Frequency Adaptive Lumped-Mass Stick Model and Its Application to Nuclear Containment Structure

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SUMMARY:
Seismic analysis of structures by using a finite element model based on the distributed mass system may provide a high accurate result. However, depending on their structural complexity and the size of the elements, it takes long time for analyzing them. To overcome such computational cost problem, a lumped-mass stick model is usually adapted. A conventional lumped mass model considering a geometric configuration like a tributary mass area can be accepted for a beam-to-column frame structure since it has specific locations to lump the mass. However, when the structure has no such specific locations, for instance tower and nuclear containment structures, the conventional model may provide a low accurate response result. In this paper, a new lumped-mass stick model is developed, based on the variations of frequencies and eigenvectors. The new model provides the same natural frequencies of actual structures. A nuclear containment building is considered for an application of the new model and its dynamic performance is evaluated through a time history analysis.

Keywords: lumped-mass stick model, frequency adaptive, nuclear containment building, dynamic response

1. INTRODUCTION

The lumped-mass stick model is usually adapted as a simplified model for complex column typed-structures, for example, nuclear containment buildings [Varma et al., 2002; Huang et al., 2010] and electric post bushings [Reinhorn et al., 2011; Roh et al., 2012]. For the lumped-mass stick model, an actual structure is discretized with a series of column elements. The lumped mass at the node is determined from the portion of the weight that can reasonably be assigned to the node, which is called “tributary area consideration”. For the stiffness evaluation of each stick element of the lumped-mass stick model, the static and geometric methods are normally used [Varma et al., 2002; Soni et al., 1987]. The static method uses an arbitrary static load applied to a single layer of the full (3D) finite element model, like a pushover analysis, while the geometric method considers the geometric shape of the cross-section to calculate sectional moment of inertia and shear-coefficients. The description above is the typical considerations for the conventional lumped-mass stick model. In this study, the equivalent stiffnesses of the lumped-mass stick model are evaluated based on the conventional methods. However, the mass evaluation is based on the eigen-properties such as natural frequencies and eigenvectors of actual structure. The new technique of the lumped-mass stick model developed in the present study provides the same natural frequencies of actual structures and a high accurate performance in the static and dynamic responses.

2. FREQUENCY ADAPTIVE LUMPED-MASS STICK MODEL

The new method developed in this study determines the lumped mass locations by investigating the mode-shapes of the structure. The number of the lumped mass locations is the same as the number of frequencies or target modes of the structure which is based on either a modal participation mass ratio or a maximum frequency required in the design spectrum. Regarding the modal mass participation ratio, normally above 90% is accepted, but it can be increased higher than 90% if more accurate responses are required. Fig. 1 shows the procedure to determine the lumped mass locations. First to fourth-mode shapes are presented for an example. The mode shapes or eigenvectors are obtained through an eigenvalue analysis for the finite element (FE) modeling of an actual structure. Since several nodes are existed on each layer of the FE model, an averaged eigenvector is considered as a representative value for each layer. Considering the deflection shapes of each mode and performing a linear interpolation between two adjacent vertexes such as points “a” and “b: in Fig. 1, new mass location (point “c”) is obtained. Considering the result (three nodes in Fig. 1), such linear interpolating is continued for the next mode (4th mode in Fig. 1). From the procedure, the number of the mass nodes is equal to the number of the modes considered.

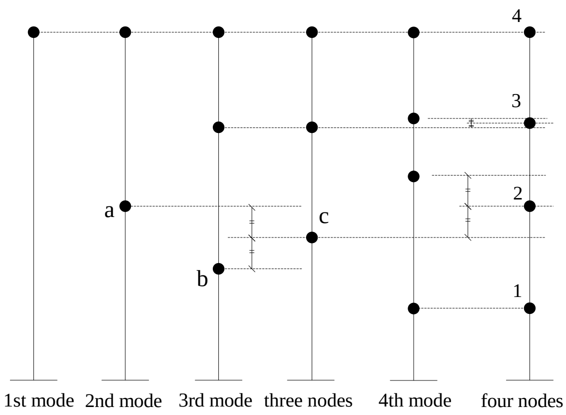

Figure 1. Interpolation procedure for lumped mass locations

In the lumped-mass stick model, the equivalent bending and shear stiffnesses are considered to evaluate the stiffness variation of the actual structures. If the sectional shape is uniform or a regular shape, then the stiffnesses are calculated by closed-form formulas. If the actual structure has a non-uniform cross-sectional shape, the total lateral stiffness, k_l, evaluated from the pushover analysis has the relationship with the bending and shear stiffnesses as following.

$$k_l = k_b \cdot k_s / \left(k_b + k_s\right) \tag{2.1}$$

where, k_b and k_s are the equivalent bending and shear stiffness, respectively. In the frame structural analysis platform like SAP2000 [Computers and Structures, 2011] and IDARC2D [Reinhorn et al., 2009], such stiffnesses are identified with the equivalent flexural and shear rigidities, EI_{eq} and $GA_{s,eq}$, respectively.

