUT GROUND MOTIONS

Single-degree-of-freedom (SDF) system having various yield strength were analyzed. The mass of the system was 1 ton and initial period  $T_0$  was 0.22s, which is equivalent of middle high stories of R/C building. Viscous damping factor was taken as 0.03. Six kinds of yield base-shear, from 0.3 to 0.8, were examined. As the force-displacement relation, degrading trilinear model as shown in Figure 2.1 was used. The rigidity degrading ratio at yield point is  $\alpha_y = 0.3$  and cracking strength is 1/3 of yield strength  $F_y$ . Under the yield point, unloading rigidity is taken as origin-oriented. Over the yield point, unloading rigidity is given by  $K_y/\sqrt{\mu}$  and sign-inversed rigidity is oriented to experienced maximum point.



a) Skeleton curve

Figure 2.1. Hysteresis model with tested SDF system

For input ground motions, six observed strong ground motions and four simulated earthquakes were used. These observed ground motions are records of El Centro NS (1940 Imperial Valley), Taft EW ( 1952 Kern County), Hachinohe harbor NS (1968 Off Tokachi), Tohoku University NS (1978 Off Miyagi), Sylmar County Hospital NS (1994 Northridge) and Japan Meteorological Agency (JMA) at Kobe NS (1995 Hyogoken-Nanbu). Simulated earthquakes are based on the target spectrum specified in the Japanese building design code. And their phase characteristics were El Centro NS, JMA Kobe NS and Tohoku University NS. These tenth ground motion were normalized as 50cm/s of peak ground velocity (PGV) and peak ground acceleration (PGA) values are shown in Table 2.1.

	Name	Earthquake	PGV $(cm/s^2)*1$
Observed Earthquake	El Centro NS	1940 Imperial Valley	485.4
	Taft EW	1952 Kern County	536.3
	Hachinohe NS	1968 Off Tokachi	338.8
	Tohoku Univ. NS	1978 Off Miyagi	348.8
	Sylmar NS	1994 Northridge	322.0
	JMA Kobe NS	1995 Hyogo-ken Nanbu	452.3
Simulated Earthquake	The Building Center of Japan		331.2
	ART El Centro		422.6
	ART Tohoku Univ		463.5
	ART Kobe		375.7

\*1 Normalized as 50cm/s of PGV

#### **3. OUTLINES OF ESTIMATION METHOD**

In our proposed method, maximum response ductility factor is estimated in such a way to correlate maximum momentary input energy with hysteretic energy dissipation according to assumed hysteretic shape during a half cycle. Here, seismic energy is represented as equivalent velocity in the following discussion. Besides, our method needs initial period  $T_0$ , viscous damping factor  $h_0$  and yield strength  $F_y$  of the examined system in order to estimate its seismic response. Our proposed method is as following:

- STEP 1 : A demanded hysteresis energy spectrum contained in maximum momentary input energy is calculated.
- STEP 2 : A hysteretic energy dissipation corresponding to peak displacement is calculated according to the given hysteresis model (*e.g.* degrading trilinear model in this research) of the system.
- STEP 3 : An amplitude ratio spectrum is calculated. Here, the amplitude ratio is defined as the ratio of both peak displacements during a half cycle of response.
- STEP 4 : Then a hysteretic energy dissipation spectrum is calculated as explained above.

The outline of our method is showing in **Figure 3.1**. The broken line shows the demanded hysteretic energy spectrum contained in maximum momentary input energy. The solid line shows the hysteretic energy dissipation spectrum according to the given hysteresis shapes and amplitude ratio spectrum. In this figure, the cross point of two lines shows the estimated the maximum response ductility factor (the maximum response displacement) of the system.



