# **Response of Base Isolated Structures Considering Inelastic Behavior of Superstructure**

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## SUMMARY:

This study investigates on the response of base isolated structures considering yielding of superstructure. The objective is to clearly show the influence of yielding on the response of the structures, and to develop theoretical rules to predict ductility demand. A 2DOF model has been developed which idealizes a typical 5-story base isolated building. The superstructure is characterized as elastoplastic while the isolation system is assumed as linear elastic with various isolation system natural periods and damping ratios. Results indicate that a decrease in superstructural strength significantly increases ductility demand, but leads to a reduction in the maximum displacement at the isolation level. For the estimation of ductility demand of base isolated structures considering nonlinear behavior of superstructure, theoretical considerations are described based on the steady-state vibration of 2DOF systems subjected to harmonic excitations. This theoretical framework shows good agreement with results from response history analysis.

Keywords: Base isolation, inelastic behavior, steady state response

## **1. INTRODUCTION**

In recent years base isolation has become an increasingly applied structural design technique. Many types of structures have been built using this approach. The ideas behind the concept of base isolation are quite simple, by adding horizontally flexible elements called isolators between the structure above and its base, thereby isolating the building from the horizontal components of ground motion. As a result, earthquake motions are not transmitted up through the building, or at least greatly reduced (Naeim and Kelly 1999).

Response of base isolated structure is significantly different from that of the fixed base structure. Fig. 1.1(a) schematically shows the expected behavior of the isolated structure. Large relative displacement is concentrated at the isolation level and the structure above is considerably rigid. Therefore, a sufficient separation distance between the structure and surrounding moat walls must be provided to prevent pounding. Previous studies on base isolated structures have primarily focused on this case. For an earthquake greater than the design basis, insufficient seismic gap may cause pounding, as shown in Fig. 1.1(b), which can causes high impact forces and significant damage especially for the acceleration-sensitive components (Kasai et al. 1990, Tsai 1997). On the other hand, yielding of superstructure may occur as in Fig. 1.1(c) and affect the response behavior.

Designed according to modern seismic provisions, base isolated structures are required to remain essentially elastic in the expected Design Basis Earthquake (DBE). Additionally, a separation distance must be provided to accommodate expected displacement in a Maximum Considered Earthquake (MCE). Therefore, yielding of superstructure is supposed to occur before pounding with surrounding walls. As yielding happens, reduction of stiffness allows superstructure to take part of the energy. As a result, the base ceases to move further and pounding is unexpected. However, little consideration is given to this situation. Therefore, objective of this study is to investigate the response of base isolated



Figure 1.1. Possible modes of behavior of base isolated structures

structure having nonlinear behavior of superstructure. The effects of yielding of superstructure on the displacement of the isolation level will be described. Moreover, some theoretical considerations to the estimation of ductility demand of the structures will be explained. This is to provide an effective tool for practice engineers in the design of base-isolated structures.

#### 2. ANALYSIS MODEL

In this study, a typical base isolated structure is idealized as a 2DOF system. The analysis model is depicted in Fig. 2.1. Assuming equal floor masses with one additional isolation level, isolation system is idealized as a viscoelastic system with stiffness  $k_b$  and damping coefficient  $c_b$ . Superstructure is assumed elastoplastic with superstructural stiffness  $k_s$ , damping coefficient  $c_s$ , superstructural yield strength  $f_{sy}$ , and ratio of post-yield stiffness to the elastic stiffness of superstructure p. Masses and stiffnesses are selected to obtain the desired periods and damping ratios from the following equations:

$$T_s = 2\pi \sqrt{\frac{m_s}{k_s}} \qquad T_b = 2\pi \sqrt{\frac{m_s + m_b}{k_b}}$$
(2.1.a,b)

$$\zeta_s = \frac{c_s T_s}{4\pi m_s} \qquad \qquad \zeta_b = \frac{c_b T_b}{4\pi (m_s + m_b)} \qquad (2.2.a,b)$$

The design strength of base isolated structure is significantly reduced from that of the fixed base structure. Fig. 2.2 depicts a design spectrum which clearly explains the difference; superstructural yield strength  $f_{sy}$  of a base isolated structure is designed based on the structure period which is considered close to the isolation system period  $T_b$  and damping ratio  $\zeta_b$ .