In the conventional lumped-mass stick model, the amount of nodal masses is evaluated by considering the structural configuration (tributary area consideration). However, the natural frequencies of the model are normally not the same as those of the actual structure. In this study, a new technique is developed to obtain the nodal lumped masses providing the same natural frequencies as the actual structure. The mass matrix is named here “frequency adaptive lumped-mass matrix”. For the first step of the technique, the eigenvalue analysis is performed for the structure prepared in the finite element (FE) model. Obtaining the target frequencies (eigenvalues) and vertexes of the corresponding each mode-shape, the preliminary lumped mass matrix $\mathbf{M}_{\mathrm{L}}^{0}$ is calculated as following.

$$\mathbf{M}_{\mathrm{L}}^{0} = \left[\left(\boldsymbol{\phi}_{\mathrm{d}}^{\mathrm{T}}\right)\left(\boldsymbol{\omega}_{\mathrm{t}}^{2}\right)\left(\boldsymbol{\phi}_{\mathrm{d}}^{\mathrm{T}}\right)\right]^{-1}\mathbf{K}_{\mathrm{L}} \tag{2.2}$$

where, $\mathbf{K}_{\mathrm{L}}$ is the static condensation stiffness matrix of the stick model and $\boldsymbol{\omega}_{\mathrm{t}}$ the eigenvalue matrix of the actual structure which is a target eigenvalue matrix for the lumped-mass stick model. The matrix ϕ_d is the corresponding maximum normalized eigenvector matrix for the same mass locations of the lumped-mass stick model (vertexes of each mode shape). The preliminary mass matrix $\mathbf{M}_L^0$ obtained from Eqn. 2.2 is neither the symmetric nor the diagonal matrix since the eigenvector matrix is not orthogonal. In order to obtain a diagonal mass matrix, the summation of the each row of the mass matrix is conducted. The diagonal mass matrix ($\mathbf{M}_L^0$) is named here "initial shooting mass matrix" and symbolized as $\mathbf{M}_L^*$. Performing the eigenvalue analysis using the stiffness matrix ($\mathbf{K}_L$) and the initial shooting mass matrix ($\mathbf{M}_L^*$) provides a new engenvalue matrix ($\boldsymbol{\omega}_L$) and the corresponding eigenvector matrix (ϕ_L). These matrixes are related as shown below.

$$\mathbf{K}_L = \left(\phi_L^T\right)^{-1}\left(\boldsymbol{\omega}_L^2\right)\left(\phi_L\right)^{-1} \tag{2.3}$$

$$= \left[\left(\phi_L^T\right)^{-1}\left(\boldsymbol{\omega}_L^T\right)\left(\boldsymbol{\omega}_t^T\right)^{-1}\right]\left[\left(\boldsymbol{\omega}_t^T\right)\left(\boldsymbol{\omega}_t\right)\right]\left[\left(\boldsymbol{\omega}_t\right)^{-1}\left(\boldsymbol{\omega}_L\right)\left(\phi_L\right)^{-1}\right] \tag{2.4}$$

where, the matrix ϕ_L is a mass normalized eigenvector matrix. Introducing a new eigenvector matrix (ϕ_{new}) which is scaled with $\boldsymbol{\omega}_t/\boldsymbol{\omega}_L$, Eqn. 2.4 is rearranged as following.

$$\mathbf{K}_L = \left(\phi_{new}^T\right)^{-1}\left(\boldsymbol{\omega}_t\right)^2\left(\phi_{new}\right)^{-1} \tag{2.5}$$

$$\phi_{new} = \left(\phi_L\right)\left(\boldsymbol{\omega}_L\right)^{-1}\left(\boldsymbol{\omega}_t\right) \tag{2.6}$$

Using the new matrix ϕ_{new}, the corresponding new mass matrix ($\mathbf{M}_L$) is calculated as following.

$$\mathbf{M}_L = \left(\phi_{new}^T\right)^{-1}\left(\phi_{new}\right)^{-1} \tag{2.7}$$

However, the new mass matrix ($\mathbf{M}_L$) is still not diagonal since the eigenvectors are scaled with $\boldsymbol{\omega}_t/\boldsymbol{\omega}_L$. Selecting only the diagonal terms of the new mass matrix, the eigenvalue analysis is performed with the stiffness matrix ($\mathbf{K}_L$) to get the new eigenvalue and eigenvector matrixes. Repeating Eqns. 2.5 to 2.7 leads to a zero value of the off-diagonal terms in the mass matrix. Once the mass matrix is diagonal, the eigenvalues are equal to the target eigenvalues and the mass matrix is named here "frequency adaptive lumped-mass matrix".