Figure 3.1. The outline of estimation for maximum response

In this method, the equivalent period  $T_{eq}$  is corresponding to the ductility factor  $\mu$ . Here, the equivalent period  $T_{eq}$  is defined as:

$$T_{eq} = \begin{cases} \alpha \cdot \sqrt{\frac{\mu}{\alpha_y}} \cdot T_0 & \mu \ge 1\\ \\ \alpha \cdot \sqrt{\frac{\mu}{\alpha_y(\gamma + \lambda)}} \cdot T_0 & \mu < 1 \end{cases}$$
(3.1)

where

$$\gamma = \frac{1 - \lambda}{1 - \lambda \alpha_y} \left( \eta \mu - \lambda \alpha_y \right) \tag{3.2}$$

The parameter  $\alpha$  is designed to convert secant stiffness to practical value corresponding to ductility factor  $\mu$  as following:  $\alpha = 0.9$  at  $\mu \leq 1$ ,  $\alpha = 0.75$  at  $\mu \geq 2$  and linearly-interpolated value of  $\alpha$  at  $1 < \mu < 2$ .

### 4. MAXIMUM MOMENTARY INPUT ENERGY

To express the seismic characteristics, elastic response spectrum (acceleration, velocity and displacement) is frequently used. In this method, it is necessary to evaluate the precise maximum momentary input energy spectrum utilizing these elastic spectrums, though previous researches have been conducted under the condition that maximum momentary input energy spectrum is given in some way. Recently, board attention has been paid to the research of momentary input energy. Ishimaru et al. (1997) have defined the seismic input energy ratio as the maximum value of the input energy during time interval of the earthquake record. In our research, maximum momentary input energy  $\Delta E_{\text{max}}$  is defined as a seismic input energy during a half cycle of response when it reaches maximum displacement. Inoue et al. (1999) have suggested that maximum momentary input energy  $\Delta E_{\text{max}}$  spectrum is independent of a plastic level of maximum response. According to that, we have evaluated the momentary input energy spectrum using elastic pseudo-velocity response spectrum  $_PS_V$ . Figure 4.1 shows  $\Delta E_{\max}$  spectrum and  $_PS_V$  of an elastic system with 10% viscous damping factor. The maximum momentary input energy spectrum agrees closely with  $_{P}S_{V}$ . This closely agreement has been given in all cases of seismic ground motions in this research. Therefore, we estimate the maximum momentary input energy using by the pseudo-velocity response spectrum in this method, however physical explanation of these agreements need to be discussed.



Figure 4.1. Maximum momentary input energy spectrum and pseudo-velocity response spectrum

#### 5. HYSTERETIC ENERGY DISSIPATION

#### 5.1 Hysteretic Energy Dissipation of The System

Maximum response ductility factor is estimated corresponding to the energy dissipation, which matches the half cyclic hysteresis energy given by the  $\Delta E_{\text{max}}$ . Figure 5.1 shows the five kinds of assumed hysteretic types according to pre-yield and the peak displacement before half cycle from maximum point (as the amplitude ratio). Here, the point before a half cycle (it's represented as  $\eta \mu \delta_y$ ) is oriented to the same point of the reverse after the sign of the load changed.

The hysteretic energy dissipation  $\Delta E_{\mu}$  is expressed as following

$$\Delta E_{\mu} = F_y \cdot \delta_y \cdot f(\mu, \eta) \tag{5.1}$$



Figure 5.1. Assumed hysteresis response types according to the amplitude ratio

where  $f(\mu, \eta)$  is a function of ductility factor  $\mu$  and amplitude ratio  $\eta$ , which is independent of base-shear and yield displacement of the system. The function corresponding to each hysteresis type (as shown in **Figure 5.1**) is given as following

$$f(\mu,\eta) = \begin{cases} \mu - \sqrt{\eta \cdot \mu} & \text{as TYPE 1} \\ \frac{(\gamma + \lambda + 1) \cdot (1 - \eta \cdot \mu)}{2} + (\mu - 1) & \text{as TYPE 2} \\ \frac{\lambda - \lambda \cdot \alpha_y - 1}{2} + \mu - \frac{\eta^2 \cdot \mu^2}{2\alpha_y} & \text{as TYPE 3} \\ \frac{(\gamma + \gamma' + 2\lambda) \cdot (\mu - \eta \cdot \mu)}{2} & \text{as TYPE 4} \\ \frac{(2\mu - \lambda \cdot \alpha_y)\lambda + (\mu - \lambda \cdot \alpha_y)\gamma'}{2} - \frac{\eta^2 \cdot \mu^2}{2\alpha_y} & \text{as TYPE 5} \end{cases}$$
(5.2)

 $2\alpha_y$ 

where  $\gamma$  and  $\gamma'$  are functions of F', which is a parameter defined as a load of one point on the second path, as following

$$F' = (\gamma + \lambda)F_y \quad \text{or} \quad (\gamma' + \lambda)F_y$$
(5.3)

Thus,  $\gamma$  and  $\gamma'$  can be treated as the ratio between load increment from the first piece-wise and yield strength  $F_y$ .