To avoid scatterness of the response data, superstructural yield strength is varied based on the maximum superstructural elastic force recorded from time history result for each particular ground motion. Then, for the selected strength reduction factor R, superstructural yield strength  $f_{sy}$  is computed based on the definitions as follows:



Figure 2.1. Analysis model

Figure 2.2. Example of (R=2) Designed yield strength of superstructure

where  $f_{s0,el}$  is the maximum superstructural force subjected to each particular earthquake.

Note that the same R gives different values of superstructure yield strength  $f_{sy}$  for different combinations of  $T_s$ ,  $\zeta_s$ ,  $T_b$ ,  $\zeta_b$  and ground motion depending on the elastic responses.

Responses considered in this study are deformations of isolation system and superstructure  $u_b$  and  $u_s$ , respectively. The latter is discussed in form of ductility demand of superstructure  $\mu$ , which is considerably an effective indicator of damage of the structure. Ductility demand of superstructure can be defined as:

$$\mu = u_{s0}/u_{sy} \tag{2.4}$$

In this study, superstructural period  $T_s$  is assumed as 0.5 second and superstructure damping ratio  $\zeta_s$  is 0.02. Six isolation systems are considered, with period  $T_b$  equals to 2, 3, and 4 sec., and damping ratio  $\zeta_b$  of 0.10 and 0.30. The SAC suites of ground motions for the Maximum Considered Earthquake (MCE) level for the Los Angeles area, LA21-LA40, are used. Following modern US building code provisions, the superstructure is expected to remain elastic for the expected Design Basis Earthquake (DBE), an event whose response spectrum is 2/3 of the MCE (McGuire 2004, Somerville et al. 1998). Thus, the structure designed according to the code is represented by the model having R = 1.5.

## **3. YIELDING OF SUPERSTRUCTURE**

#### 3.1. Time history results

Fig. 3.1 shows two example cases of dynamic response of a base isolated structure having  $T_b = 2$  sec.,  $\zeta_b = 0.10$ , and p = 0 is subjected to the LA29 excitation. Peak ground acceleration of this ground motion is 0.81g. Fig. 3.1(a) and (b) show respectively superstructure displacement  $u_s$  and base displacement  $u_b$ . Solid lines represents the behavior when R = 1.0 in which no yielding occurs. The superstructure performs an elastic behavior and superstructure displacement is relatively small compare to the base displacement, which is the expected behavior of base isolated structures. For this particular cases (LA29), maximum elastic superstructure displacement  $u_{s0,el}$  is 2 cm and maximum isolation displacement  $u_{b0,el}$  is 29 cm.

Dotted lines represent the case when R is increased to 1.5 in which superstructural yield strength is 2/3 of the maximum superstructural force observed in the elastic case. Superstructural yield displacement



Figure 3.1. Response history results of a  $T_b = 2$  sec.,  $\zeta_b = 0.1$  structure subjected to LA29 excitation

 $u_{sy}$  can be computed by dividing the maximum elastic superstructural displacement  $u_{s0,el}$  by strength reduction factor *R*, gives  $u_{sy}$  for this case 1.33 cm. As superstructural strength reduces, a single yield excursion occurs at 12.3 sec, as observed in Fig. 3.1(a), resulting in the maximum superstructure displacement  $u_{s0}$  at 11.3 cm and a large residual structural displacement. However, Fig. 3.1(b) shows that yielding of the superstructure ceases the base to move further so the maximum base displacement  $u_{b0}$  is reduced from 29 cm in the elastic case to 25.3 cm. This indicates that weaker structure decreases the possibility of pounding.