3. CASE STUDY OF NON-PRISMATIC COLUMN

The development procedures described in the previous section are presented using a non-prismatic column. Through a time history analysis, the dynamic performances of the new lumped-mass stick model are investigated. Geometry and material property of the non-prismatic column are shown in Fig. 2 (a). The column is a symmetric structure which has a constant thickness of 1 meter and modeled with finite solid elements using the computational platform SAP2000. The natural frequencies obtained from the eigenvalue analysis are summarized in Table 1, which are used as a target eigenvalue matrix ($\boldsymbol{\omega}_t$) for the iterations. The number of modes considered for the stick model is five since the modal participation mass ratio is over 90% from the fifth mode. Considering the linear interpolation between the adjacent points as described in the section 2, the locations of the lumped mass are determined as shown in Fig. 2 (b). Using the stiffness matrix of the stick model which is evaluated through a pushover analysis (static method) and the iterations presented in the section 2, Figs. 3 and 4 show the variation of the eigenvalues and masses. The lumped mass and stiffness matrixes provide the same natural frequencies as the actual column.

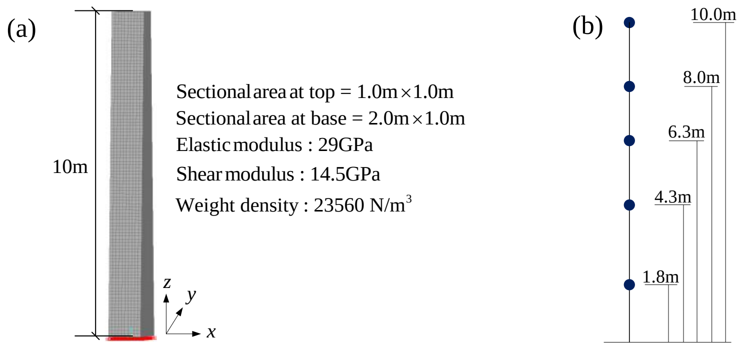

Figure 2. One-way symmetric non-prismatic column: (a) FE model and (b) lumped-mass stick model

Table 1. Eigenvalues of the non-prismatic column (y direction)

Mode	Eigenvalue (rad/sec)	Natural frequency (Hz)	Modal participating mass ratio (%)
1st Mode	75.31	11.99	52.50
2nd Mode	334.72	53.27	73.65
3rd Mode	781.77	124.42	82.92
4th Mode	1339.75	213.23	88.12
5th Mode	1966.55	312.99	91.33

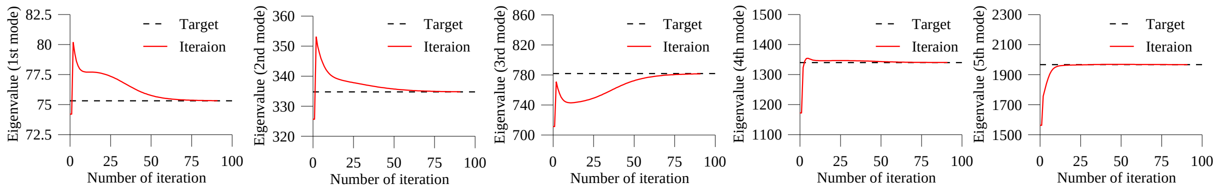

Figure 3. Variation of eigenvalues during iterations [unit of eigenvalues: rad/sec]