As is seen the above relations are summarized in **Table 5.1** and **Figure 5.2**. It can be seen that hysteretic energy dissipations are given by TYPE 1 or TYPE 2 totally in the case of  $\mu > 1$ . In the case of  $\mu < 1$ , they are given by almost TYPE 4 because TYPE 5 can be recognized rare cases in the view point of extremely lower or higher amplitude ratio.

Hysteresis type	Ductility factor condition	Amplitude ratio condition
TYPE 1		$\eta \geqq 1/\mu$
TYPE 2	$\mu \geqq 1.0$	$\lambda \alpha_y / \mu \leq \eta < 1 / \mu$
TYPE 3		$\eta < \lambda \alpha_y / \mu$
TYPE 4	u < 1.0	$\eta \geqq \lambda \alpha_y / \mu$
TYPE 5	$\mu < 1.0$	$\eta < \lambda \alpha_y / \mu$

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Table 5.1. Hysteresis type condition according to  $\mu$  and  $\eta$ 

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Figure 5.2. Hysteresis type condition according to  $\mu$  and  $\eta$ 

#### 5.2 Amplitude Ratio as A Function of Equivalent Period

The amplitude ratio  $\eta$  is important factor for the precise seismic estimation. Therefore, we proposed the amplitude ratio model using by only response spectrum (without elasto-plastic time history analysis). Previous research has suggested that the amplitude ratio shows the same tendency regardless of elastic or elasto-plastic response. According to that, we have proposed the amplitude ratio spectrum model as a function of pseudo-velocity response spectrum  $_PS_V$  as following

$$\eta \left( T_{eq}, {}_{P}S_{V}, \mathrm{PGV} \right) = \begin{cases} \frac{0.3}{\mathrm{PGV}} \cdot \frac{{}_{P}S_{V}}{T_{eq}} & T_{eq} \leq 0.6\\\\ \frac{1}{2\mathrm{PGV}} \cdot {}_{P}S_{V} & T_{eq} > 0.6 \end{cases}$$
(5.4)

**Figure 5.3** shows estimated amplitude ratio spectrum and the results of analysis containing elastic and several levels of elasto-plastic response). However the amplitude ratio spectrum has high uncertainty, they shows the same tendency regardless of their plasticity level. Our proposed models averagely agree with the results of time history analysis.



Figure 5.3. Estimated Amplitude ratio spectrum and the results of time history analysis

### 6. RESULTS OF ESTIMATION

**Figure 6.1** shows estimated maximum ductility factors vs. results of time history analysis plotted. Here, the numbers of estimated cases is sixties (sixth base-shear models by tenth earthquakes). It can be seen from this figure that our proposed method can estimate precise maximum response. Exceptionally in a small number of cases, estimated responses are disaccorded with time history analytical results. It is conjectured that the precision of the demanded hysteresis energy spectrum have a major effect on the estimated results under the large ductility factor ( $\mu > 3$ ). Though, these results suggest that the propose method can obtain the maximum response displacement within practical objectives.



Figure 6.1. Estimated maximum ductility factors and results of time history analysis

#### 7. CONCLUSION

A simple method of estimating seismic maximum response without non-linear time history analysis utilizes relationships between maximum momentary input energy and maximum displacement has been proposed. By sixties cases of SDF system, the effectiveness of the evaluation precision has been investigated. As a result, it is found that the demanded hysteresis energy spectrum contained in maximum momentary input energy can be estimated utilizing elastic pseudo-velocity response spectrum. Hysteretic energy dissipation can be expressed by a function of ductility factor and amplitude ratio. The hysteretic energy dissipation spectrum can be obtained using by proposed amplitude ratio spectrum model. The proposed method can obtain the maximum response displacement of an inelastic single degree freedom system without non-linear time history analysis.

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