#### 3.2. Trends of superstructure ductility demand

Strength reduction factor *R* is varied from 0.1 to 4.0 to study the ductility demand. Results indicate that yield strength has a significant effect on resulting ductility demand in the superstructure. Fig. 3.2 presents the ductility demand observed from the model for isolation system period  $T_b = 2$  sec. Each graph corresponds to a distinct isolation system damping ratio  $\zeta_b$ . Responses from 20 earthquakes are plotted along with the median shown as red lines. The trends are clearer explained in Fig. 3.3. As superstructural strength decreases, ductility demand of the superstructure increases dramatically. When  $R \leq 1.0$  in which the superstructure remains in the elastic range, ductility demand is equal to strength reduction factor *R* for every isolation period and damping ratio. When R > 1.0, lateral force acting to the superstructure exceeds its elastic capacity, therefore, yielding occurs and isolation period  $T_b$  and damping ratio  $\zeta_b$  begin to take effects. An increase in isolation period (softer isolator) resulted in an increase in ductility demand. Moreover, an increase in isolation damping results in minor reductions in ductility demand, particularly long period cases.

Fig. 3.4 presents the reduction of the base displacement for different isolation systems. Each data point represents median of the ratio of the maximum base displacement to the maximum base displacement for elastic superstructure response,  $u_{b0}/u_{b0,el}$ , observed from each earthquake. The maximum base



Figure 3.2. Ductility demand and median response from 20 ground motions

Figure 3.3. Effect of superstructural strength on ductility demand without considering pounding



**Figure 3.4.** Reduction of  $u_{b0}/u_{b0,el}$  ratio observed from the median responses

displacement  $u_{b0}$  is apparently reduced from the elastic response when yielding of superstructure occurs.

## 4. ESTIMATION OF DUCTILITY DEMAND

For better understanding, steady state response of base isolated structure due to harmonic excitations with various amplitudes and excitation frequencies is investigated. In this chapter, theoretical considerations of the system will be developed and then compared with time history response observed from the model.

#### 4.1. Equivalent viscoelastic system

For an elastoplastic system, the dynamic properties of this structure vary according to its velocity and displacement. Therefore, the equivalent viscoelastic system is developed to have the same maximum superstructure deformation  $u_{s0}$ . The area with in the loop is also identical, to maintain the same amount of energy dissipation. Consider a structure with initial stiffness  $k_s$ , damping coefficient  $c_s$ , post-yield stiffness ratio p, and yield strength  $f_{sy}$ , having maximum ductility demand  $\mu$ . Fig. 4.1 presents hysteresis curve of the elastoplastic system and the equivalent viscoelastic system. Equivalent superstructural stiffness and damping coefficient,  $k_{s,eq}(\mu)$  and  $c_{s,eq}(\omega,\mu)$  can be defined as a function of excitation frequency  $\omega$  and ductility demand  $\mu$  (Kasai and Kawanabe 2005).

The equivalent stiffness  $k_{s,eq}(\mu)$  is reduced from the initial stiffness  $k_s$ , and is geometrically obtained from the hysteresis loop (i.e. secant stiffness at  $u_{s0}$ ). Then, equivalent circular frequency  $\omega_{s,eq}(\mu)$  is obtained as

$$\omega_{s,eq}(\mu) = \omega_s \sqrt{\frac{1+p\mu-p}{\mu}}$$
(4.1)

Or written in terms of equivalent vibration period,

$$T_{s,eq}(\mu) = T_s \sqrt{\frac{\mu}{1 + p\mu - p}}$$
(4.2)

While, strain energy stored at the maximum deformation is

$$E_{s}(\mu) = \frac{k_{s} u_{sy}^{2}}{2} \mu (1 + p\mu - p)$$
(4.3)

The energy input to superstructure is dissipated by two sources, viscous damping of the system  $E_{\nu_2}$  and



(a) Elastoplastic system (b) Equivalent viscoelastic system **Figure 4.1.** Hystereses of the elastoplastic system and the equivalent viscoelastic system

hysteretic energy absorbed by plastic deformation  $E_p$ , which causes permanent deformation and damage to the structure. Both energy dissipations can be approximated as

$$E_{v}(\omega,\mu) = \pi c_{s} \omega \mu^{2} u_{sy}^{2}$$

$$\tag{4.4}$$

$$E_{p}(\mu) = 4k_{s}u_{sv}^{2}(\mu-1)(1-p)$$
(4.5)