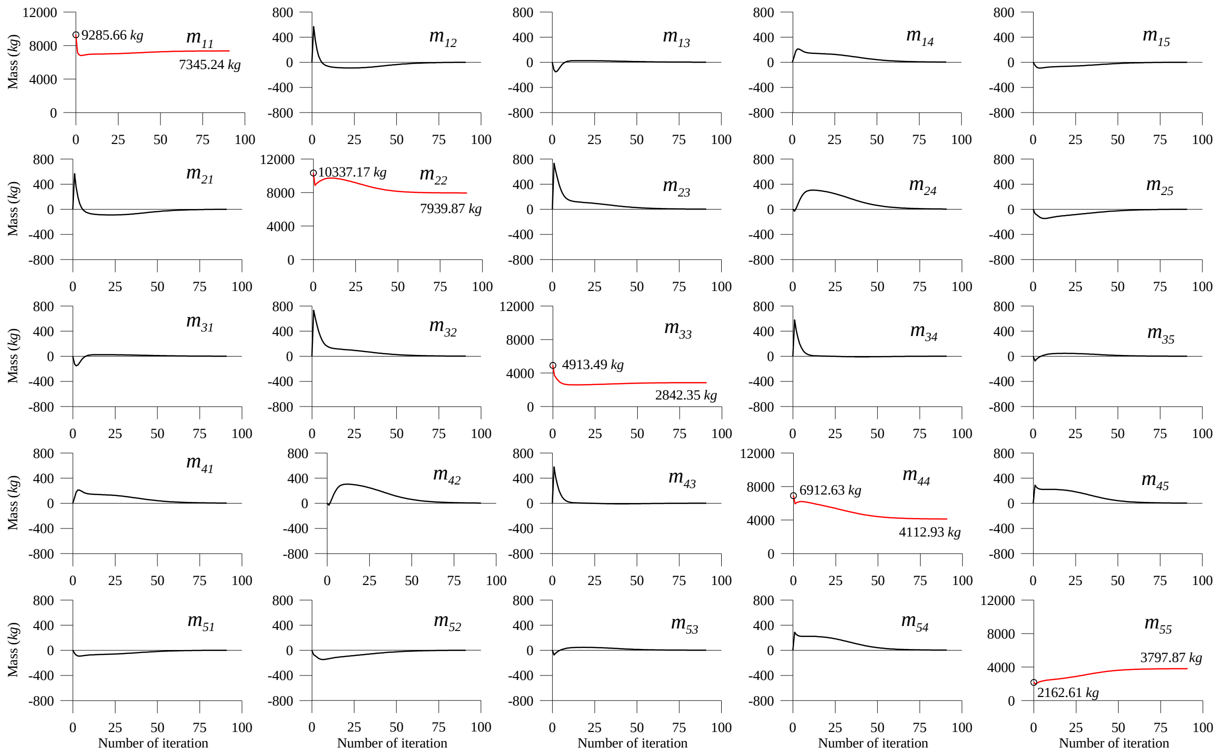

Figure 4. Variation of the mass matrix and frequency adaptive lumped-masses (diagonal terms)

The amount of the lumped masses obtained from the iterations is 26038.25kg which is 72.2% of the actual column. A time history analysis is conducted to investigate the dynamic performance of the new lumped-mass stick model. The horizontal ground motion is prepared on basis of the design spectrum presented in the ASCE 7-10 [ASCE 7-10, 2010]. Fig. 5 shows an artificial ground motion generated by the program SIMQKE [Gasparini and Vanmarcke, 1976]. The time history responses are shown in Figs. 6 and 7 obtained from both the FE model and the lumped-mass stick model, considering 5% modal constant damping ratio. As shown in Fig. 6, the phase of the displacement response is identical since the two models have same natural frequencies until the fifth mode. Similar result is found in the acceleration response, as shown in Fig. 7, which indicates that the LMS model provides an exact same phase as the actual column (FE model). Fig. 8 compares the peak responses of the LMS model to those of the FE model. As shown in Fig. 8 (a), the peak displacements are close to the results of the FE model. Such difference is caused by the consideration of limited target modes or natural frequencies. Similar pattern is found in the peak acceleration response, as shown in Fig. 8 (b).

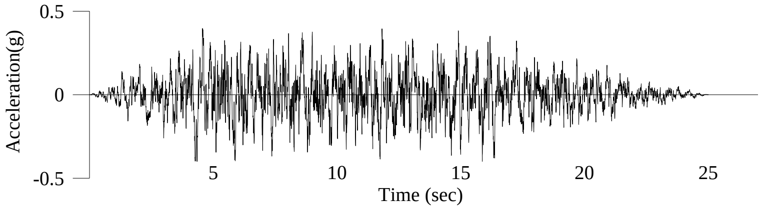

Figure 5. Artificial ground motion (5% damping)