And the equivalent damping ratio  $\zeta_{s,eq}(\omega,\mu)$  is estimated from the following equation:

$$\zeta_{s,eq}(\omega,\mu) = \frac{E_{\nu}(\omega,\mu) + E_{p}(\omega,\mu)}{4\pi E_{s}(\mu)} \cdot \frac{\omega_{s,eq}(\mu)}{\omega} = \zeta_{\nu}(\mu) + \zeta_{p}(\omega,\mu)$$
(4.6)

Therefore, equivalent damping ratio and equivalent damping ratio of plastic deformation is

$$\zeta_{\nu}(\mu) = \zeta_{s} \sqrt{\frac{\mu}{1 + p\mu - p}} \tag{4.7}$$

$$\zeta_{p}(\omega,\mu) = \frac{2(\mu-1)(1-p)}{\pi\left(\mu^{1.5}\sqrt{1+p\mu-p}\right)}\frac{\omega_{s}}{\omega}$$
(4.8)

It should be note that  $\zeta_{\nu}(\mu)$  is a function of ductility demand  $\mu$  only, it is independent to excitation frequency  $\omega$ . However,  $\zeta_{\nu}(\omega, \mu)$  is a function of both ductility demand  $\mu$  and excitation frequency  $\omega$ .

## 4.2. Steady state response of base isolated structures

Response of base isolated structure is significantly different from that of the fixed base structure. When yielding occurs, stiffness of superstructure decreases. For fixed base structure, stiffness degradation results in significant energy dissipation and its effective frequency will move away from the excitation frequency that is causing the damage. Unlike the fixed base structure, superstructural yielding has only a minor effect on the system frequency and the overall isolated structure frequency is dominated by the elastic frequency of the isolation level. This agrees with previous study by Kikuchi et al (2008).

Fig. 4.2 presents time history result of the steady state response of the 2DOF model having  $T_b = 2$  sec. and  $\zeta_b = 0.1$ , subjected to a harmonic motion  $a_g sin\omega t$  with ground acceleration ratio  $a_g/a_s = 0.5$ , where  $a_s$  is the yield acceleration of superstructure, and frequency ratio  $\omega/\omega_b = 1$ . The natural frequency that governs the response of superstructure is the elastic isolation circular frequency  $\omega_b$ . Hystereses of the superstructure and isolation system for this example case are shown in Fig. 4.3. If the superstructure remains in elastic range, the isolation system will performs a perfectly ellipse hysteresis. However, yielding of superstructure results in a slight distortion of the isolation system hysteresis, but the ellipse still remains. Therefore, the superstructure is considered as a lumped mass system subjected to a sinusoidal excitation  $\ddot{u}_b = \ddot{u}_{b0} sin\omega t$  with the circular frequency  $\omega$  close to the natural frequency of the isolation level  $\omega_b$ .

Fig. 4.4(a) presents the equivalent period as a function of ductility demand  $\mu$ . Each line in the graph represents different value of post-yield stiffness p that is 0, 0.1, and 0.2, from top. As the structure yields, equivalent period increases. Also, equivalent viscous damping ratio is shown in Fig. 4.4(b) for the initial damping ratio  $\zeta_s = 0.02$ . Fig. 4.4(c) and 4.4(d) present respectively the trends of the equivalent damping ratio of plastic deformation  $\zeta_p(\omega, \mu)$  and total equivalent damping ratio  $\zeta_{eq}(\omega, \mu)$ , of the base isolated structure assuming  $\omega = \omega_b$  at which  $T_b = 2$  sec. Both  $\omega_s$  and  $\omega_b$  are constant and generally  $\omega_s$  is significantly larger than  $\omega_b$  for short building. Therefore, overdamped may happened because  $\zeta_p(\omega,\mu)$  is propagated, especially when the superstructure is just a little yielding (e.g.  $\mu = 3$ ).



Figure 4.2. Steady stated response of a 2DOF system subjected to a sinusoidal excitation



Figure 4.3. Hystereses of the steady state response of base isolated structure



Figure 4.4. Equivalent period and damping ratio as a function of ductility demand



Figure 4.5. Comparison of the estimated and median ductility demand observed from time history model

#### 4.3. Maximum displacement

Consider the superstructure behavior as a SDOF system subjected to a sinusoidal excitation  $\ddot{u}_g = \ddot{u}_{g0} sin\omega t$ , the maximum displacement can be estimated as the following equation (Kasai and Kawanabe 2005).

$$u_{0}(\omega,\mu) = \frac{u_{g0}}{\sqrt{\left(\omega_{eq}^{2}(\mu) - \omega^{2}\right) + \left(2\zeta_{eq}(\omega,\mu)\omega_{eq}(\mu)\omega\right)^{2}}}$$
(4.9)

Base on the findings that the superstructure response is significantly governed by the elastic frequency of the base, the superstructure is considered as a lumped mass system subjected to a sinusoidal excitation  $\ddot{u}_b = \ddot{u}_{b0} \sin \omega t$  with the circular frequency  $\omega$  close to the natural frequency of the isolation level  $\omega_b$ . As a result, we can estimate ductility demand of the superstructure as follows:

$$\mu = \left(\frac{\omega^2}{\sqrt{\left(\omega_{s,eq}^2(\omega,\mu) - \omega^2\right)^2 + \left(2\zeta_{s,eq}(\omega,\mu)\omega_{s,eq}(\omega,\mu)\omega\right)^2}}\right) \left(\frac{u_{b0}}{u_{sy}}\right)$$
(4.10)

However, the maximum base displacement  $u_{b0}$  is apparently reduced from the elastic response when yielding of superstructure occurs. Therefore, using  $u_{b0}$  observed from time history response, we can estimate ductility demand by iterations combining Eqn.(4.1), (4.5) and (4.7).

Each graph in Fig. 4.5 describes the ductility demand as a function of strength reduction factor R, for each isolation system damping ratio  $\zeta_b$  considered. Each line in the graph corresponds to a distinct isolation system period  $T_b$ . Solid lines represent the ductility demand observed from time history model with post-yield stiffness ratio p = 0, from the suite of SAC ground motions. Broken lines represent the ductility demand increases significantly. The estimation is underestimating, especially for high damping case, but still, showing the same trend. Note that the resulting ductility demand is found highly sensitive to the  $u_{b0}/u_{b0,el}$  ratio which causing errors in the estimation especially when R is small.

## **5. CONCLUSIONS**

Parametric studies of the response of base isolated structures considering yielding of the superstructure have been described. Result proves that this simple model could be used to study the responses of

earthquake-induced pounding of base-isolation structures.

A decrease in superstructural strength significantly increases ductility demand but the base ceases to move further so the maximum displacement at the isolation level reduces. As a result, to prevent pounding, the separation distance should be provided to accommodate the isolation level displacement which results in a shear causing yielding of the superstructure. The appropriate separation distance can be estimated depending on the dynamic properties of the structure. If the sufficient separation distance cannot be provided, reducing superstructural strength can reduce the possibility of pounding; however result in an increasing of ductility demand.

An increase in isolation period (softer isolator) resulted in an increase in ductility demand. An increase in isolation damping results in minor reductions in ductility demand, particularly for long period cases. These trends can be theoretically explained.

For the estimation of ductility demand of base isolated structures considering nonlinear behavior of superstructure, theoretical considerations have been described and showing the good trend along with the history results from the model. Steady state response shows that superstructural yielding has only a minor effect on the system frequency and the overall isolated structure frequency are dominated by the elastic frequency of the isolation level. Yielding of superstructure results in a slight distortion of the isolation system hysteresis but its ellipse hysteresis still remains. However, more improvements are needed, especially for high damping cases.

Future study will be focused on a complete method estimation of ductility demand from the elastic response spectra. In addition, a method of selecting a good combination of superstructural strength and separation distance to obtain the minimum damage as indicated by ductility demand of the base isolated structure will be considered.

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