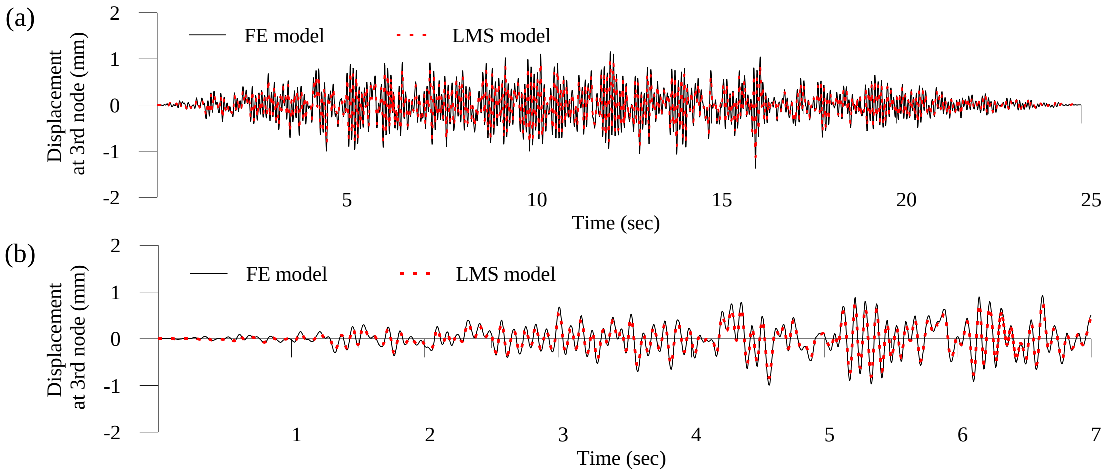

Figure 6. Displacement responses: (a) entire duration and (b) up to 7 second

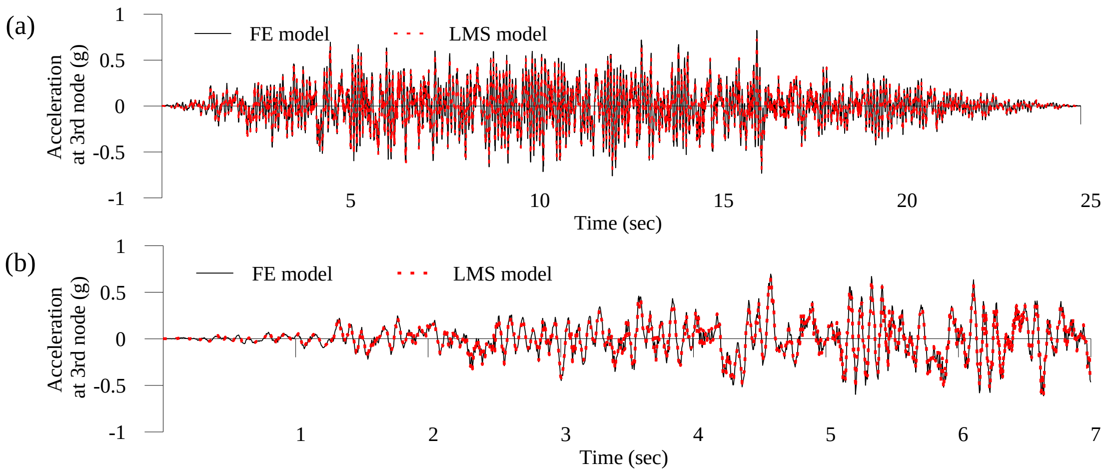

Figure 7. Acceleration responses: (a) entire duration and (b) up to 7 second

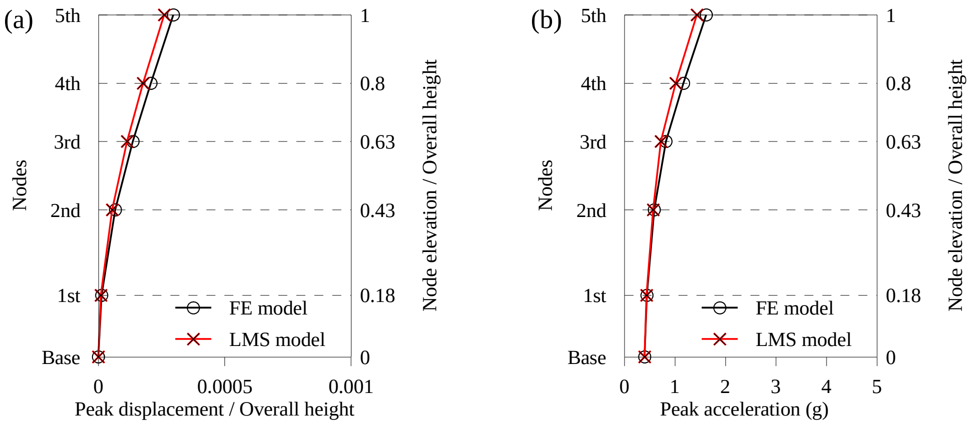

Figure 8. Comparison of peak responses: (a) displacement and (b) acceleration

4. APPLICATION TO NUCLEAR CONTAINMENT BUILDING

For an application of the lumped-mass stick (LMS) model to a practical civil structure, a nuclear containment (NC) building is considered. The geometry of the NC building considered in the study is shown in Fig. 9. The height of the NC building is 77.27m and the inner diameter is 45.72m with 1.22m of the wall thickness.

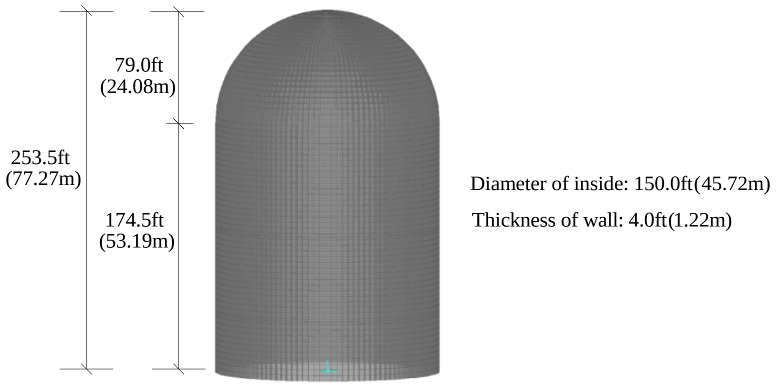

Figure 9. 3D-Finite element modeling of the nuclear containment building

Solid elements are used for three dimensional finite element modeling of the structure. The Young's and shear modulus applied are 29.16GPa and 14.58GPa, respectively, assuming to be elastic. The total mass of the NC building is 33,089,588kg. From the eigenvalue analysis of the FE model, the first-mode natural frequency is found as 3.90 Hz, as shown in Table 2. The modal participating mass ratio is over 90% from the third mode. In the present LMS model, the first-fourth modes are considered since the design spectrum for the NC building requires up to 33Hz for the seismic design [Varma at al., 2002; Regulatory Guide, 1973]. Fig. 10 shows the LMS model of the NC building. The node number 3 is the same location where the dome part starts.

Table 2. Eigenvalues and modal participating mass ratio

Mode	Eigenvalue (rad/sec)	Natural frequency (Hz)	Modal participating mass ratio (%)
1st	24.48	3.90	67.71
2nd	75.56	12.03	88.30
3rd	152.45	24.26	91.20
4th	190.75	30.36	92.20

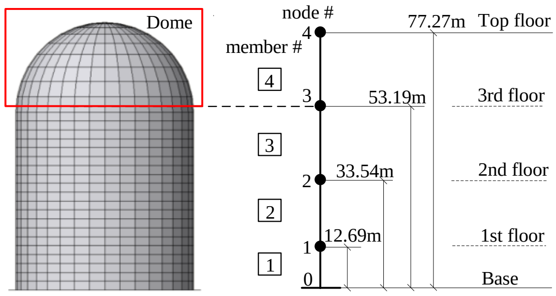

Figure 10. Lumped-mass stick model of nuclear containment structure

The variation of the eigenvalues during the iterations is shown in Fig. 11. Table 3 compares the converged eigenvalues and natural frequencies obtained from the iterations. The result of the eigenvalues is the same as the actual building (FE model). Fig. 12 shows the variation of the mass matrix during the iterations. All off-diagonal terms of the matrix become zero and the total amount of the diagonal terms is 27776894kg which is about 84% of the actual building.

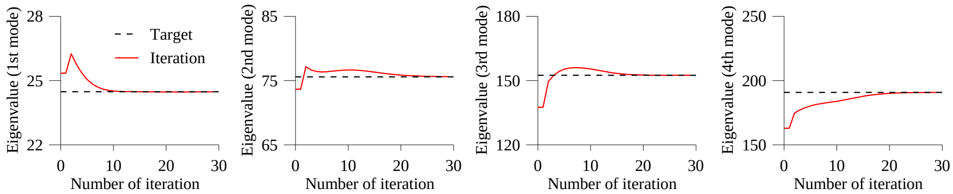

Figure 11. Variation of eigenvalues during iterations [unit of eigenvalues: rad/sec]

Table 3. Comparison of natural frequencies

Mode	Eigenvalue (rad/sec) / Natural frequency (Hz)	
	FE model (Actual structure)	LMS model
1st	24.48 / 3.864	24.47 / 3.895
2nd	75.56 / 12.031	75.59 / 12.026
3rd	152.45 / 24.263	152.44 / 24.262
4th	190.75 / 30.359	190.70 / 30.351

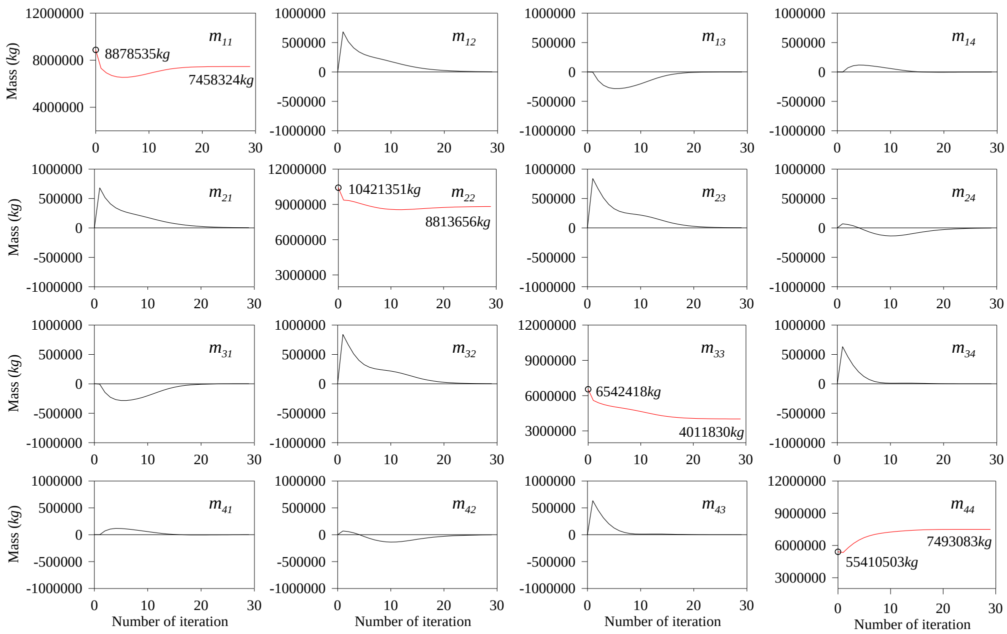

Figure 12. Variation of mass matrix during iterations

The ground motion shown in Fig. 13 is generated based on the US NRC RG 1.60 design spectrum which is normally applied for the seismic design of the nuclear containment (NC) building [Regulatory Guide, 1973]. The peak ground acceleration (PGA) of the horizontal excitation is 0.3g for a Safe Shutdown Earthquake.

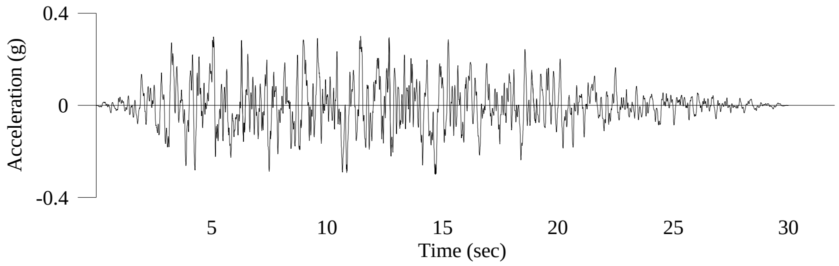

Figure 13. Artificial ground motion (0.3g PGA)

The time history analysis is performed with 0.005 second of the time increment, considering 5% of the damping ratio. Figs. 14 and 15 compare the displacement and acceleration responses. The both responses of the LMS model are almost identical to the responses obtained from the FE model. Figs. 14 (b) and 15 (b) capture the responses up to 10 seconds in order to show the similarity of the response phase. Fig. 16 compares the peak responses of each floor. The maximum difference of the peak displacement between the two models (FE model and LMS model) is found at the third floor while the responses of the other floors are very close. Fig. 16 (b) shows the peak floor acceleration response. Except third and top floors, the peak responses are almost identical. The differences of the peak acceleration at the third and top floors are only 0.08g and 0.04g, respectively.

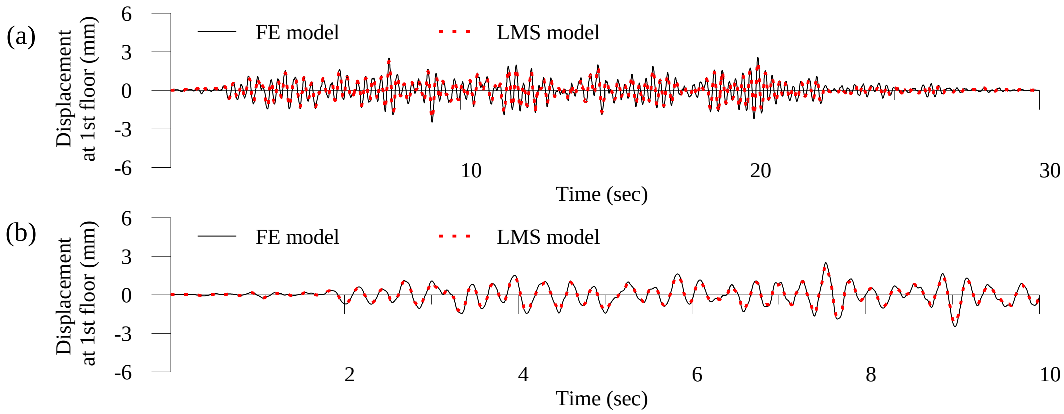

Figure 14. Displacement time history response of NC building: (a) entire duration and (b) up to 10 second

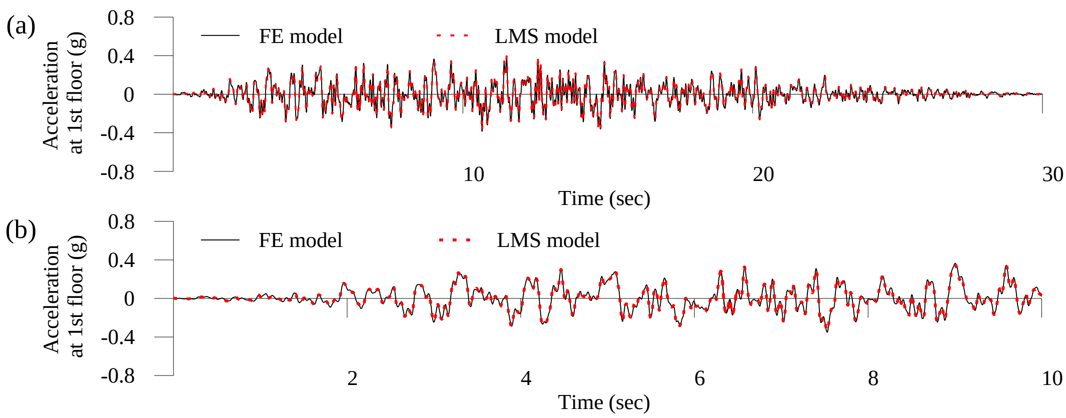

Figure 15. Acceleration time history response of NC building: (a) entire duration and (b) up to 10 second

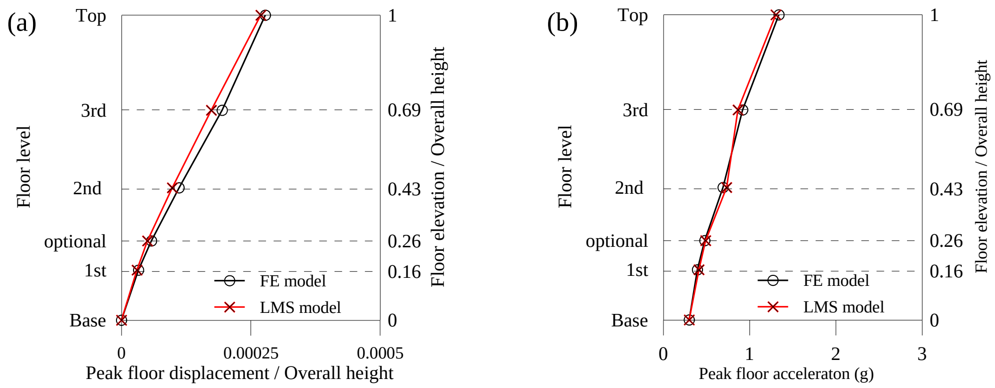

Figure 16. Peak floor responses: (a) displacement and (b) acceleration.

5. REMARKS AND CONCLUSIONS

Using the non-prismatic column and the nuclear containment building, detailed developing procedures of the new lumped-mass stick model are presented. A linear time history analysis is performed for the new stick model of the both structures in order to investigate its numerical performance. Artificial ground motions incorporating the seismic design spectrum are used as an input base excitation. The seismic responses resulted from the new stick model, such as floor displacement and acceleration including its peak values and phases, are almost identical to those of the finite element model representing the actual structures. Based on the overall results and comparisons, the new lumped-mass stick model can be suitable for analyzing the seismic responses of the structures. Adding more modes or natural frequencies improves the numerical performance of the new lumped-mass stick model.

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